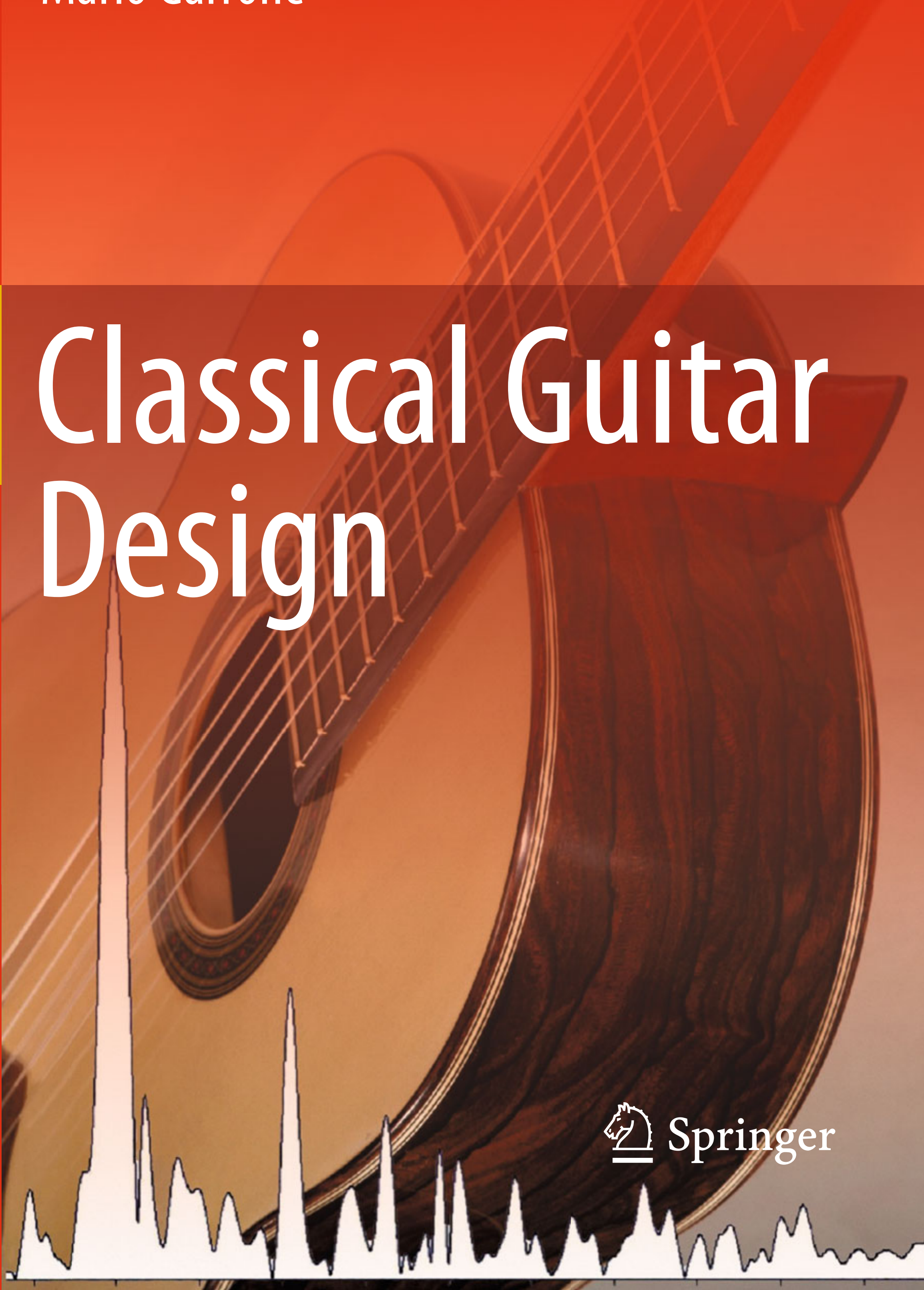
Giuseppe Cuzzucoli
Mario Garrone

Classical Guitar Design



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felix qui potuit rerum cognoscere causas
Virgil (70–19 BC) *II Georgic*

Preface

The miracle of creating an outstanding musical instrument cannot be explained by science. We kept that in mind while writing the first part of this book (Chaps. 1–6) where we illustrated the physical and acoustic workings of the classical guitar. The miracle of creating an outstanding musical instrument arises primarily from the experience, sensibility and creativity of the luthier.

However, we believe that all of us (guitar-makers included) can go beyond personal experience, still taking advantage of it. If we do not reach out of what we learned, and we always did well, experience—though endowed with creativity and sensibility—will turn into a cage. On the contrary, if we grab the chance to explore new paths, then knowledge (if not science) will be the guide beyond our limits. This is the purpose in the second part of the book: trying to connect knowledge with experience, never disregarding the value of personal creativity and sensibility.

The title may be surprising: *Classical Guitar Design*. We normally think of a bridge, or a car engine, being designed. Commonly, a musical instrument must be just built.

The starting point in designing is our idea of the features and performances we expect from the final object. Secondly, we manufacture it according both to experience and to processing of new information. In order to get a detailed knowledge of the object, we must use proper models and simulations. We also need to apply suitable technological tools to assess the potential performances in pre-construction laboratory tests. This will enable us to take the steps required to fulfil our expectations.

Of course, we need a good deal of sensibility and creativity. What we suggest here is in the first place an intellectual and cultural approach, which will guide us through the guitar manufacturing procedure.

All guitar-makers have an 'expected performance': the concept of the sound they demand from their instruments. How can we express this concept in measurable parameters? We firstly examined the guitar sound and developed quality evaluation criteria that can be applied to any instrument. Then we got a deeper knowledge

of the instrument to be created: the implementation of proper models and measurement methods (both hardware and software) allows in-progress evaluation of the instrument performance and reveals the necessary structural adjustments. We examined the resonator elements separately, and then we assembled them in order to estimate the guitar resonator as a global object. New tools, and how they assist traditional construction techniques are also illustrated here. For example, a particular shape for in-progress fine-tuning of the back and soundboard. Working on the soundboard when mounted on the frame (without the back) is a very important and useful procedure because, this way, all internal elements can be easily reached and modified. An in-depth study of the backboard response is also presented in the text, as when properly sized, this component can favourably affect the quality of sound.

Furthermore, quantitative parameters are provided for the evaluation of timbers employed in lutherie. Finally, the same method is applied to the analysis of finished guitars, some of which being very important in the guitar-making history. This *reverse engineering* process provides much noteworthy information about engineering guidelines that inspired the construction of these guitars.

This book is mainly addressed to guitar-makers. Many among them feel the need to go beyond the limits of their experience and gain a better understanding of the workings in the instrument they are manufacturing. This turns into a more conscious and rational experimentation. If this book contributes to enlarge this category of luthiers, a first and most important goal will be achieved.

Secondly, this book is addressed to guitar-players—whether professionals, amateurs or students who will maybe find an answer to some questions they have about the guitar sound. Moreover, they will cast a look upon the 'magic' realm of guitar-making (which is not so 'magic', actually). Maybe, following the indications exposed here, some of them will try to build up a guitar of their own, even if their aim is not to become professional luthiers. Step by step, they will be conquered by how some wooden parts change into a guitar. They will learn how to design, experiment, act on this object under construction right to the finished, playable instrument! If the book answers questions and encourages initiative, our second purpose will be fulfilled.

Last but not least, the book is addressed to readers having a technical education in Physics or Engineering, who are curious about how some basic notions can explain the acoustic workings in a musical instrument. We apologize for inaccuracies or omissions they may find in the text, hoping they will report them to us. Mathematical formulae were reduced to a minimum and demonstrations omitted, being the whole subject available in specific textbooks and essays.

After all, this book is not 'just' concerned with classical guitar building, nor is the mere description of a physical model, disconnected from the manufacturing practice. In both cases, plenty of publications are available. We tried to develop theoretical analysis and manufacturing practice as two parallel melodic lines that intertwine, interact and sometimes superpose to build the harmony of a piece of music. If the reader shares this feeling—to some extent, at least—our third goal will be reached.

We tried to offer the most detailed description of the hardware and software tools employed in the examination and fine-tuning of instruments. Should our explanations appear incomplete or, hopefully, arouse new questions, we are available for constructive discussion.

The following list of sources is just a selection of the most important publications that support our investigation. Readers willing to expand their knowledge of the subjects discussed in the book can refer to these authors. On our part, we wish for research aiming to forward the knowledge and engineering of the classical guitar.

Pianezza, Italy Giuseppe Cuzzucoli
Acqui Terme, Italy Mario Garrone

Suggested Readings

1. Caldersmith, G. (1978). Guitar as a reflex enclosure. *Journal of the Acoustical Society of America*, *63*(50).
2. Caldersmith, G., Freeman, E. (1990). Wood properties from sample plate measurements. *Catgut Acoustic Society Journal*, *1*(5).
3. Christensen, O. & Vistisen, B. B. (1980). Simple model for low frequency guitar function. *Journal of the Acoustical Society of America*, *68*(30).
4. Cremer L. (1984). *The physics of the violin*. Cambridge: The MIT Press.
5. Cuzzucoli, G., Lombardo, V. (1999). A physical model of the classical guitar, including the player's touch. *Computer Music Journal*, *23*(2).
6. Fletcher, N. H. & Rossing, T. D. (1991). *The Physics of acoustical instruments*. New York: Springer-Verlag.
7. Garrone, M. (1994). *La costruzione della chitarra classica*. Ancona: Edizioni Bèrben.
8. Jansson, E. V. (1971). A study of acoustical and hologram interferometric measurements on the top plate vibrations of a guitar. *Acustica*, *25*.
9. Jansson, E. V. (2002). Acoustics for violin and guitar makers. www.speech.kth.se/music.
10. Meyer, J. (1985). *Akustik der gitarre in einzeldarstellungen*. Frankfurt: Verlag E. Bochinsky.
11. Morse, P. M. & Ingard, K. U. (1968). *Theoretical acoustic*. New York: McGraw-Hill.
12. Righini, P. & Righini, G. U. (1974). *Il suono*. Milano: Tamburini Editore.
13. Rossing, T. D. (1983) An introduction to guitar acoustic. *Guild of American Luthiers Quarterly*, *11*(4).
14. Rossing, T. D., Popp, J. & Poltein, D. (1985). Acoustical response of guitars. *Proceedings SMAC 83*. Stockholm: Royal Swedish Academy of Music.
15. Sali, S. & Kopac, J. (2002). Positioning of braces on a guitar soundboard. *Proceedings IMAC* XX 709–715.

Contents

Chapter 1
The Sound

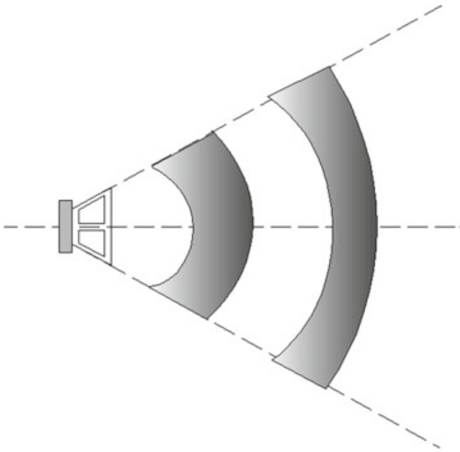
Abstract The first chapter introduces the reader to the sound production by the guitar. It establishes basic concepts of musical acoustics like pitch, intensity, transients. Important subjects like the relationship between frequency and time domains, 'attack' and 'sustain' are discussed. Various representations of the sound are introduced, like spectrum, time domain waveforms, partials, waterfall, thirds of octave graphs; finally the guitar timber is discussed, focusing on the quality parameters, as well as on the correlation between the qualitative perception of the timber and physical, measurable parameters. Values for a reference guitar are given as an example. In Appendix audiogram, SPL, exponential, Fourier series are introduced.

When we hear a bell tolling, a phone ringing, or a note played on guitar, what we perceive we call it 'sound'. This perception is due to a specific physical phenomenon. Even though we use the same word *sound* to refer to both the physical phenomenon and the effect it produces on our senses, they are two different issues and must be considered separately. Based on experience, we can identify the source of the sound we hear ('the telephone rings', 'I listen to the guitar', 'I hear a bell tolling') but we normally are not able to describe the underlying physical phenomenon.

Except for some hints to physiological aspects, this chapter deals with the physical phenomenon.

If we gently lay a finger on a loudspeaker cone, we feel its vibration and, if the emitted tone is very low, we can even see a periodical back-and-forth movement of the cone. This movement is the origin of the sound we hear: when the cone moves towards the observer, the surrounding air is compressed; vice versa, when the cone moves away from the observer, the air is sucked in by the source, and pressure diminishes in the surrounding air. This phenomenon is depicted in the following image.

G. Cuzzucoli and M. Garrone, *Classical Guitar Design*,
https://doi.org/10.1007/978-3-030-32992-1_1



So, from the viewpoint of physics, sound is generated by a small variation in barometric pressure, which in turn is brought about by the vibration of a surface that oscillates around its equilibrium position. The unit of pressure is *Pascal* (1 Pa $= 1\ \mathrm{N/m^2}$). The alternation of compressions and depressions in the air surrounding the source (here the loudspeaker cone) generates a pressure wave: this wave travels through the air at sound velocity, which at 20 °C (68 °F) is about 343 m/s (1129 ft/s).

When the sound pressure wave reaches a person's ear, it sets the eardrum in vibration and, through other mechanisms we are not going to deal with, sound perception takes place.

In much the same way, when a sound pressure wave hits a microphone membrane, it turns into an electric signal output that represents sound *as a physical phenomenon.* Today even low-performance computers are able to record and process the signal coming from a microphone, allowing objective analysis of sound phenomena.

1.1 Fundamental Sound Parameters

This book only deals with *musical sounds*. An object falling on the ground produces a *non*-musical sound. Likewise, the blowing of a fan or the uproar of a jet taking off are not musical sounds: we rather call them *noises*. On the contrary, a tolling bell, a ringing telephone, or a note played on guitar are musical sounds, as they bring a fundamental tone which is well distinguished by the senses and corresponds to a musical note. Not necessarily, as these examples show, a musical sound has to be produced by a musical instrument.

First of all the sound (the adjective 'musical' will be hereafter implied) and so the fundamental tone have a distinctive *pitch*: a whistle generates a much higher tone than the open fourth string of a cello. As a physical phenomenon, the pitch in the fundamental tone depends on the number of oscillations carried out by the source in one second. This is called *oscillation frequency*: high tones correspond to high

oscillation frequency of the sound source and, conversely, low tones correspond to low frequency.

The oscillation rate of the source is measured in Hertz (Hz): 1 Hz corresponds to 1 oscillation per second, and 1000 Hz correspond to 1000 oscillations per second, abbr. 1 kHz. For instance, the fundamental tone produced by the open fifth string of the guitar is an A at 110 Hz. Summarising,

$$\begin{aligned} Pitch\,of\,the\,Tone &\Rightarrow Number\;of\;Source\;Oscillations\;per\;Second \\ &\Rightarrow Frequency\;(Hz\,or\,KHz) \end{aligned}$$

From now on, tone pitches will be expressed in the book in terms of frequency (Hz or KHz).

Loudness, or intensity, is the second attribute of sound. From a physical point of view, the larger the vibrating surface the greater the *intensity* of the sound radiation. A large vibrating surface displaces a large volume of air, meaning that a great amount of energy is involved. In short,

$$\begin{aligned} Intensity\,of\;the\;Sound &\Rightarrow Extent\;of\;the\;Radiant\;Surface \\ &\Rightarrow Volume\,of\;Air\;Displaced \end{aligned}$$

The dynamic range of sound intensity is very wide, spanning from a hardly perceptible whisper up to the clamor of an airplane taking off and beyond (when it becomes damaging to the auditory system). On a linear scale, it would be impossible to distinguish the sound produced by a soloing instrument (no matter how loudly played) from the sound produced by a big orchestra. Physicists describe phenomena of such wide-ranged dynamics with a logarithmic scale, and define the intensity of sound as SPL (*Sound Pressure Level*), which is expressed in *decibels* (dB). See Appendix 1.2 for mathematical formulation.

The following table reports the SPL values of some common sounds.

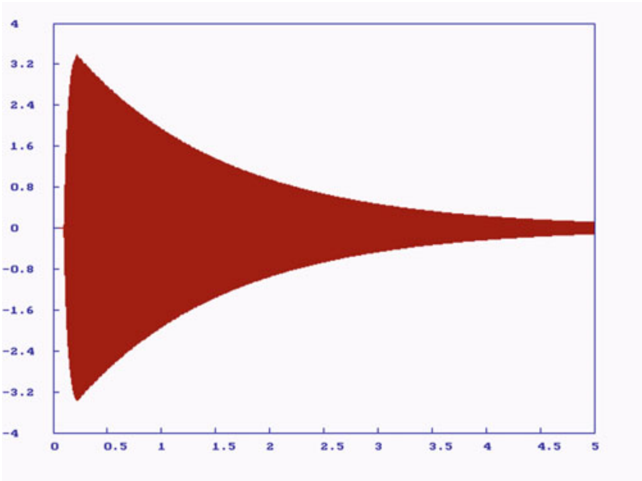
Sound	SPL (dB)
Threshold of hearing	0
Hum	20
Conversation	60
Heavy traffic	80
Big orchestra	98
Jet take-off	120
Threshold of pain	130
Permanent damages to the auditory system	150

Frequency and intensity define sound as a physical phenomenon. For a description of sound as a physiological phenomenon, please refer to specialized publications.

We only wish to point out that the human hearing is especially sensitive to sounds around 1000 Hz, and far less sensitive under 200 Hz or above 12 kHz. This means that we perceive the lowest and highest tones as weaker than they really are—see the *audiogram* shown in Appendix 1.1. What is more, high tone sensibility diminishes with age. As a term of comparison, a good audio amplifier faithfully replicates electric signals in the bandwidth between 20 Hz and 20 kHz.

1.2 Elementary Sounds

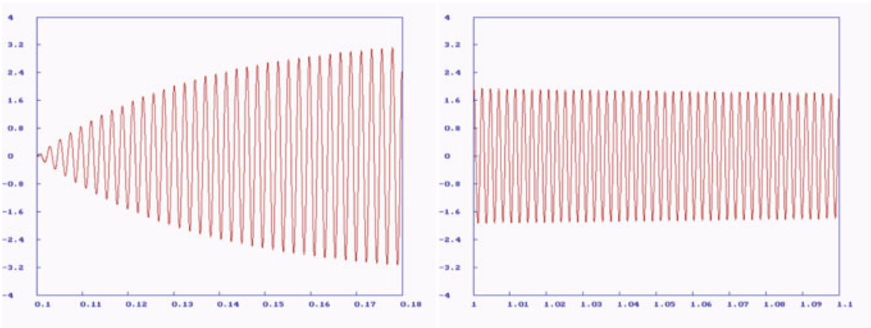
The following figure is the *waveform* of a sound like the one of a tuning fork tuned at 440 Hz. The waveform represents the *sound pressure variation over time* corresponding, for instance, to the electric signal provided by a microphone placed in front of a source excited by a hammer stroke.



In the figure, the intensity of sound is represented in a conventional scale, while the time scale covers the interval between 0 and 5 s (s). Notice that, after the opening interval of 0.1 s, the source begins to send out sound and reaches maximum emission after about 0.2 s.

Now the emission amplitude enters a slow decreasing stage, and is very low at the end of the observation window, 5 s after the beginning.

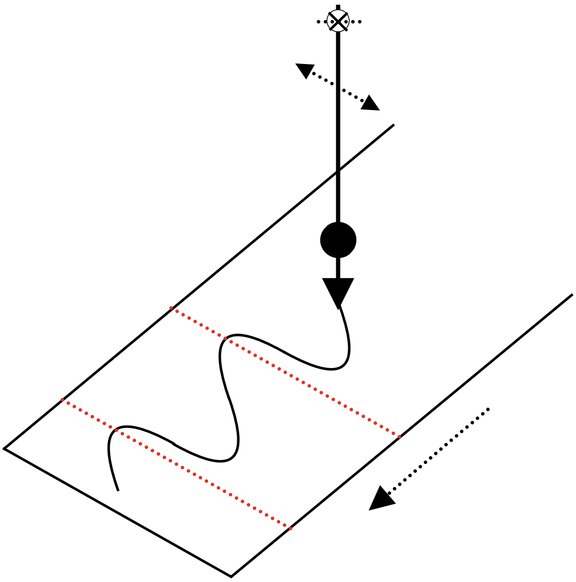
For a detailed observation of the development of sound intensity, the time scale was extended in the two following figures.



The left panel is a ‘snap-shot’ of the sound intensity development in the first 180 ms, while the right panel depicts the sound between 1 and 1.1 s.

Both in the increasing (left panel) and decreasing (right panel) stage, the sound pressure periodically oscillates between a positive maximum and a negative minimum, following a sinusoidal law.

The sinusoid (or sine wave) as mathematical function is very important in acoustic phenomena, but *instead of giving a mathematical representation*, we will rather try to understand its nature by observing the following figure.



The figure illustrates a pendulum (the simplest oscillating system) that oscillates with a certain period, marking a tape whose forward movement simulates time. The plot depicts a set of sinusoids that periodically alternate (in the figure a sinusoid is encompassed between the two red lines) with the same period as the pendulum oscillation. The values at the two ends of the sinusoids correspond to the maximum

pendulum displacement from its rest position. The number of cycles (or sinusoids) completed in one second is the frequency (in Hz) of the oscillation.

This illustration clearly displays the close relation between the sinusoid and the *harmonic motion* of a pendulum. The sinusoid is the development over time of the oscillation amplitude in a system that moves with harmonic motion. As we will see later, sinusoidal (or *harmonic*) oscillations can describe any complex periodic motion.

Now we have all the elements needed to understand the nature of our elementary sound.

- In the first part of the waveform, the source is not excited. The tuning fork is at rest and no sound is generated.
- After 0.1 s the source is excited by the hammer, and the sound starts rising in accordance with a regular, *exponential law*. It does not rise immediately because of some *inertia* of the source, which is 'slow' in reacting to the impulse given by the hammer. In radiating surfaces, inertia normally depends on mass. The period of time when the sound is rising is called *initial transient* or—in musical terminology—*attack time.*
- Once the excitation is extinguished, at the end of the *attack transient* the sound starts decreasing by an exponential law, but the descent is much slower than the rising. The reason why the intensity of sound decreases is that only a fraction of the energy accumulated by the source, as a consequence of the excitation, converts into sound; the remaining part is dispersed both because of friction between the source and the air, or due to losses occurring inside the source. The decreasing stage of the sound is called *decay transient.* See Appendix 1.3 for mathematical description of the laws governing *transient periods*.
- Both during the attack and the decay transient, the sound oscillates in accordance with a sinusoidal law. The oscillation frequency (440 Hz) is the frequency our tuning fork is tuned at, and determines the tone we perceive when the tuning fork is set in motion by the hammer.

The elementary sound generated by a tuning fork has been considered in detail because many of its features also occur in more 'complex' sounds, like those produced by musical instruments. These sounds too, generally show attack and decay transients but—as we will see—they result from a combination of sinusoids with different frequencies (not from a single sinusoid, as is the case with the tuning fork). Yet 'continuous' sounds exist, e.g. those produced by bowed or wind instruments. These sounds show transients too, although their particular kind of excitation mechanism periodically supplies new energy, which restores the lost energy: if the two are equal, the level of the continuous sound remains constant; if not, it rises or decreases depending on which of the two prevails. This way the intensity of the sound can be controlled.

Among continuous sound generators, electronic oscillators must be mentioned. They are used for particular laboratory measurements and, as we will see later on, they also serve our purposes.

1.3 Sound Representations

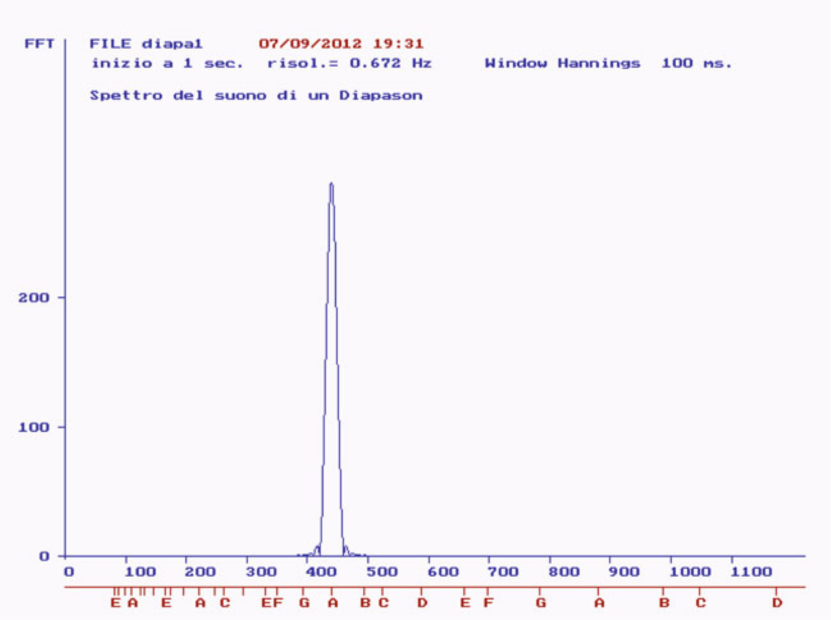
The waveform describes amplitude variations occurring in the sound picked up by a microphone at any instant or, in other words, amplitude variations in the *time domain.*

In 19th century, mathematician Jean Baptiste Fourier pointed out that *a periodic, continuous and limited signal* (like those we are dealing with) *can be described as the sum of sinusoids having suitable amplitude and multiple frequency of the fundamental* f_0.

This means that the signal produced by a complex sound can be broken up into a series of sinusoidal oscillations (or harmonics) which, taken together, compose the so called 'spectrum' of the signal. We give the mathematical expression of Fourier's Theorem in Appendix 1.4.

We can envision this phenomenon if we think of a white light beam which, similarly, splits into beams with different colours (and wavelengths) when passing through a prism. Special electronic analysers (or *spectrographs*) were once used for the decomposition of a sound, being today replaced by specific computer software for processing the *Fast Fourier Transform* (or FFT). This is an algorithm that will help us to analyze such complex sounds as those produced by the guitar.

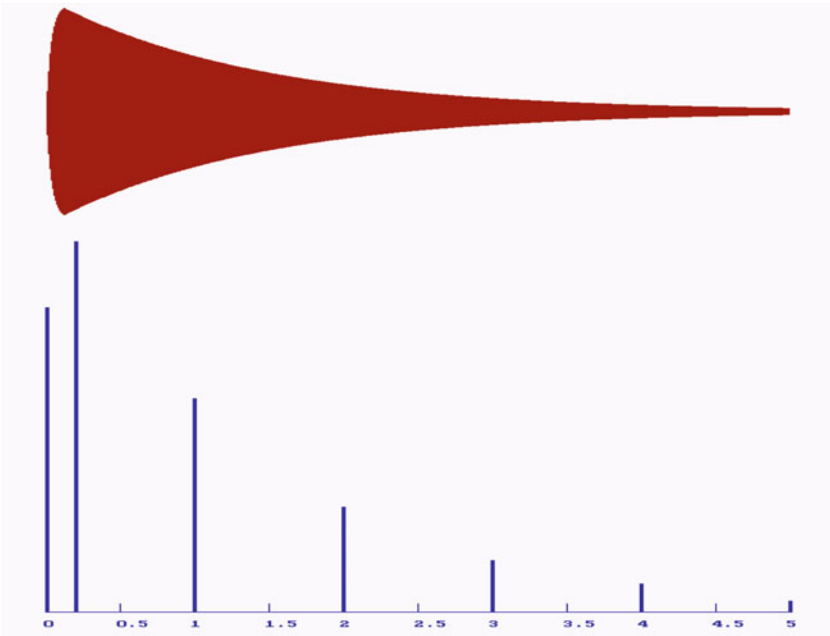
The next figure reports the spectrum of the tuning fork sound. We note that only one spectral line is present at 440 Hz (our tuning fork frequency).



So the spectrum is an alternative representation of the sound *in the frequency domain.* The time representation shows when specific events occur (when the sound begins, when it reaches maximum level, when it has decreased to 50%, etc.) but gives

no information about the tones that compose the sound. Conversely, the frequency representation shows what tones are included in *the sound* ‘*recipe*’, determining the timbre of a particular sound. Still the frequency representation is incomplete. It is clear that in very articulate sounds like those of the guitar the recipe varies over time: certain components die away faster than others do, because they faster lose energy. For that reason, we need to understand the timing and amplitude of the single components involved or, in other words, *how the sound recipe varies over time*.

The following figure illustrates the *spectrogram* of our sound, consisting of a single harmonic component at 440 Hz.



Each spectral line is comprised in the scale between 0 an 5 s, and the height of the lines indicates the sound level on the same scale. For instance, the line at 1 s represents the height of the spectral component at 440 Hz, calculated in a period of sound starting from 1 s. Notice how, during the attack transient, the height of the lines rises as the system gains speed, while during the decay transient it drops owing to losses in oscillatory energy.

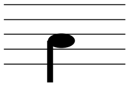
Therefore, the spectrogram is a representation of spectral components on a time scale, that is to say a time/frequency representation. Combined with the waveform, it provides full representation of a sound.

We will see how to apply these notions to much more articulate sounds, like those produced by the guitar, and what kind of information can be obtained.

1.4 The Sound of the Guitar

1.4.1 The Waveform

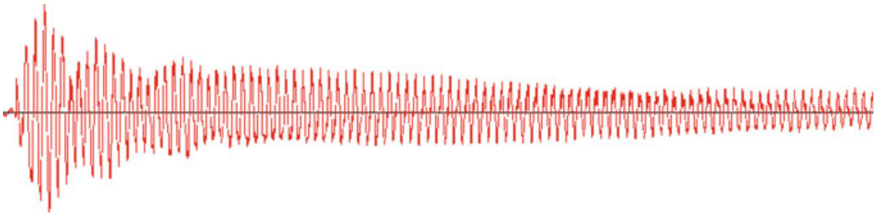
The figure below is the waveform of the sound produced by a guitar on the second fret of the third string. This note is marked on the staff as



The waveform illustrates the sound in the interval between 0 and 2 s.

The figure emphasizes a very rapid growth opposed to a very slow decrease in the sound level. The reason is that when a note is played on the guitar, the fingertip firstly dislocates the string from its rest position, and then releases it. The initial sound level depends on the degree of the string displacement from its rest position, hence on the force applied. After release, no more energy is provided to the string which, from now on, oscillates free and transfers part of the energy to the guitar body (the *resonator*) via the bridge; the residual fraction is dispersed in losses due to *viscous friction*. These phenomena will be discussed in detail afterwards, being the reason why the sound fades out after a few seconds—as shown in the waveform—with a *decay transient* that resembles the one observed in the tuning fork case.

The waveform of a sound produced by the guitar is actually much more complicated than the elementary sound produced by a tuning fork. To understand why, we must examine the waveform of the same sound in a reduced time scale, specifically between 0 and 0.5 s.



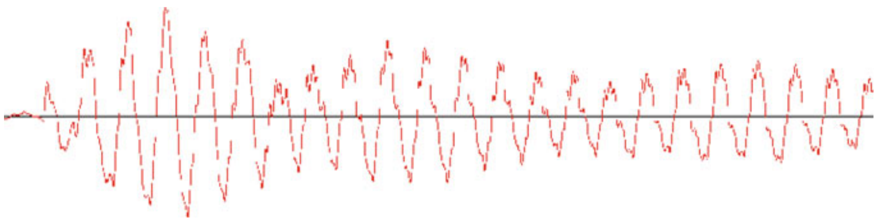
The attack time is evident in the diagram, and is similar to the one observed in the tuning fork case, but now the maximum level of the sound is reached in just 20 ms.

The purpose in the next chapter is to examine the string motion: we will see that the transmission of elastic energy from the string to the resonator depends on a crucial parameter, the *resonator impedance.*

We like to point out that, both in this waveform and in the previous one, after the attack transient and differently from the tuning fork, the sound does not decay by a regular (or *exponential)* curve, but shows level undulations at comparatively low frequency.

This is because, in the guitar, several surfaces can radiate sound (the soundboard, the hole, the back), being components of oscillating systems that are somehow connected to each other by elastic bonds. When, after release, the string starts oscillating, these systems are also set into motion and, since they are connected to each other, they start passing the stored energy to one another, like two tennis players knocking up the ball. This mechanism of *coupled oscillating systems*, aside from musical instruments, is also a feature of other acoustic systems (for instance, Bass Reflex speakers). This issue will be discussed in detail later on. We just point out once again that the modulation of the sound level shown in the waveforms is due to this exchange of energy between coupled oscillators.

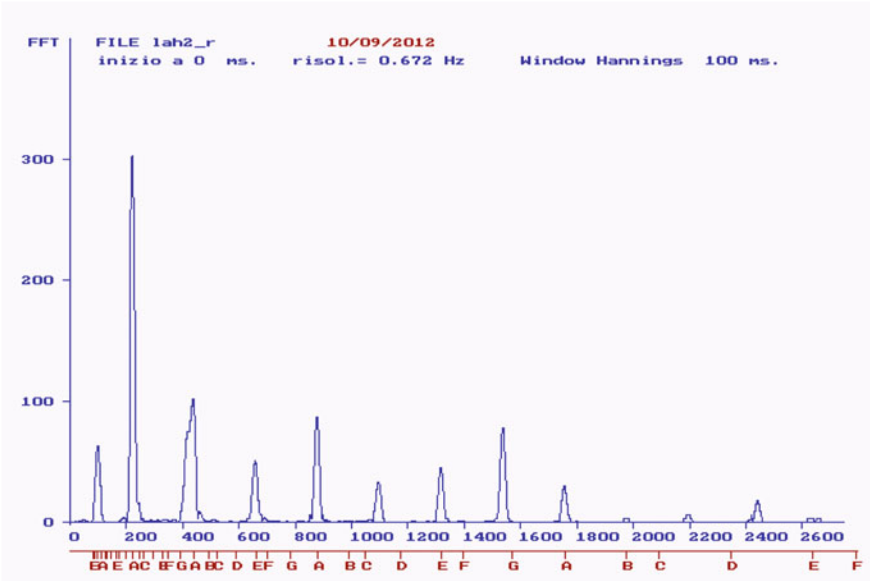
If we further enlarge and observe the waveform in the first 100 ms after release, we will see that the sound oscillations do not agree with the single sinusoidal law governing the tuning fork sound. Here, as can be seen in the following figure, oscillations are much more articulate and appear to be consisting of several components with different frequencies. If we further enlarge and observe the waveform in the first 100 ms after release, we will find much more articulate oscillations than those highlighted in the tuning fork sound. Here oscillations appear to be consisting of several components with different frequencies.



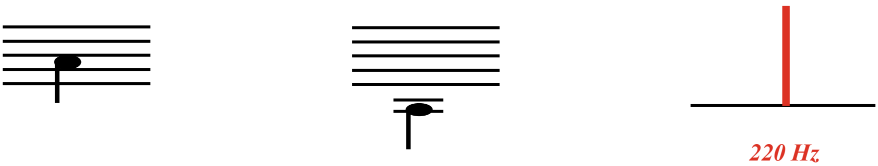
For a detailed study of these oscillations the sound spectrum is essential: this brings us out of the *time domain* we have been in so far, and takes us into the *frequency domain*.

1.4.2 The Spectrum

The following graph results from the frequency analysis (the spectrum) executed on the same sound that was illustrated in the last paragraph by the waveform in the time domain. The red line indicates the position of notes with relation to the frequency scale.



A wide spectral line stands out at 220 Hz. This is an unexpected result: while, according to international conventions, the height of A in the third space of the staff corresponds to 440 Hz, here the fundamental frequency of the sound is 220 Hz. This apparently bizarre condition is because the guitar-generated sounds are an octave lower than the staff notation.



Staff ***Yielded Sound*** ***Fundamental***

In other words, *guitar music is written an octave above the real pitch of the yielded sounds*. Therefore the guitar is an *octave transposing instrument*: this does not affect in the least the engineering and construction of the guitar, but we must take it into account, in order to prevent misinterpretation of the information resulting from the spectral analysis.

Most important to notice in the diagram above, there are spectral lines (*harmonics*) corresponding to whole multiples of the fundamental 220 Hz frequency, i.e. to 440, 660, 880, 1100 Hz, and so on. Starting from the fundamental, the graph reports the amplitude of harmonics up to the 11th. If we further enlarged the frequency scale, we would see harmonics recurring even beyond 4000 Hz, though presenting much smaller amplitudes.

By adding to the fundamental, these harmonic components (their presence and relative amplitude) determine the timbre of the sound produced by the instrument.

We will talk exhaustively about the function of these components in the next chapter. Here we just mention that, in the guitar, all begins when the player excites the string, which is when the string transmits force to the resonator. Also the force conveyed by the string to the resonator can be divided up into harmonic components of the *string motion* recipe. These components are multiple of the fundamental and depend on the way and place of plucking.

The harmonic components of the force produced by the string are someway filtered by the resonator, which generates in the surrounding environment a sound pressure, whose composition (*the recipe of the instrument sound*) is affected by the resonator impedance, that is to say by its resonances and anti-resonances. The amplitude of the fundamental and of the subsequent harmonics (as shown in the figure), depends both on the string motion (see next chapter) and on the resonator response (which will be the subject of later chapters). Both the force provided by the string and the resonator response contribute to the sound quality of an instrument.

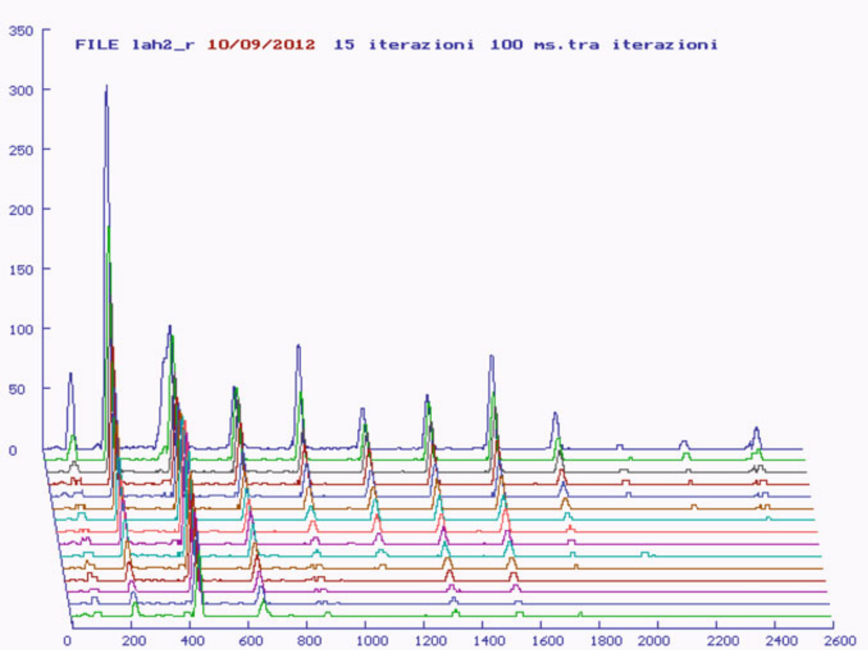
The graph also shows a spectral line with considerable amplitude at about 94 Hz, in correspondence with the *tuning note* of the instrument—as it is commonly defined by luthiers and players. If we beat the soundboard of a guitar with knuckles, we can hear a sound that normally lies in the span between F (at about 87 Hz) and G# (at about 104 Hz). In some instruments the tuning note can be as high as A (at about 110 Hz) or almost as low as E (at about 82 Hz). This tone corresponds to the *first fundamental resonance* or *air resonance*, which is caused by the interaction between the soundboard and the air contained in the guitar body. Quite interestingly, this tone also results from a fundamental at 220 Hz, which is positioned more than an octave above. Also the fundamentals lying two or three octaves above can excite the air resonance (although with smaller amplitude). This means that the tuning note is always present in the recipe of the guitar sound, at least in low-mid registers, and serves as a background on which the musical texture intertwines. The *timbre* (or *tone color*) will be darker or lighter, depending on the pitch of the tuning note. Amplitude, as reported in the sound spectrum, defines the degree of *presence*.

1.4.3 The Sound Spectrogram and the Variation of Harmonics Over Time

In Sect. 1.3 we have considered the variations over time of the single spectral component produced by a tuning fork at 440 Hz. The amplitude graph (the spectrogram) represents this spectral component on a time scale. Combined with the waveform, it provides a comprehensive description of the sound.

As shown in the previous image, the guitar case is more complicated, since the sound produced by pressing a string in any position contains several harmonics. So the spectrogram is a *3D representation* of the sound, whose dimensions are the frequency, amplitude and variation over time of each harmonic.

Among the different kinds of representation available, we prefer the one called *Waterfall*, which is again a 3D image of sound components as they vary in *amplitude*, *frequency and time*, but allows simultaneous observation of the spectra resulting from the same signal in different moments. The following image is the Waterfall chart of our signal, or the map of the sound produced by pressing the third string of a guitar on the second fret.



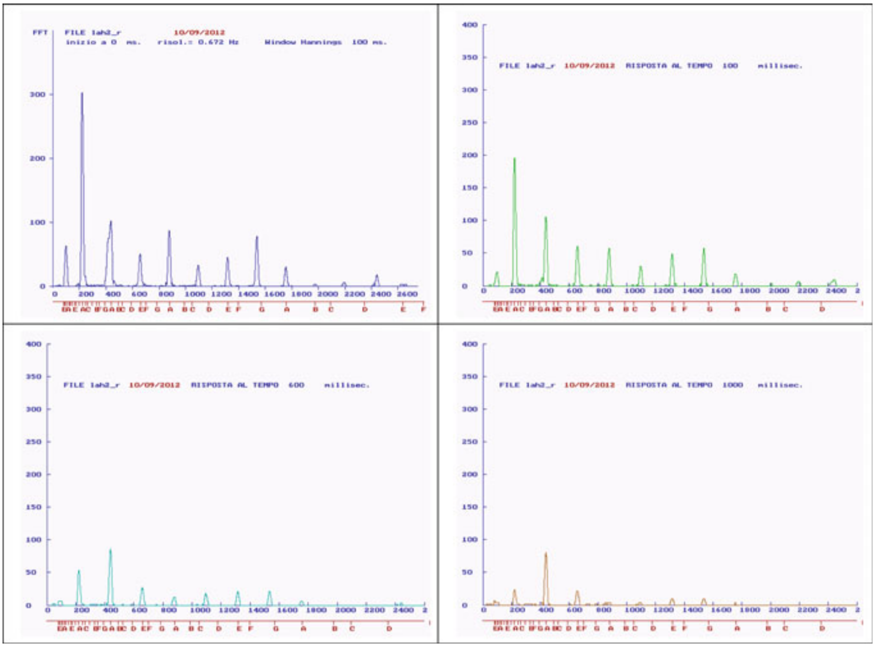
The Waterfall chart shows a cluster of spectra that result from the same signal, but in different moments. In our case, we created 15 interacting spectra, starting from the spectrum at 0 ms seen in the previous graph. Each consequent spectra is calculated 0,1 s after the previous, and the whole spectrogram describes the development of harmonics composing the signal in the first 1.4 s—which is more than sufficient to comprehensively examine the development over time of the sound components. These spectra being related to different moments, provide an image of how the amplitude of each harmonic component dampens down.

What comes to notice in this graph is that upper harmonics (from 1760 Hz upwards) tend to die out rapidly; others, though starting from a lower initial level, persist until the end of the observation period (1.4 s). What is more, the tuning note at about 94 Hz is still present after 1 s, though considerably reduced.

To better understand the development of harmonics, we will consider some of the spectra independently, with the help of the following figure.

The upper-left panel corresponds to the previously analyzed initial spectrum, here reported for the reader's convenience.

The upper-right panel represents the spectrum at 0.1 s from the start, when a significant variation has already occurred in the distribution of amplitudes: the fundamental has dropped from 300 to 200 in comparative units; the tuning note also appears considerably diminished, while the first harmonic shows a very slight decline, and upper harmonics are still very evident in the spectrum. As a conclusion, we could say that each harmonic has a specific ‘character’, with regard to the way amplitude drops off, and this character—manifested right from the beginning of the sound phenomenon—will recur again in the succeeding moments.



Let us skip forward in time, and observe the spectrum at 600 ms (lower-left panel). The most important point in this spectrum is that the first harmonic (at 440 Hz) overcomes the fundamental, (which is now much lower). In other words, the fundamental drops off more rapidly than the first harmonic, whose amplitude, in a definite moment surpasses the fundamental. This happens because the resonator of this guitar responds with a high resonance peak at about 400 Hz (not far from the frequency of the harmonic), which ‘supports’ the oscillation of the first harmonic. We will see these phenomena in detail later on. The dominance of the first harmonic over the fundamental can be detected through hearing, with no negative result in the quality of the instrument—as it does not ‘clash’ with the fundamental timbre of the note.

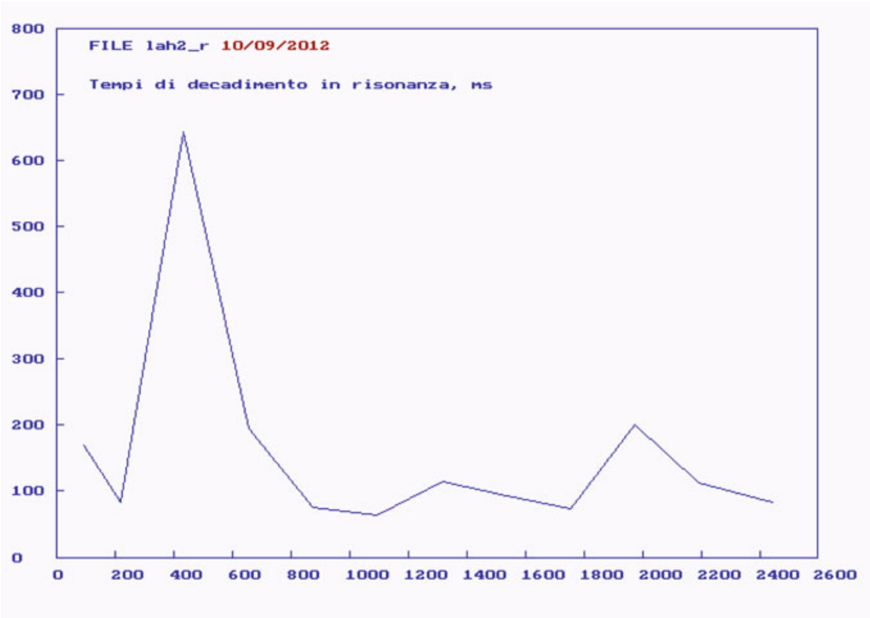
In the same image also the third harmonic at 660 Hz stands out, as it lyes close to one of the important resonances of this guitar.

Moreover, the tuning note is still present—though much reduced—while some of the high frequency harmonics are hardly visible.

After 1 s from the start (lower-right panel) the amplitude of the fundamental, of the first and the second harmonic is prominent. The amplitude in higher frequency

harmonics is very small, if present at all, and the tuning note is also much reduced. As a conclusion, after 1 s the sound recipe primarily depends on what remains of the fundamental and of the first two harmonics.

These qualitative considerations are summed up in the following graph, which depicts the *decay time* of the components of the signal, i.e. of the tuning note, of the fundamental at 220 Hz and of subsequent harmonics.



A mathematical assessment of the decay time is given in Appendix 1.3. Summarizing: *the longer the decay time of a harmonic component, the slower the decrease in the amplitude of that component*. Another way of stating this, is that components with long decay time drop off more slowly, and consequently persist longer in the sound composition.

The amplitude of the harmonic at 440 Hz drops so slowly as to eventually prevail over the fundamental; this is clearly seen in the graph, where the decay time of the harmonic at 440 Hz is about 650 ms, while for the fundamental is just around 83 ms.

It must be furthermore recognized that also the tuning note, the second harmonic at 660 Hz and the fifth at 1320 Hz have long decay times, persisting in the sound composition until the end of the observation period.

On the other hand, also the ninth harmonic at 2200 Hz has a slow decay, but since it starts from a low initial value, quickly fades out.

The graphs we have proposed (the spectrum of a note at the initial instant, the Waterfall, the progression of decay times) fully describe the dynamic development of the components of a signal taken as an example. As the timbre depends on what harmonic components are present in the sound recipe at any instant, and on their

relative amplitude, it is easy to understand how sharply the colour of the guitar sound changes, while fading out, after the string has been released.

A tuning fork delivers a poor, not very enjoyable sound. In fact, the *amount of information* reaching our brain is very limited: we perceive a note with constant pitch (or frequency) that slowly fades out. In other words, a monotonous (single toned) sound, which explains the presence of a single spectral line due to the very simple nature of the tuning fork, working as an elementary oscillator (a 'pendulum').

Quite differently, even a single note played on the guitar can get a completely different colour, depending on the playing technique and on the instrument. In terms of acoustics, the sound of the guitar is very complex, involving several factors that we have partly exposed and now we will summarize.

By observing the waveform we see that, just after release, it takes some oscillation periods for the resonator to reach maximum oscillation amplitude.

During the *attack time* the sound components (the tuning note, the fundamental and the harmonics) grow in the recipe of the sound according to the resonator own inertia: faster the high frequency components than the low frequency ones. Consequently, the longer the attack time—i.e. richer in low frequency components—the softer the attack; on the contrary, the shorter the attack time, the harder the attack of the sound as we perceive it. If instead, during a short attack transient the sound amplitude rapidly reaches very high values, the sound is perceived as 'percussive'.

With bowed and wind instruments, the vibration can be protracted until the end of the note, whereas in the guitar the sound level quickly fades out after the attack transient. The duration of a note (the *sustain*) as perceived by the ear depends on the *decay time*; however, in the guitar (differently from the tuning fork) the decrease in sound occurring after the attack does not follow a regular exponential curve, but goes through level fluctuations at comparatively low frequency. This amplitude modulation is due to the interaction between the different oscillating systems that compose the guitar resonator; once activated, they pass the energy stored in the string to one another.

This mechanism of *coupling between different oscillating systems* is the core of the guitar resonator functioning, which will be discussed in detail afterwards.

If we enlarge the waveform and observe it in the first 100 ms after release, we notice that pressure level oscillations do not agree with the simple sinusoidal law governing the tuning fork sound; they are much more complex and apparently constituted by a number of components at different frequencies. To understand this phenomenon we must leave the time representation and enter the frequency domain; this allows us to separately examine the spectral lines forming the sound. The first important graph presented here is the spectrum of the sound immediately after release. A theoretically endless sequence of harmonics at multiple frequencies of the fundamental goes along with the fundamental itself. The amplitude of the fundamental and of subsequent harmonics depends both on the string motion—as determined by the playing technique—and on the resonator response.

As previously observed, the tuning note is also present in the spectrum, and defines the *tone colour*—whether bright or mellow—with relation to its pitch.

The sound composition undergoes dynamic variations in the instants following release, as we can see in the Waterfall chart: this illustrates how some of the harmonics

(especially high-pitched ones) owing to their shorter decay time dampen down faster than others. Other harmonics protract instead until the end of the observation period (1.4 s in our example): these are to be found in proximity (not coincidence) of some important resonance of the guitar body that sustains them. The Waterfall chart is a 3D image of sound components as they vary in *amplitude*, *frequency, and time*. The graph highlights some distinctive (and somewhat unexpected) phenomena: for instance, in the specified case, the amplitude of the first harmonic after release prevails on the fundamental and is clearly intelligible on hearing. Similarly, in the long term, the second harmonic at 660 Hz approaches the fundamental.

The suggested inspection method is also applicable to different situations, that is to say to other notes, fingered in different ways, and on different guitars. By applying these measurements to other cases different results will probably come out, but still conforming to the analysis expounded here.

The final question is: what in this analysis really matters for the guitar-maker's purposes? We said that sound characteristics depend on the harmonic components (their presence and relative amplitude) which, by adding to the fundamental, determine the timbre. These components form the string motion recipe and, to some extent, they are determined by the way and place where the string is plucked. Obviously, the luthier cannot act on these aspects. The harmonic components of the force generated by the string are filtered by the resonator which produces, in the surrounding ambient, a sound pressure conditioned by its resonances and anti-resonances, on which the luthier can intervene. The next chapters deal with the instrument response (its resonances and anti-resonances), and how the instrument design affects it.

1.5 The Quality of Timbre in a Guitar

When we ask a guitar-player to estimate the quality of sound in a guitar, we usually get generic information, as 'loud' or 'quiet', 'light' or 'dark', 'brilliant' or 'dull'. The same definitions are normally used by listeners. Both the player and the listener tend to define the quality of timbre in emotional terms, trying to communicate the perceptions brought about by the instrument sound. If we submit to the same player two high-quality instruments, he (or she) will make a choice based on the same emotional criteria. A listener or another player could express a completely different opinion.

The case complicates when we submit a variety of instruments with different quality to a group of guitarists. We (as other authors) did it: the feedback reveals that

- Players clearly distinguish poor-quality instruments, and share the same negative opinion.
- As to high-quality instruments, players do not always agree in their choice.

These responses are coherent: every guitarist recognizes in a poor instrument the lack of one or more of the features that distinguish a good instrument. Differently, in the appraisal of high-quality instruments, every player tends to prefer certain timbre qualities, according to a personal 'concept of sound'. We must not forget that music

is an art, and all artists choose the instrument that best matches their playing style and technique.

Players also know that some kinds of instrument 'work' better with some tunes than with others. If the tune is a slow monody (i.e. one with a single melodic line prevailing) an instrument with warm sound and good sustain is generally preferred. If the tune is rich in chords and parallel melodic lines, an instrument with brilliant sound, allowing each line to stand out without superposing to others, is generally more appropriate.

However, the assessment of quality in an instrument is not just a matter of personal preference. All excellent instruments have measurable, objective characteristics. To identify these attributes, we asked our guitar-players what qualities they prefer. We got a rather homogeneous response:

- Most players consider *projection* as the hallmark of a concert guitar. In our opinion, this generic word denotes a combination of loudness and sound persistence (or *sustain*). A guitar having powerful output and good sustain 'projects' loud and persisting sounds in an auditorium.
- *Sustain* is highly appreciated, for instance in recording studios, even when the sound is not very loud; on the other hand, a powerful but poorly sustained sound is not very esteemed.
- *Balance* in fundamental registers (bass, middle, treble) is considered a most important feature, since this allows a natural output of the sound in all registers, and the player is not forced to boost up any of them in particular.
- Balance goes along with *dynamic extension*: all notes (especially high and very high-pitched ones) must stand out clearly and no one must too rapidly 'fall' with respect to the notes lying close in the musical scale.
- *Timbre* is a most important feature in the guitar, though depending on a very personal appreciation. It is very important for a player to have *control of timbre* along the whole dynamic range of the instrument.
- The *attack transient* and the *promptness of response* are also very important, both depending on the instrument response in the first instants following the string excitation.

These quality parameters are more than simple, emotional criteria on which the evaluation of instruments is often based. Yet they are not directly equated with the acoustic properties and the design of a guitar. In order to make this correlation possible, we must recall what previously observated about the characteristics of the guitar sound. The next step—design optimization for an excellent result—will be the subject of subsequent chapters.

1.6 Relation Between Quality and Physical Parameters

The waveform clearly shows that, immediately after release, it takes some oscillation periods for the vibrating surfaces of the resonator to reach maximum oscillation amplitude. During the *attack transient* (which depends on the resonator impedance

seen by the string) the tuning note, the fundamental and the harmonics (or the sound components) grow and increase in the sound recipe.

Our brain only recognizes the pitch (or frequency) of a tone if it lasts for some periods; if not, the sound is perceived as 'noise'. A tone at 100 Hz has a period T = 1/f, corresponding to 10 ms, hence it cannot be perceived before 20–30 ms; therefore, low-frequency sound components (chiefly the tuning note) will only be perceived in an attack transient lasting a minimum of 20–30 ms.

In other words, the attack of the sound is smooth if the attack transient is long (i.e. richer in low frequency components) while a short transient causes a harder, though cleaner attack of the sound. The promptness of response too, is related to the attack transient: the longer the transient, the smoother but slower the attack, and vice versa. Nevertheless, if the sound level rapidly grows to very high values, the response is prompt and the sound is perceived as 'percussive'—especially if the subsequent decay time is short.

In the previous sections we have seen how, after the attack transient, the sound level progressively dies out. During this *decay transient,* the elastic energy initially stored in the string is dispersed because of losses due to different phenomena: friction within oscillating systems (string and resonator) and sound radiation from the same systems to the surrounding environment. Anyway, for the listener, the *sound persistence* (or *sustain*) depends on the time it takes for a tone to die out.

We noticed how, during this transient, the sound level shows undulations at relative low frequency, chiefly owing to the above-mentioned coupling mechanism (between the board and the air in the body) which will be thoroughly investigated later. These amplitude variations confer the sound a richer character than the monotonous tone provided by a constant amplitude source.

In the signal spectrum observed during the decay transient, we noticed the presence of the tuning note, that is the resonance of air in the resonator; this defines the *basic colour* of the timbre, which will be dark or light according to its frequency, while amplitude designates the degree of *presence*.

We also observed that the fundamental and subsequent harmonics of a note—which are present at the first instant in the sound composition—die away in different moments. A dynamic variation in sound composition occurs in the instants following release, as can be seen in the sound spectrogram (the Waterfall chart): some of the higher frequency harmonics fade more rapidly while others, lying close to an important resonance of the guitar body, persist longer. Therefore, what we called the 'sound persistence', which can be assessed by the decay time in the waveform, actually depends on the way the different harmonics of a guitar tone decay. We presented the decay time diagram of each component of a tone, including the fundamental and the tuning note.

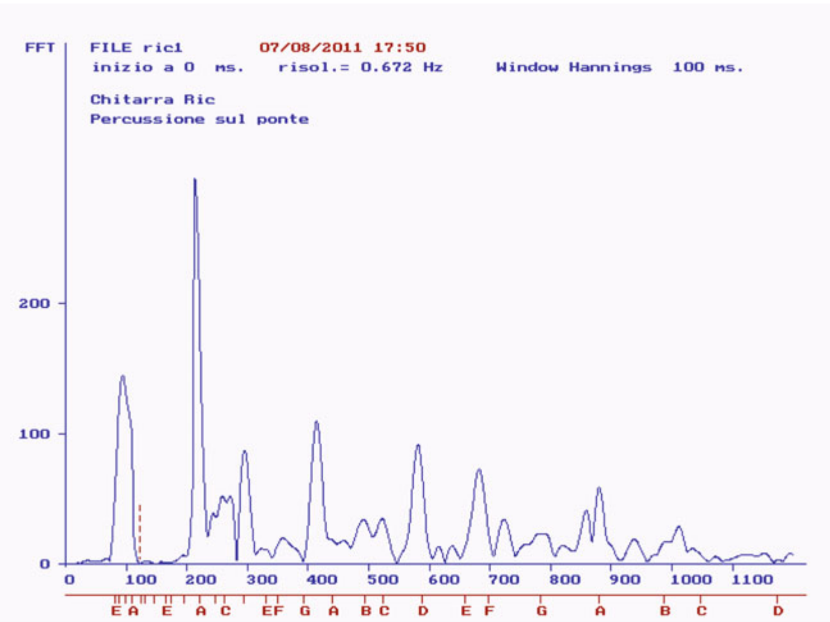
The presence and equilibrium of a specific sound component depend on the proximity to a resonance of the body: if corresponding to a resonance, the initial volume is high and the decay is fast; on the other hand, a resonance close to, but not coincident with a harmonic, boosts its initial sound level, but also provides a slow decay and a good sustain. Later on, when dealing with the guitar resonator, we will investigate these aspects in detail.

As previously mentioned, the waveform and the spectrogram gather many of the characteristics of a guitar sound:

- The attack transient and its duration affect the promptness of response and the attack of the sound (its sharpness, smoothness, or percussiveness).
- Presence, amplitude, and persistence of the tuning note in the spectrum affect the tone colour, hence the timbre.
- The decay transient affects sustain, as well as projection.
- Variations in sound level, taking place during the whole duration of a note because of the coupling between the resonator components, have a special influence on timbre, conferring it the typical richness and nuances of the guitar sound.
- The decay time of each harmonic (which is related to the position of body resonances) affects sustain but, above all, dynamic extension and balance between different sound components.

The waveform and the spectrogram of a single note has been our subject so far. To identify other properties of the instrument that can be related to quality criteria, a study of the guitar resonator properties, regardless of the excitation method, is required. The *resonator response* is the attribute representing these characteristics in terms of frequency.

The following image is the resonator response in a 'reference' guitar. The graph shows *resonances* (at maximum emission frequencies) and *anti-resonances* (at minimum emission frequencies).



The origin of these resonances and anti-resonances, and how they can be partially controlled through proper design and construction of the instrument, will be comprehensively discussed in the following chapters. Now some general observations should be sufficient.

Apart from our research, previous investigations also reveal the importance of the *first fundamental resonance* or *air resonance* (at about 94 Hz in this instrument). This resonance determines the above-mentioned tuning note. The resonance should be large enough to ensure a good presence in the sound composition, and should have a significant bandwidth to be excited by the fundamentals of much higher frequency tones as well. A broad bandwidth also ensures a good presence of the tuning note in the sound. In straightforward terms, the bandwidth is broad if this resonance is significantly extended in frequency and the peak looks rounded; it is narrow when appearing like a slim and pointed bell.

Equally, the *second base resonance* or *table resonance*—occurring in this instrument at 213 Hz—deeply affects both timbre and sustain. In order to stimulate the first harmonic of the basic note, to enhance its presence and the sound duration, this resonance should arise at about an octave above the first resonance, i.e. at twice its frequency (which is not the case with this instrument). It should also be large and have a narrow bandwidth, so as to determine the behaviour of the instrument in the low-mid register. However, even in some good instruments the table resonance appears to be too high and too narrow; consequently, though very powerful in the beginning, sound components lying close to this resonance quickly die away, resulting in a percussive, nasal timbre: the attack is 'prompt', but scarce modulation control is left to the player.

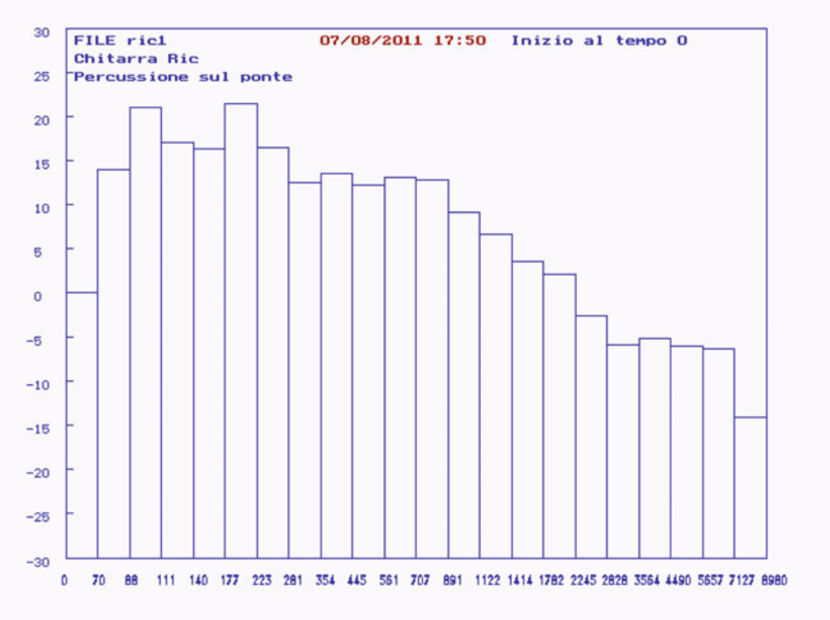
Higher frequency resonances (up to 400 Hz) depend on the soundboard and, mostly, on the back. They support the fundamentals of notes in an intermediate register; hence, they favourably affect timbre and, above all, sound equilibrium.

In good or excellent guitars we always find a resonance at about 400 Hz, owing to the soundboard and its interaction with air in the body. This resonance, concerning the upper-mid register, should surpass the nearby ones in amplitude; it favourably affects timbre, equilibrium and sustain, since the fundamentals and the harmonics lying close to this resonance are enhanced, and persist in the sound recipe for a long time. See the example shown above when observing the spectrogram of an A at 220 Hz, played on the second fret of the third string.

Subsequent resonances are also due to the interaction between the soundboard and the air in the body. Their presence and amplitude distinguish the instrument responses in upper registers, so they influence clarity and transparency, balance and dynamic extension of the sound. It is interesting how this instrument provides a very articulate and lively response above 900 Hz (beyond the high A).

In the previous diagram, an axis reports the position of notes in the musical scale with relation to the response diagram. This axis helps finding notes that match some resonance (or anti-resonance) occurring in the response of the guitar resonator. Generally, any correspondence between important resonances and notes of the scale must be prevented, because resonances tend to absorb energy from the fundamental of the notes, leading to a sound initially powerful which, however, decays rapidly.

Another way of representing the resonator response is the *thirds of octave* graph. The following diagram, concerning the reference guitar, is an example.



Let us analyse the diagram structure. The horizontal axis can be compared to the piano keyboard: this is composed of octaves, each one occupying an equal section. The same spacing (in cm) is reported along the horizontal axis: in the previous figure, each octave covers about 2 cm on the axis (we remind that an octave is the span between two tones when one has twice the frequency of the other). Dividing this space in three parts, the extension of each band will be *one third of octave*: in the figure, each band spans $2/3 = 0.66$ cm on the horizontal axis. Then a frequency value is assigned to the centre of each band. An international convention allows comparison between results from laboratories all over the world; the convention established the centre of the 'reference' band at 1000 Hz. This way, both the central value of each band spanning 1/3 octave, and the frequency values corresponding to the ends of each band, can be easily determined.

Passing over mathematical details, we can say that frequencies are arranged on a logarithmic scale on the horizontal axis. The previous diagram (in accordance with the standard) covers frequencies spanning round 70 Hz to about 9000 Hz, and reports the two ends frequencies of each band. Then the middle frequency (mentioned in Appendix 1.5) and a specific frequency interval $\Delta f = f_2 - f_1$ (where f_2 and f_1 are the frequency limits of that band) are assigned to each band. In musical terms, an interval Δf of 1/3 octave is defined as a *third major interval.*

The height of each band corresponds to the *average value of the spectrum between the frequency limits* f_2 and $f_{1.}$ Therefore the 1/3 octave diagram is still a depiction

of sound in the frequency domain, or a spectrum representation, but here the axis is partitioned in intervals, each one assigned to an average frequency response value. Another way of expressing this is that spectral density, rather than detailed values, is the focus here.

Though normally implemented by software right from the resonator response, it is important for the reader to be familiar with this diagram, in order to estimate many of the features that contribute to the timbre quality of an instrument.

By the expounded reasoning we deduce that each of these intervals defines the level of emission in that band: for instance, the previous diagram reveals stronger emission in the bands containing fundamental main resonances.

Through a calculation of the average value in different intervals, we can assess the average amplitude value over a larger frequency range, thus obtaining information about the features and quality of a specific instrument.

Below are listed and explained some of this 'extended' ranges, the value of the reference guitar serving as an example:

- *Average value of the thirds spanning 80–1000 Hz.* 14.91
  This is the average sound level in the frequency range comprising the fundamentals and the harmonics that are essential for a good sound. In a sense, it also defines the efficiency of the instrument in the related frequency range: given constant excitation energy, the higher this value, the better the instrument exploits the available energy to turn it into sound radiation.
  This parameter also gives a reliable indication of the sound *projection* in a guitar: projection is elevated when a high value of the thirds 80–1000 Hz goes with a great persistence of the sound, i.e. a long decay time after release.
- *Value of the thirds between 1250 and 3150 Hz.* 0.692
  This is the average level in the frequency range comprising harmonics that are crucial for clear and brilliant sound, hence for timbre and dynamic extension too. This level is much lower than the previous, because the performance of the guitar in the corresponding frequency range is generally modest. On the other hand, we observed that an exceedingly high value results in a harsh, metallic, disagreeable sound.
- *Value of the thirds between 80 and 125 Hz.* 17.27
- *Value of the thirds between 160 and 250 Hz.* 18.05
- *Value of the thirds between 250 and 400 Hz.* 14.13
  Each of the last three values sums up the sound emission level in a restricted frequency range, where one or more fundamental frequencies are prominent. In the first interval (80–125 Hz) the basic air resonance is dominant. In the second interval (160–250 Hz) the fundamental resonance of the soundboard, the second resonance of the table, and the fundamental resonance of the back, are the most outstanding elements. In the third interval (250–400 Hz) the upper resonances of the back and, above all, the third resonance of the soundboard prevail. To ensure a powerful and balanced response of the instrument, the values of the thirds should be high and very homogeneous.
- *Value of the thirds between 800 and 1250 Hz.* 9.46

This is the average level in the frequency range comprising the fundamental and the first harmonics of the sounds in the extreme-high register, as well as upper harmonics of notes played in the treble register. This level is important for clarity and transparency of the sound, for equilibrium and dynamic extension.

1.7 Conclusions

In this chapter we have examined the characteristics of the guitar sound both in the time domain (the waveforms) and in the frequency domain (the spectrum and the spectrograms).

Then we listed the main properties of good and excellent instruments as defined by listeners and players, neglecting emotional evaluations that bring scarcely helpful information.

The next step was to equate subjective quality criteria with objective sound parameters. As a result, it is evident that each quality criteria is linked to more than one physical parameter of the sound: in other words, each quality criteria results from a compromise between more than one characteristic of the sound, each one in turn affecting more than one quality criteria.

Moreover, probably no instrument is the absolute best for every player, every listener, every kind of music, and every location where the instrument is played.

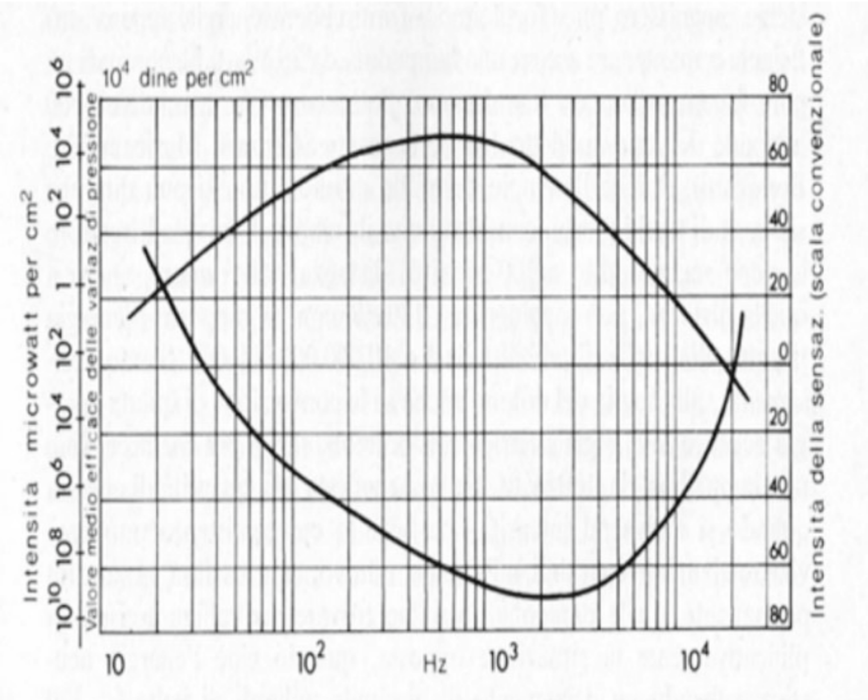
For these reasons, we did not suggest optimal values for the observed physical parameters, but we limited our considerations to the response of an excellent 'reference' guitar (the same that will later serve for the study of the resonator).

We tried to give detailed explanation of an investigation method, and of the outcoming results: luthiers can apply this to their instruments and appraise the outcome according to their own 'concept of sound'. Obviously, transposing this concept into reality is quite a difficult task, concerning both the design and the construction process which will be carefully described afterwards.

Of course the luthier can also apply this method to instruments produced by other guitar-makers, in order to figure out their 'concept of sound' and understand what designing criteria they followed to reach their targets.

Appendices

Appendix 1.1: Standard Audiogram



Lower curve = threshold of hearing
Upper curve = threshold of pain

From Pietro Righini—Giuseppe Ugo Righini.

Il Suono
Tamburini Editore—Milano 1974.

Appendix 1.2: Sound Pressure Level (or SPL)

SPL (*Sound Pressure Level*) is the parameter commonly used to denote the power of a sound, as defined by the formula

$$SPL = 20\, Log_{10}\left(\frac{\text{Acoustic Pressure}}{20 \times 10^{-6}}\right)$$

In this formula

- *Acoustic Pressure* is the physical value of the pressure generated by the source of the sound, and it is measured in Pascal (1 Pa = 1 N/m^2).
- The constant 20×10^{-6} Pa indicates the hearing threshold, or the acoustic pressure corresponding to the faintest audible sound to which, based on this formulation, sounds generated by a specific source are referred.
- The acoustic pressure level, as defined by this formula, is measured in decibels (*dB*).
- The function $y = Log_{10}(x)$, which is applied in the SPL formula, is the *logarithmic representation* of a generic value (x). To understand how this function transforms the value x into its logarithmic representation $y = Log_{10}(x)$ see the following table where, beside some values of the x, we find the values of its logarithmic representation y and the corresponding values in dB.

x	$y = Log_{10}(x)$	Value in dB
1	0	0
10	1	20
100	2	40
1000	3	60
10,000	4	80
100,000	5	100

If we reported onto an A4 sheet the values of a variable x ranging between 1 and 100,000 on a linear scale, the smaller values would be so crowded in the initial part of the scale as to be hardly distinguishable. Otherwise, we should use an extremely extended, hence unmanageable scale.

The logarithmic representation contracts the greater values, so that they can be reported on a scale that clearly displays both the minimum and the maximum values of the parameter x, and their mutual relationship. The representation in dB enlarges the scale of the logarithmic representation.

Please note that the SPL value in dB is zero when the acoustic pressure equals the hearing threshold pressure (20×10^{-6} Pa), which corresponds to the faintest audible sound level.

Appendix 1.3: Initial Transient and Decreasing Transient

The observed representation of the tuning fork sound shows that the amplitude of the tone at 440 Hz firstly increases, then decreases over time. The first part is the *rising transient*; the next is the *decreasing transient.*

The mathematical formulation of the amplitude during the rising time is:

$$A(t) = A_0 \sin(2\pi f t)(1 - e^{-t/\tau_s}) \quad (e = 2.71828)$$

where

- t is the time variable.
- $A(t)$ is the instantaneous amplitude value.
- A_0 is the asymptotic value that would be reached by amplitude after a very long time.
- f is the tone frequency (440 Hz in this case).
- τ_s is the *rising time constant* of the transient.

The curve connecting the sinusoid peaks (the *envelope*) is a growing exponential curve that starts from the initial time ($t = 0$) and tends to A_0, a value that would be reached after a very long time. This exponential curve is determined by τ_s, that is the value of the *rising time constant*.

After a time t_1, when amplitude reaches the value A_1, and the effect of the excitation is over, the decreasing transient begins. This is defined by the following formula:

$$A(t) = A_1 \sin(2\pi f t)(e^{-(t-t_1)/\tau_d})$$

Now the envelope starts from A_1 and tends to zero, the decreasing time constant being τ_d. Notice the different formulation of the exponential function.

The two time constants τ_s and τ_d dominate the progression of the envelope curves in the rising and decreasing stages, and are related to the physical characteristics of the source (hence not related to excitation nor to tone frequency):

- the rising time constant τ_s depends on the source *inertia*, i.e. on how promptly the system reacts. It is generally very short (in our case is $\tau_s = 35$ ms).
- On the contrary, the decreasing time constant depends on how quickly the energy stored in consequence of the excitation is dissipated, due to the internal viscous losses and to sound radiation towards the surrounding environment. It is generally much longer than the rising time (in our case $\tau_d = 1.4$ s).

Appendix 1.4: Fourier's Theorem Formulation

Fourier's Theorem tells us that a periodic signal (as those produced by a plucked string) can be approximately assessed by summing up sinusoidal functions at multiple frequency of the fundamental:

$$f(t) = \frac{A_0}{2} + A_1 \cos \omega t + B_1 \sin \omega t + A_2 \cos 2\omega t + B_2 \sin 2\omega t$$
$$+ A_3 \cos 3\omega t + B_3 \sin 3\omega t + \cdots$$

- The constant A_0 is proportional to the continuous component of the signal (when present).
- The constants A_n and B_n (or the *Fourier coefficients*) are the *amounts* that each sinusoid at the frequency $f(t) = \frac{n\omega t}{\pi}$ contribute to the whole *in amplitude* and *phase*.
- If we transfer onto a diagram the amplitude and phase of each of the sinusoids resulting from the decomposition of the original signal $f(t)$, we get the *frequency representation*, or *spectrum*, of the signal.

The expression above shows the time representation $f(t)$ of the periodic signal and the constants A_n and B_n that allow construction of the signal spectrum. Consequently, this expression is—in a sense—the bridge linking the *time domain* to the *frequency domain.*

Under certain conditions, it is possible to extend the Fourier's Theorem formulation to non-strictly periodic signals, as those representing the guitar resonator response. However, in this case the sinusoidal components of the spectrum will not be integer multiples of a fundamental. Later on in the text we will investigate the relationship between spectrum, resonances, and vibration modes of the resonator.

Different algorithms based on the Fourier series are available for constructing the spectrum of a signal. The algorithm of the Fast Fourier Transform (FFT), published in 1965, is probably the most powerful tool we have today for the analysis of signals.

We implemented this computing method in our software, and we used the consequent results to analyse the response of the examined systems.

For details on implementation, please refer to specialised texts.

Appendix 1.5: Center Values of Octaves and Thirds of Octave Bands.

Octave Bands (center frequency—Hz)	Third of octave bands (center frequency—Hz)
63	63
	80
	100
125	125
	160
	200
250	250
	315
	400
500	500
	630
	800
1000	1000
	1250
	1600
2000	2000
	2500
	3159
4000	4000
	5000
	6300
8000	8000

Chapter 2
The String

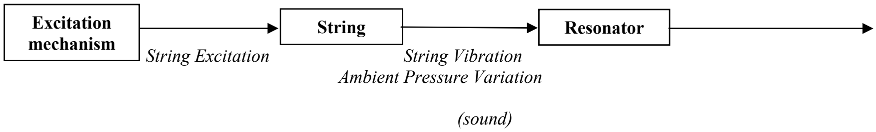
Abstract The sound begins with the string. The sound producing system is based on the 'excitation mechanism' (the plucking process), the generator (the string), the 'resonator' (the body of the guitar). The second chapter presents the basic equations of the ideal string, relating tension, density, velocity to the pitch of the fundamental. The distribution of the normal modes (harmonics) as a function of the plucking point is discussed here. Some important subject like the effects of the playing technique on the string motion, or the string making technology is not covered, as considered beyond the scope of this book. A second section deals with the 'real' non ideal string, the damping due to internal losses, the impedance interface and the reflection coefficient at the bridge.

As a simplified pattern, the classical guitar is composed of three basic parts.

- The *sound-producing system* consists of different elements whose vibration causes an ambient pressure variation that we perceive as 'sound'. In the classical guitar the sound-producing system, called *resonator*, consists of two vibrating surfaces (the soundboard and the back) joined by a semirigid frame (the sides or ribs).
- The *generator* is the element that activates the *sound-producing system.* In the classical guitar this function is carried out by the strings.
- The *excitation mechanism* is the input element setting the generator in motion. The guitar belongs to the big family of stringed instruments like piano, violin, mandolin, harp and others. One of the most crucial differences among these instruments is indeed the string excitation mode: in the piano by the hammers, in the violin by the bow, in the mandolin, as in the electric guitar, by the plectrum. In the classical guitar the string oscillation is brought about by the player's nails and fingertips. The player can also vary both the plucking point and the way he acts on the string. So any class of instruments employs a specific kind of excitation mechanism, which is crucial to the sound characteristics like timbre, sustain, dynamics and damping.
  We will not talk again about the function of the excitation means, as this is more related to the playing technique than to acoustics or engineering and construction of the guitar.

G. Cuzzucoli and M. Garrone, *Classical Guitar Design*,
https://doi.org/10.1007/978-3-030-32992-1_2

The following scheme illustrates the logical relation linking the three fundamental parts of the guitar, and can be applied to any stringed instrument.



2.1 The Ideal String

A string is stretched between two ends which we suppose to be rigid (in the guitar the *bridge* and the *nut*), and undergoes a longitudinal tension **T** (i.e. a force pulling the string in the length direction).

The string is then displaced from its position of equilibrium by the fingertip, and finally released. This way two travelling waves are generated: one that runs from the plucking point towards the bridge, and the other from the plucking point towards the nut. On reaching the ends, the waves completely reverse and start travelling each towards the opposite end; here they reflect back again, the one from the bridge towards the nut, the other from the nut towards the bridge.

We can visualize and understand this phenomenon by thinking of what happens when we cast a stone into a pond: we will see waves expanding from the *point of impact* towards the water's edge; on reaching this, they reverse and reflect back to the opposite end, where, again, they replicate.

If we toss a piece of paper into the pond, we will see it alternately going up and down (with an *undulating motion* to and from the water level). On the other hand, it will not move horizontally towards the edge or the centre, which means no dislocation of substance is involved. We can also observe that the wave peaks move at a definite *velocity of propagation* **c**.

Analogous phenomena take place by plucking a string with nail and fingertip: two waves run in opposed directions along the string; once reached the ends, they reflect backwards, displaying an alternate and continuous movement of wave-peaks. These propagate at a definite velocity without any dislocation of substance. The wave reflected on one end is a mirror image of the incoming wave, that is to say it brings a reversed—positive or negative—polarity.

The propagation velocity along the string depends on the physical properties of the string: the heavier the weight, the slower the propagation; the higher the tension, the higher the propagation velocity.

The propagation velocity of the travelling waves along the string is expressed by the following equation (we do not provide mathematical proof as this can be found in acoustics textbooks).

$$c = \sqrt{\frac{T}{\mu}}$$

T represents the degree of tension applied to the string, **μ** stands for *mass per unit length*, that is the weight of the string referred to its length.

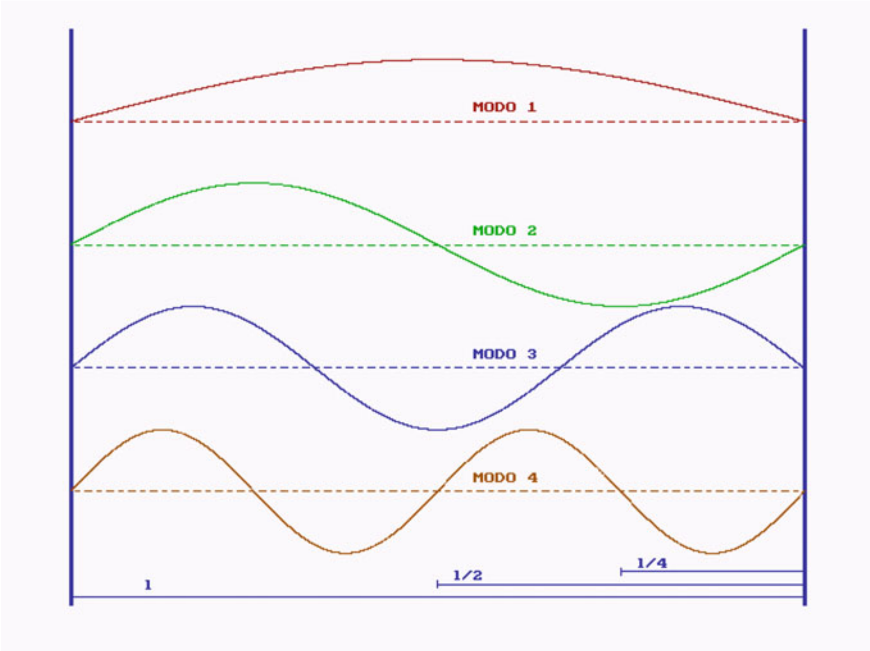
This parameter depends on the density of the material from which the string is made, on its diameter (or *gauge*), but not on its length.

The two travelling waves interfere with each other along the string: in some points they add together (here we have the maximum string oscillation amplitude); in some other points they tend to mutually nullify (here we have the minimum string oscillation amplitude). The consequence is a very complex motion and, at any time, the displacement of any point of the string from its rest position depends both on the mechanical parameters of the string (tension, length, mass) and on where (and how) it is excited.

Fortunately, the complex physical phenomenon here taking place is one that can be divided into many simpler and more intelligible physical phenomena. Let us think about a white light beam passing through a prism. On its way out, the beam generates luminous emissions with different wavelengths that our eyes perceive as a rainbow of colours. In our case the prism, allowing us to divide the string motion into elementary phenomena, is a proper *analyser*. This is an instrument (possibly mathematical) capable to detect the ingredients—and their different amounts—composing the motion 'recipe'.

The following diagram shows how the string motion can be divided into some elementary, *sinusoidal*, oscillating motions. These are the string *vibration modes*: at any instant, the string displacement from its equilibrium position is the sum of the displacement due to mode 1, mode 2, mode 3 and so on. The two ends in the illustration correspond to the nut and the bridge, namely the clamping points of the string.

Most important to be observed, the mode 2 frequency doubles the mode 1 frequency. In other words, the amplitude in mode 2 goes through a full variation cycle between the ends, while in mode 1 it goes through just half a sequence. The other modes behave similarly: for instance, mode 4 completes two entire cycles, whereas mode 1 (hereafter called *fundamental mode*) only accomplishes half a cycle. Accordingly, the frequency in mode 4 is four times that of the fundamental mode.



From the above-illustrated arguments, we can deduce that each mode frequency is in constant proportion to the fundamental mode frequency:

Frequency (mode 2) = 2 $\times$ Frequency (fundamental mode)
Frequency (mode 3) = 3 $\times$ Frequency (fundamental mode)
Frequency (mode 4) = 4 $\times$ Frequency (fundamental mode).

and so on.

We can say that, in the oscillating motion of the string, a *harmonic relation* subsists between the mode 1 frequency and the 'higher' vibration modes.

Now what is the practical meaning in these assumptions? The note the string is tuned to goes with the frequency of the fundamental mode—that is exemplified by the following formula (without demonstration).

$$F_1 = \frac{c}{2l} = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$$

The string is tuned to a definite pitch by varying its tension: in the guitar at least, this is the only parameter we can adjust, given fixed mass per unit length $\boldsymbol{\mu}$ and vibrating length $\boldsymbol{l}$ (or *scale length*, corresponding to the distance between the ends). By the way, we must report that some instruments, such as the harp, also allow vibrating length adjustment. Tuning a string means modifying the frequency of both the fundamental mode and the higher modes, though their harmonic relation remains unaffected.

The different string making technologies were developed out of this fundamental frequency equation. In the course of centuries, strings have been made by using materials with different density (as gut, metal, and nylon) which, in order to properly work on the instrument, require diverse combinations of gauge and tension. Today, for instance, the three bass strings of the guitar (tuned as D, A, E) are generally composed of a core, made of nylon threads, and wound with thin wire. This stratagem increases the mass unit per length. Differently—in order to obtain a proper tuning—either an excessively low tension or an over-sized diameter should be imposed to these strings. Either way, playing them would result in a very challenging task.

Consider again the illustration above: in the fundamental mode we have a zero oscillation amplitude only at the ends while, in higher modes, points of no displacement lie along the string as well. More accurately, the oscillation amplitude is nullified at half the length in mode 2, at 1/3 and 2/3 of the length in mode 3, at 1/4, 2/4 and 3/4 of the length in mode 4, and so on for other vibration modes. These points, where we always have zero oscillation amplitude, are called *nodes* of that mode. The points, where the vibration amplitude is maximum, are called *antinodes*. These, in any vibration mode, are located midway between the nodes.

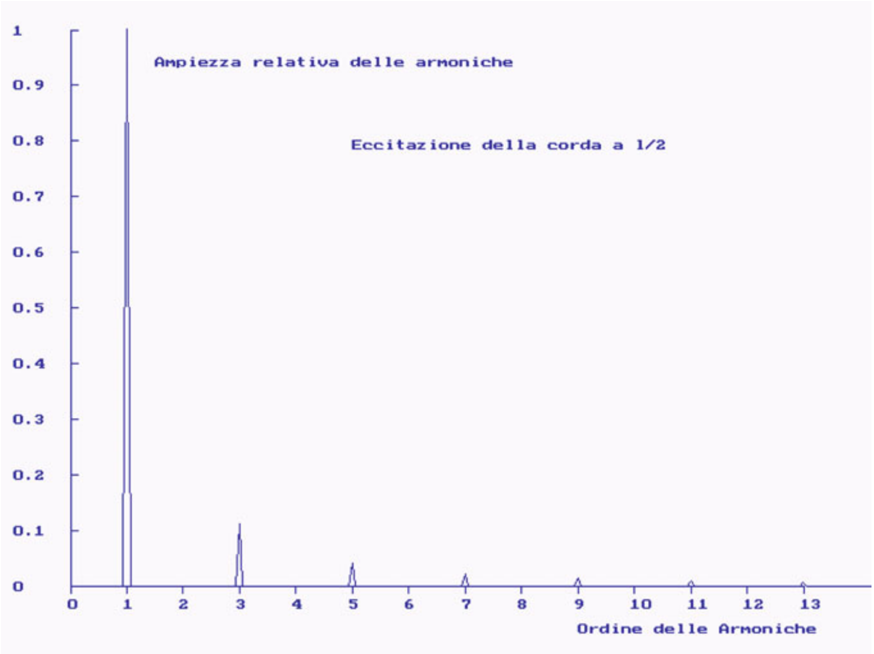
As previously stated, the fundamental vibration mode determines the string intonation, and so its pitch. Nevertheless, higher modes are only *potentially* comprised in the 'recipe' of ingredients composing, in different amounts, the string motion. In fact, they are completely absent in particular conditions: specifically, *vibration modes containing a node on the plucking point are absent*.

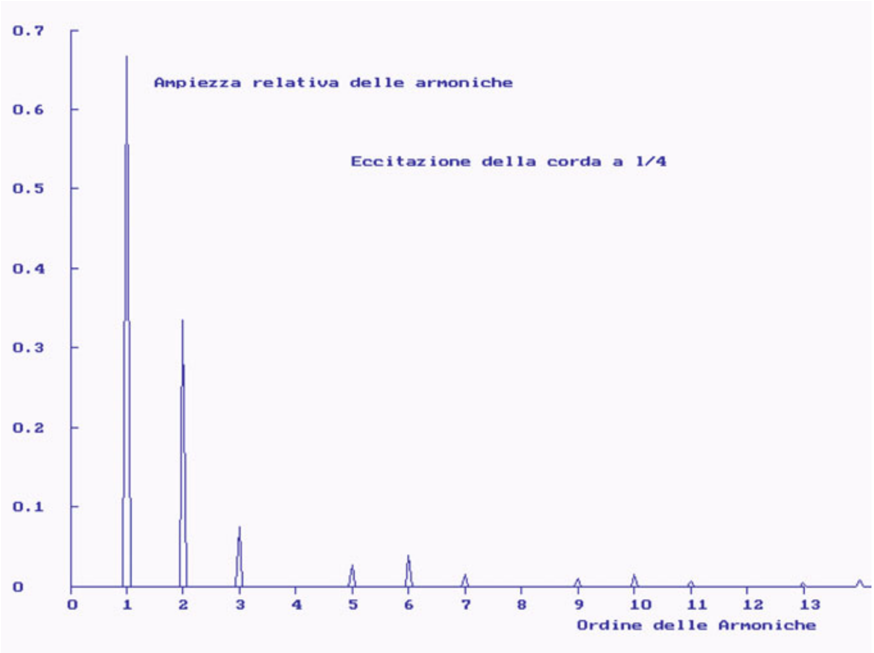
The following Figures summarize our reasoning so far. By means of a *spectrum analyser*—a mathematical instrument in this occurrence—we examined the string motion in three plucking points (l/2, l/4, l/8). Then we transferred the relative amplitude of harmonics composing it up to the 13th harmonic onto the graph, along with the relative position (or *rank*) of each harmonic with regard to the fundamental.

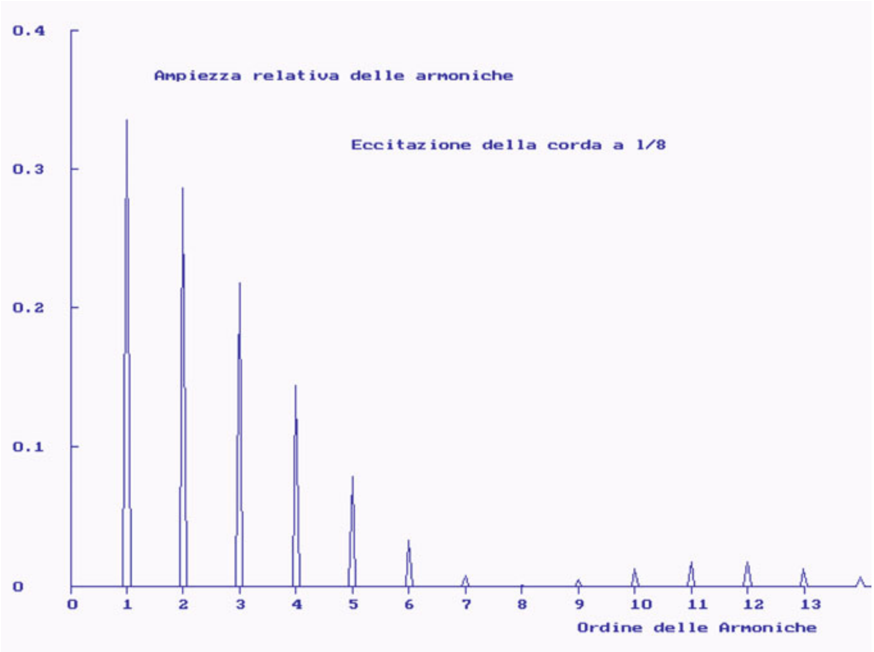
It must be noticed that, if we pluck the string at half its length, the mode 2 will be absent in the string motion recipe. In the same way, the mode 4, the mode 6, and all even modes—since they include a node in the middle of the string length—will be omitted. Please note that, in the fundamental, the amplitude is much larger than in other harmonic elements.

If we pluck the string at 1/4 of its length, the mode 4, the mode 8, and all other modes including a node at 1/4 of the length, will be missing in the motion recipe. Similar considerations apply to a string plucked at 1/8 of the length, or in other positions.

Equally, the amplitude of vibration modes (that is to say the amount of ingredients composing the motion recipe) depends on the point where the string is plucked. With reference to the fundamental, the degree of higher harmonic components increases as the excitation moves towards the bridge; as a result, the sound becomes brighter and more 'aggressive' than the mellow, full sound we obtain when we pluck the string at 1/4 of the length. Last but not least, the significance of excitation at half the length of the string is essentially theoretical as, in fact, it produces a nasal, quite unpleasant sound, due to the dominance of the fundamental mode over the other vibration modes involved in the process.


Ampiezza relativa delle armoniche
Eccitazione della corda a 1/2
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0 1 2 3 4 5 6 7 8 9 10 11 12 13
Ordine delle Armoniche


Ampiezza relativa delle armoniche
Eccitazione della corda a 1/4
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0 1 2 3 4 5 6 7 8 9 10 11 12 13
Ordine delle Armoniche



2.2 The Real String

In the previous paragraph we presented an ideal model of the string behaviour, which helps us describing the main characteristic phenomena of its motion. The real behaviour of the string is different in at least two aspects:

- The motion of the standing waves along the string does not develop in an alternate and endless oscillation between the ends. In the real behaviour, the string motion damps down progressively and—as confirmed by experience—after a certain time the string returns to its position of equilibrium. Damping is due to three different loss mechanisms: internal damping, damping in the surrounding air, and conveyance of energy to the supports.
  Along with mechanical properties (gauge, length, and mass per unit length), the viscoelasticity of the material also characterizes the string. During motion—at microscopic magnitude—the string grows longer and its diameter reduces; then it shortens, while diameter dilates, and so on. These variations exploit the potential elastic energy stored at the moment of excitation.
  This phenomenon causes an *internal damping*, which is independent from gauge, length, or tension. This kind of damping is generally very limited in metal strings, but crucial in gut or nylon strings (especially wire-wound ones). It is commonly recognized that sound damping is slower in metal strings than in nylon ones.

The *decay time* due to this mechanism is shorter if phenomena causing energy losses recur more frequently. Therefore, damping is higher at highest frequencies. In other words, among guitar generated sounds those having higher frequency (fundamental and harmonics included) fade faster than the low frequency ones.

The string alone is a very poor sound generator, and this for two reasons: the surface moving air is very narrow as compared, for instance, to the soundboard surface; furthermore, it produces air compressions and rarefactions that tend to nullify each other. We can recognize this, for example, when we play an electric guitar without audio amplifier.

Nevertheless, the string *cutting* the surrounding air is influenced by a *radiation* resistance, which causes dissipation of the elastic energy, so contributing to the motion damping. The damping due to radiation resistance depends both on the string dimensions (length, gauge) and on frequency, regardless of the material.

- The real string is clamped to two flexible supports: the bridge, fastened to the body (or resonator) of the instrument at one end, the nut at the other end. This two elements—chiefly the resonator—can oscillate under a stimulating force that sets them in motion.

  Ignoring the nut for the moment, let us consider the bridge as part of the resonator: in other terms, we temporarily refer to the system composed by the instrument body and the bridge as the *resonator*, leaving to further discussion the role of the bridge as an autonomous element.

When reaching the resonator and before reversing back towards the nut, the standing wave coming from the nut delivers a force impulse. This impulse sums up in its shape (duration and amplitude) all information contained in the string motion, therefore the frequency and amplitude of the fundamental, as well as information about the whole series of associated harmonics and, accordingly, information about the way and place of plucking. Thus, the force impulse encloses what we called the string motion 'recipe'.

If the resonator were rigid the force would produce no effect, and the wave would be totally reflected. On the contrary, owing to the resonator *mobility*, the wave is only partially reflected towards the nut, the remaining part being 'captured' by the resonator. This phenomenon involves energy transfer from the string to the flexible support, thus contributing to the progressive damping of the string motion.

Because of the above-described loss mechanisms, the effect of the energy impulse on the resonator also drops off as the string motion dies away, until the string recovers its rest position and, consequently, does not influence the resonator any more.

The distinguishing feature of the resonator is the connection between the force applied to the bridge and the velocity of movement of the latter. This relation is called the *resonator impedance*. The term derives from electrical engineering, where impedance defines the relationship between voltage and electric current, and is used in acoustics because of many analogies between electric and acoustic phenomena. The resonator impedance is given by the formula

$$Z_r = \frac{Force\,applied\,on\,the\,Bridge}{Velocity\,at\,the\,Bridge}$$

If, in consequence of a given force, the bridge and the connected resonator move faster, this indicates greater *mobility* of the bridge/resonator system (or greater propensity to oscillate) and, conversely, minor resonator impedance. On the contrary, if velocity is nil for any force value, impedance is immense, which means the support is rigid. In this case, mobility is nil and the force cannot produce any displacement at the bridge. These concepts are summed up in the following scheme.

$$\text{Force from the String} \Rightarrow \text{Resonator Impedance} \Rightarrow \text{Velocity at the Bridge}$$
$$\Rightarrow \text{Mobility}$$

This impedance depends upon a very complex concurrence of factors, namely the physical nature of the resonator elements and their mutual interaction; the corresponding graph is rich in high peaks that alternate with very-low-value areas.

At certain frequencies, where impedance is very low, velocity is higher. As a premise to subsequent reasoning, we call them *resonant frequencies of the resonator*, where the resonator is more 'inclined' to use the energy of the string and to give it off in the air as sound pressure. On the other hand, there are frequencies where the impedance is high and velocity is very low; at these *anti-resonances* the resonator is 'reluctant' to return the string energy as sound pressure.

The string has its own impedance too, defined as the ratio between force (the tension **T**) and velocity **c**, which in this case is the propagation velocity of the standing waves. See the previous paragraph for the formulation of **c**.

The string impedance is

$$Z_c = \frac{Longitudinal\,Tension}{Propagation\,Velocity} = \sqrt{\mu T}$$

This impedance (more properly called the *characteristic impedance* of the string R_c) does not depend on frequency, so it is the same for all of the motion recipe components; it depends instead on the mechanical properties of the string (precisely tension and mass per unit length).

We have now all the elements required to ascertain how much of the wave received by the resonator is reflected, and how much of it is captured and used by the resonator. The *reflection coefficient* represents the reflected part of the wave, in contrast with the incoming wave, and is given by the following formula:

$$\rho = \frac{R_c - Z_r}{R_c + Z_r}$$

The reflection coefficient ρ is frequency-dependent (since the resonator impedance Z_c also depends on frequency). At anti-resonances (where the resonator impedance is high), the reflection coefficient is -1. This means that the incoming

wave completely reverses without absorption by the resonator (the sign—indicates that the incoming wave undergoes a mirror-like reflection and takes the opposite sign). On the contrary, at resonant frequencies (where impedance is low), the incoming wave is mostly absorbed by the resonator, and its energy is returned as sound pressure through oscillation of the resonator surfaces.

In fact, this analysis tells us that the harmonic components of a string motion (depending on where and how the string is plucked) are somehow 'filtered' by the resonator, depending on its impedance at the bridge.

Some components are not thoroughly exploited, because they match some anti-resonance; accordingly, the resonator only absorbs a modest part of the incoming wave and transforms it into sound pressure, while the rest keeps wandering between the bridge and the nut. The resulting sound is weaker, but longer-lived.

Other components, lying close to some resonant frequency, are enhanced. It is not correct to say that the resonator *amplifies* these components, since the concept of 'amplification' implies the contribution of an external source of energy. Here instead the enhancement of the string fundamental (or of some associated harmonics) is due to superior *efficiency* in translating elastic energy into sound pressure at the resonance frequencies. At these frequencies, however, the resonator absorbs most of the incoming wave, and just a little part of it goes back to the nut. Consequently, the sound is louder in the beginning but short-lived (*percussive* sound). In drastic conditions, this phenomenon causes nasty sound response as—for example—the notorious *wolf tone*.

In summary, the string *motion recipe* depends on where and how the string is plucked. Somehow, the resonator filters the harmonic components, and generates a sound pressure whose composition (the *sound recipe* of the instrument) is affected by the resonator impedance, i.e. by its resonances and anti-resonances. As partly observed in Chap. 1, and as we will further discuss in following Chapters, their position crucially contribute to the quality of an instrument.

Chapter 3
Oscillating Systems

Abstract This chapter introduces the reader to the fundamentals of oscillating systems, starting from the simplest one (the pendulum) and discussing the effects of stiffness and mass on natural frequencies. This chapter also presents the analysis of vibrating modes of plates via the Chladni patters and via FEM models. The basic properties of a vibrating plate are discussed considering both the boundary conditions and the elastic (Young's modulus) and mechanical (density) properties. Then, in preparation for next chapters, the effects of arching and bracing on the normal modes of a simple plate are discussed. In last section the resonance phenomenon is deeply investigated, evidencing the parameters which define a resonance (Q-factor, bandwidth, phase, impedance) as well as the transient motion which arises when an external exciting force is applied to a resonant system (which is the normal operation in an instrument like the guitar). The transient response is described in time and frequency domains in terms of spectrum obtained via the Fast Fourier Analysis.

As seen in the previous chapter, the bridge fastens the string to the resonator, which is a flexible support. Because of the resonator impedance, only part of the wave reflects, while the remaining part is 'captured' by the resonator. The portion of elastic energy that the string transfers to the resonator sets in motion its surfaces (soundboard, back, soundhole, sides). These surfaces and their vibration are the basis of the sound producing system.

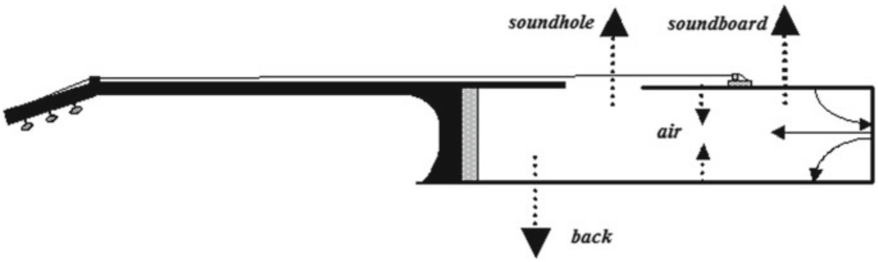
We also observed that the harmonic components of the string motion are 'selected' by the resonator: some components are enhanced (those corresponding to *resonances*), while others are reduced (those corresponding to *antiresonances*).

The guitar resonator fulfils two purposes: on one hand, it turns the energy received by the string into sound pressure, setting its surfaces in motion; on the other hand, it 'filters' relevant harmonic components.

These two functions are related: the position and characteristics of resonances and antiresonances, as well as the efficiency in transforming elastic energy into sound pressure, determine the exclusive and inimitable qualities of an instrument.

The following image exemplifies the resonator functioning phenomena.

G. Cuzzucoli and M. Garrone, *Classical Guitar Design*,
https://doi.org/10.1007/978-3-030-32992-1_3



The force that the string conveys to the resonator via the bridge sets the soundboard into motion. Soundboard oscillations generate sound pressure in the surrounding air and, at the same time, they put both the air in the body and the sides into motion. While expanding through the hole towards the environment, the air contained in the body vibrates the back. The oscillatory motion of the sides also contributes, to some extent, to the backboard vibration. The back in turn engenders sound pressure in the surrounding air, while contributing to oscillate the air in the body. The air that comes out of the hole and flows towards the environment also produces sound pressure.

This simplified scheme shows the three main oscillating surfaces of the guitar resonator: soundboard, back, hole and, to a lesser extent, sides radiate sound in the surrounding air. Taken together, they compose a *Helmholtz Resonator* (see Chap. 4). Other oscillating systems, like the sides and the neck, have a minor influence on the functioning. Differently from the violin, the bridge is not an oscillating system on its own; it is a rather stiff element, whose geometrical features heavily affect the properties of the soundboard.

The reader should not be surprised that the hole, much like the soundboard or the back, is considered as a radiant surface. If we cover the hole with cardboard, the sound becomes dull and weak; this highlights the crucial function of the hole in the overall sound emission of the instrument. But differently from the soundboard and the back, the air flowing through the hole encounters more and more resistance as the oscillation frequency grows. Above a frequency limit (about 300 Hz) which corresponds to the notes on the first string, any communication between the interior of the body and the surrounding environment is interrupted, and the air flow stops. Being the hole essentially rigid, it does not convey sounds, so only the soundboard and the back go on generating sound pressure.

The surrounding air opposes a *radiating resistance* to the swift pressure variations here taking place, contributing to progressive damping in the oscillation of the surfaces. This explains how the resonator, through its vibrating surfaces, fulfils its first function by transforming energy into sound pressure.

The *resonator response* depends both on the individual characteristics of its oscillating systems and their interaction (or *coupling*). The concept of coupling implies that two or more distinct oscillators, when combined, give rise to a new oscillating system whose response depends both on the characteristics of the original oscillators and on their interaction. Therefore the resonator response—or the property of filtering and selecting the harmonic components from the force received by the string—is determined by this mechanism of interaction between oscillating systems.

During construction, the luthier can choose and modify both the properties of the oscillating systems and their coupling. To understand how, we first need to examine some of the main acoustic qualities in these systems. In this chapter we will study the oscillating systems, from the simplest (the pendulum) to more complex ones (plates).

In the next chapter you will be shown how the functioning of fundamental oscillating systems affects the properties of the surfaces that define the resonator (back and soundboard) as well as the interaction between each other and with the air contained in the body.

3.1 Natural Frequencies in Oscillating Systems

All objects have a degree of elasticity. A sudden excitation (the beating of a drumhead, the plucking of a string, the blowing in a mouthpiece, etc.) makes them oscillate with one or more distinctive *natural frequencies*.

The simplest oscillating system is the *pendulum*, composed of a non-extendable thread with length **l** and point mass **m** fixed at one end. When we displace the mass from its equilibrium position, and then release it, it starts oscillating with a certain period; then amplitude diminishes until the mass returns to its rest position. The *oscillation period* **T** depends on the length of the thread **l** and on the gravitational acceleration **g**. This can be written as

$$T = 2\pi\sqrt{\frac{l}{g}}$$

The pendulum accomplishes **f** oscillations per second. This is the *natural frequency* of the pendulum, defined (in Hz) as

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

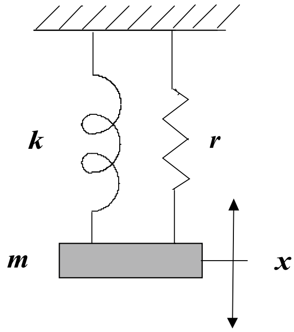
The pendulum has only one oscillation mode, therefore only one natural frequency, which depends on the geometrical features of the vibrating system (in this case the length of the thread).

The motion of the pendulum is *harmonic*: if we report the position of the mass on a paper roll moving at constant speed we obtain a perfect sinusoid. In other words, the displacement of the mass follows a *sinusoidal* law with respect to time (see Chap. 1).

All oscillating systems encounter opposing influences that absorb energy, causing gradual extinction of the amplitude. In the pendulum, *aerodynamic resistance* opposes the movement of the mass. In the tuning fork, a fraction of the oscillatory energy produces sound pressure and compensates *radiation resistance*; another share compensates internal viscous losses (greater losses correspond to higher speed of the prongs).

Another interesting kind of pendulum is composed of a mass **m** hanged to a spring with stiffness **k**. The spring-mass system is not only significant as a basic pattern for

the functioning of many mechanisms (as, for example, the suspensions of a car) but also for vibration modes of the complex acoustic systems we are concerned with, since these can be seen as groups of spring-mass systems.



The parameter **k** defines the degree of lengthening for the free end of the spring, when undergoing a longitudinal pulling force **F**. The lengthening is $x = F/k$, so the stiffer the string the shorter the lengthening. Apart from intrinsic properties of the spring, **k** depends on a series of constructing parameters, like attributes of the substance and dimensions.

The *oscillation period* in this kind of pendulum only depends on the mass and stiffness of the string. Like the thread-mass pendulum, the spring-mass system has only one oscillation mode, so only one natural frequency. This frequency is the inverse of the oscillation period and is defined by

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

This pendulum also shows *harmonic, sinusoidal* motion whose decay depends on losses brought about by viscous losses and radiation resistance. The parameter **r** is a general coefficient representing energy losses in this vibrating system.

It must be noticed that natural frequency does not depend on losses; this means that the system always oscillates at natural frequency, in spite of *damping*.

As already discussed in Chap. 2.1, the string motion can be split into infinite elementary, sinusoidal harmonic motions, or vibration modes, corresponding to the string *harmonic frequencies*, which are multiples of the tuned fundamental frequency. The amplitude of the fundamental mode (corresponding to the fundamental frequency) is only nil at the string ends, while oscillating amplitudes of upper modes are nullified at specific positions along the string. These no-displacement points are called *vibration nodes*, whereas the points of maximum amplitude are called *antinodes*. The notion of nodes and antinodes in oscillating systems is very important, and will often recur in this text for practical purposes.

The string, like the pendulum, encounters different kinds of inner and outer resistance, which absorb energy and cause progressive extinction of the amplitude. In

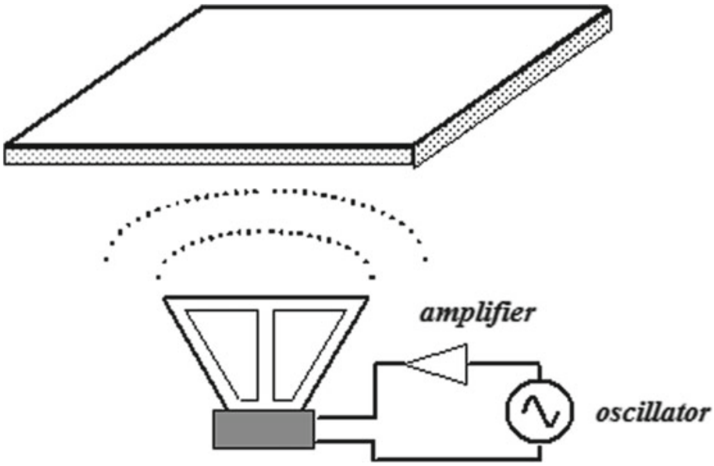
addition to aerodynamic and radiation resistance, amplitude is influenced by inner damping due to *viscous losses* in the substance.

This is the main difference between pendulum and string: the pendulum is a *one-dimensional* oscillator, whose oscillating properties are defined by the motion of a single point; the string, on the other hand, is a *two-dimensional* oscillator, with infinite points moving along its length. So the string can be seen as a line of countless pendulums vibrating each in a different way, though co-ordinated with each other.

Naturally, *three-dimensional* oscillating systems also exist. Such are *plates*, being objects with a dimension, thickness, much smaller than the other two: width and length. The boards of the guitar (back and soundboard) are comparable to plates. To examine both the natural frequencies and the oscillation modes of the plates we perform the following test, where the *Chladni pattern* method is applied. Later on we will see how to apply this method of analysis to the study and optimization of the soundboard and the back of the guitar. For the moment it is enough to observe that:

- A rectangular tablet measuring 300 × 200 × 2.5 mm, with edges lying on a foam rubber support, is mechanically excited by a sinusoidal signal produced through an adjustable frequency generator; then the signal is amplified and transmitted to a loudspeaker that generates acoustic pressure in the surrounding air.
- A thin layer of suitably sized sand is scattered on the tablet.

The test is sketched in the following figure.

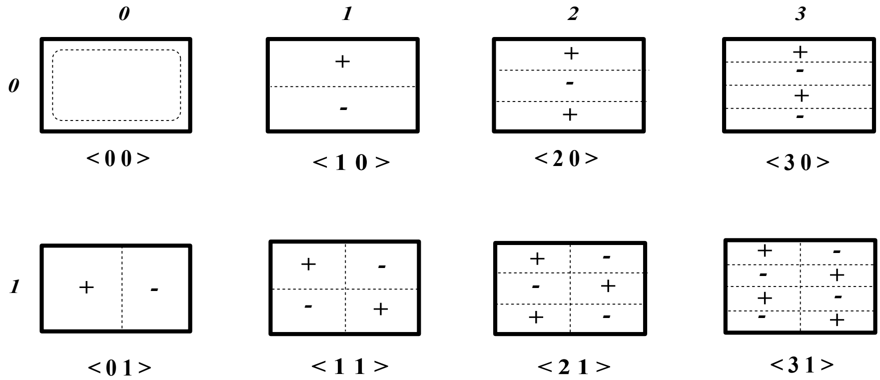


If we lay a finger on the plate when the oscillator is functioning, we perceive its vibration; the acoustic pressure wave sets the tablet into oscillation.

When, starting from low values, we progressively increase the signal frequency, on reaching the value corresponding to the *first natural frequency* of the plate, the sand particles start stirring; from maximum vibration points, the sand leans towards nil vibration points and collects in regular forms called *nodal lines*. By further increasing frequency, new nodal lines corresponding to different vibration modes will highlight the related natural frequencies.

In this example the plate rests on the edge, whose vibration is inhibited by the support. An important nodal line lies along the perimeter and defines the so called *ring mode*. This is vital in lutherie, chiefly because the ring mode encircles the most mobile area of the board. This way we determine the first natural frequency, or the *fundamental frequency* of the board.

Subsequent nodal lines develop along the longitudinal and transverse axes. So each vibration mode is associated with a number of lines stretching parallel in two directions. As a consequence, we need two indicators to define plate vibration modes instead of only one as in the string case (mode 1, mode 2 etc.). According to the standard we adopted, the first indicator refers to the number of nodal lines lying along the longer axis, and the second to the number of those lying on the shorter one. Vibration modes are therefore named as *mode* ⟨0 0⟩, *mode* ⟨0 1⟩, *mode* ⟨m n⟩. Consequently, *mode* ⟨0 0⟩ is assigned to the nodal line of the ring mode, which runs along the border of the plate. We will use the same system to identify vibration modes of the back and soundboard of the guitar. The following image illustrates the arrangement of nodal lines and the relation between their position and vibration modes.



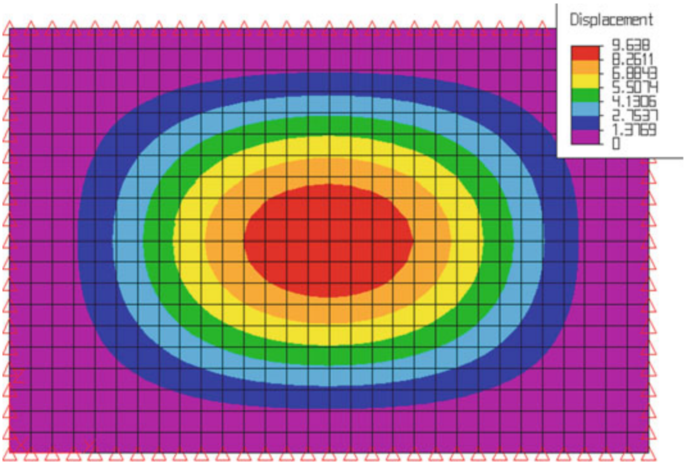
There are many outstanding analogies between *string* and *plate* as oscillators. As earlier stated, at any instant the displacement of each point of the string from its rest position is the sum of the displacements due to the different vibration modes. This is also true for plates, differing in the distribution of the points (over a surface instead of a line).

But what is the meaning of a plate vibration mode, for instance mode ⟨0 0⟩? The following figure helps understanding the development of motion in a fastened plate. The figure illustrates a simulated oscillation of the plate in mode ⟨0 0⟩, carried out by means of the *Finite Elements Method* (or FEM). This method will also turn useful in later observations. The analysis is performed by means of software applications that firstly split the examined structure into elementary elements, then calculate the forces conveyed between them under the action of an external excitation and, finally, gather all deformations occurred in the structure.

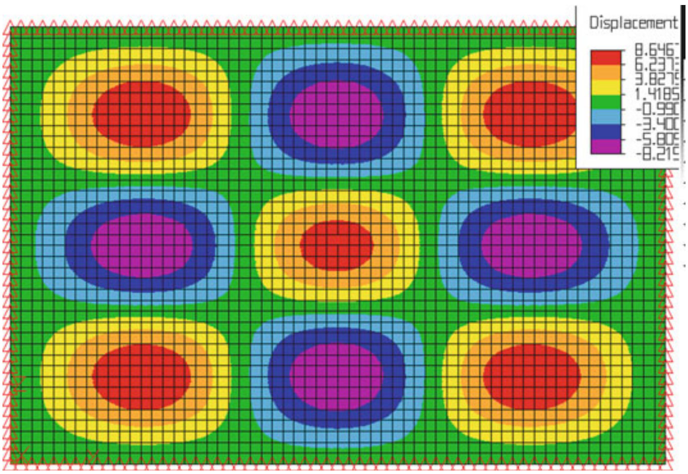
The analysis shows that the oscillation amplitude of the plate in mode ⟨0 0⟩ is nil along the edge and maximum at the centre. Amplitude decreases from the centre

towards the perimeter, running along lines that cover most of the plate surface—forming a sort of *bell* base—and define the maximum amplitude region (or *antinodal area*). Colours define bands where the amplitude is comprised between limit values specified in the figure inscription.

In the antinodal region, points alternatively oscillate up and down with respect to the equilibrium position of the plate, at the same frequency as the fundamental mode of the plate itself.



The situation complicates if we consider a higher mode, which displays more lines both in length and width. The illustration below depicts the plate motion in mode ⟨2 2⟩, as simulated by means of the FEM model.



Here we have nine ‘bells’ (antinodal areas), each one facing two nodal lines in the longitudinal and two in the transverse direction, in addition to the peripheral line where, because of the support, the plate cannot vibrate.

Most interestingly, each antinodal area vibrates in antiphase with regard to all adjacent ones. For example, the points in the central area (where the longitudinal and transverse axes intersect) move upwards from the rest position of the plate while, at the same time, the points in the four adjoining areas move downwards. The same chequer-like arrangement of oscillation phases recurs here as in the previous diagram, where phases in the quadrants are marked with $+$ and $-$. Positive phases produce air compression, negative phases produce rarefaction. At some distance from the plate, compressions and rarefactions mutually nullify; it follows that this, as in symmetrical modes in general, produces very low sound radiation. The guitar soundboard as well, being comparable to a plate, has some poorly radiating symmetrical modes. The luthier can adjust table structure features like thickness, bracing, and others, to keep these modes asymmetrical and so to enhance the radiation efficiency.

In the above-illustrated case, antinodal areas seem to be separate and independent from one another, but are actually connected, as they share the same vibration mode. If we modify the upper left area (for instance by adding a little mass in the centre) all other areas belonging to that mode are also modified. The main consequence is that the mode loses its structure regularity, and frequency too can diverge, but does not switch to other modes.

Another significant analogy exists between the plate and the string. As earlier stated, vibration modes presenting a node in the plucking point are absent in the motion recipe of the string. The same concept applies to the plate: *when the plate is excited in correspondence with the nodal line of one o more vibration modes, these are not activated, and the related frequencies do not affect the oscillatory motion of the plate. So these frequencies are not included in the sound spectrum of the plate.*

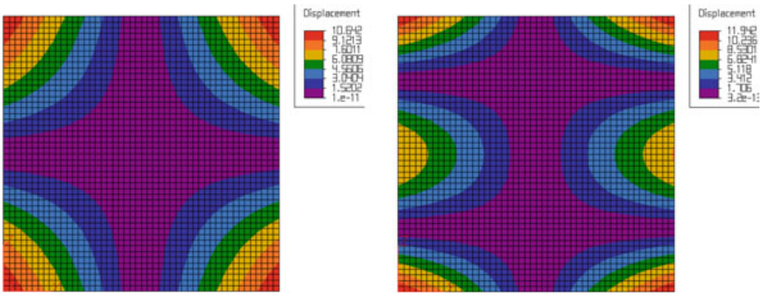
In the two previous figures, mode $\langle 0\ 0 \rangle$ is properly excited when the force acts in the centre of the antinodal area, and is not activated if the force operates on the perimeter (the nodal line); mode $\langle 2\ 2 \rangle$ is suitably excited when the force acts in the centre of any antinodal area, and not excited outside those areas.

Let us consider the practical meaning of this concept. The string excites the guitar soundboard via the bridge, which is located in an antinodal area of mode $\langle 0\ 0 \rangle$ that, as a consequence, reacts in a suitable way. Other modes, whose nodal line runs along the bridge (as, for instance, mode $\langle 0\ 1 \rangle$) are weakened. A skilled luthier modifies board structure elements (bracing, bridge, etc.) so as to obtain the best response from all of the key modes. Advice on how to reach this goal will be provided later on. For the moment we are content to point out the crucial role played by the bracing, because the braces connect the bridge area with the whole board and, in a sense, they spread the excitation from the centre to the periphery of the board.

We have been assuming so far a plate with simply supported edges. Actually there are different boundary conditions to which a plate can be subjected.

- **Clamped plate**: the plate is inserted into the supporting frame, which prevents any horizontal movement or rotation of the border. On the contrary, the simply supported edge observed above allows rotation along the border. Vibration modes are similar in a clamped plate but—intuitively—as the bond increases the stiffness of the plate, this last is compelled to vibrate at higher frequencies.

- The situation in the guitar soundboard is a compromise between the two mentioned above. The elasticity of the sides allows a rotation of the soundboard, whose extent depends on the features of the sides (solid/laminated), of the linings (kerfed/unbroken) and of the side reinforcing brackets. This issue will be discussed later on. We just point out here that the connection between sides and soundboard, apart from the soundboard motion, also influences the force that the soundboard conveys to the back, hence the vibration modes of both these plates.
- **Free plate**: in this condition there is no bond along the edge; as a consequence, the edge is not a nodal line, and the shape of the vibration modes is very different. The figure shows two vibration modes in a free plate.



The case of the free table is relevant in lutherie only for a traditional test: the luthier holds the board with two fingers in a nodal point along the edge, and beats with the knuckles an antinodal point to hear the timbre (or timbres). By the resulting sounds, called *tap tones*, the luthier estimates the board response. This method will be commented afterwards.

3.1.1 Modal Frequencies

Vibration modes represent sinusoidal oscillations of the antinodal areas in a plate, whose movements develop around nodal lines where amplitude is nil. The typical frequency of each vibration mode is called *modal frequency*.

A formula allows careful calculation of the modal frequency in all modes for the simply supported plate. This formula includes

- The *geometrical dimensions* of the plate (length, width, thickness).
- The *elastic properties* of the material.
- The *density* ρ, or the *specific weight* of the substance, expressed in Kg/m^3.

The most important elastic feature in a material is the *elastic modulus* (or *Young's modulus*) which is crucial in defining the modal frequencies.

The elastic modulus somehow resembles the above-mentioned parameter **k** that indicates the stiffness of a spring. The definition of elastic modulus answers the question: if we apply a force **F** to a surface **A** of the material under examination,

what is the lengthening $\boldsymbol{\Delta l}$ of the material compared to its initial length **l** (before the force was applied)? By definition the elastic modulus **E** (or Young's modulus) expressed in Newton per square meter (N/m^2) is

$$E = \frac{F/A}{\Delta l/l}\,[\mathrm{N/m^2}]$$

The material will be now considered, supposing a plate made of wood (specifically spruce, as for many guitar soundboards). Wood is an *orthotropic* material: elastic properties manifested along the grain are not the same as across (differently from *isotropic* materials like, for instance, aluminium). So we basically need two elastic constants: E_x along the grain, E_y in the transverse sense.

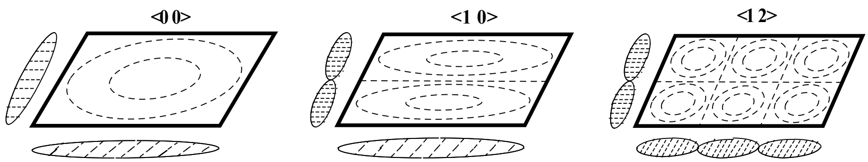
Let us examine an orthotropic plate made of Engelmann spruce measuring 300 × 200 × 2.5 mm with lengthwise grain. Average values for this kind of wood are

- $E_x = 9859500000$ (or 9.8×10^9) N/m^2
- $E_y = 1261700000$ (or 1.2×10^9) N/m^2
- $\rho = 387\,\mathrm{Kg/m^3}$.

Actual assessment of elastic parameters in a rough table for a guitar soundboard will be shown afterwards. According to the formula, the first seven modal frequencies are associated with the values listed below (extracted from a database reporting typical parameters of different timbers).

Mode	Modal frequency (Hz)
⟨0 0⟩	111
⟨0 1⟩	263
⟨1 0⟩	302
⟨1 1⟩	444
⟨0 2⟩	521
⟨2 0⟩	623
⟨1 2⟩	694

Interesting in this listing is that modal frequencies following the first are not integer multiples of the plate natural frequency associated with mode ⟨0 0⟩. This is worth the notice as differing from the string behaviour, where each mode frequency is in constant harmonic ratio to the fundamental. In fact the plate, as shown below, works like a *texture* of intersecting strings.



The figure shows the arrangement of nodal lines on the plate in three different vibration modes. Beside are sketched the longitudinal and transverse 'strings' of vibration. Antinodal areas, where the vibration amplitude is greatest, are enclosed by nodal lines. The modal frequencies of the *longitudinal strings* develop with constant ratio to a given fundamental frequency, but differently from the fundamental and the harmonics of the *transverse strings*. At particular frequencies, longitudinal and transverse 'strings' intersect and form the nodal lines, but no harmonic relation exists between these frequencies. In other terms, *no constant relation exists between the modal frequencies of the plate*. We will see that this is exactly the case of the guitar soundboard. When, for example, through the methods described afterwards, the main frequency of a soundboard in mode $\langle 0\ 0 \rangle$ is established at 140 Hz, we will not necessarily find modal frequencies at 280 Hz (2×140), at 420 Hz (3×140) and so on.

The above-mentioned formula, since it includes parameters applicable to the whole plate (elastic constants, density and dimensions) provides a global solution for the calculation of the modal frequencies of the plate. Another interpretation is also possible, comparing each vibration mode of the plate with the oscillation of a mass-spring pendulum. This way the modal frequency associated with each vibration mode depends on modal mass and modal stiffness, with the same relation seen for the mass-spring system:

$$f_{\text{mod}} = \frac{1}{2\pi}\sqrt{\frac{k_{\text{mod}}}{m_{\text{mod}}}}$$

Specific parameters for each vibration mode are applied to this method. In contrast with the previous one, we might call this a *local* solution. The whole stored and returned energy (losses excluded) is the sum of the energy of each single mode composing the plate motion.

Except from simple plates and mode $\langle 0\ 0 \rangle$ of a guitar soundboard, defining modal mass and modal stiffness of an irregular table is generally a complex duty. However, this interpretation explains how the luthier can control one or more vibration modes of an acoustic system (like the soundboard) by acting on stiffness or modal masses. Two ways to do that are adding struts and arching the boards. An overall review should be sufficient for the moment, leaving practical application to the second part of the book.

3.1.2 Adding a Longitudinal Wood Strip

Let us suppose gluing a longitudinal strip to the plate, both made of spruce and both with length l (300 mm), and suppose the strip has height h, thickness s and its longitudinal grains run parallel to the grain of the plate.

This is just to highlight variations brought about by the strip, which are partly similar to the effect of bracings that, as we will see, are used to enhance the stiffness of the soundboard and to select particular vibration modes.

The strip as a standalone element behaves as the above-examined plate. In the fundamental vibration mode ⟨0 0⟩ its natural frequency depends on modal stiffness and modal mass which, by a local examination, is expressed as

$$f_{strip} = \frac{1}{2\pi}\sqrt{\frac{K_{strip}}{m_{strip}}}$$

Regardless of detailed formulation we observe that, given length l and thickness s,

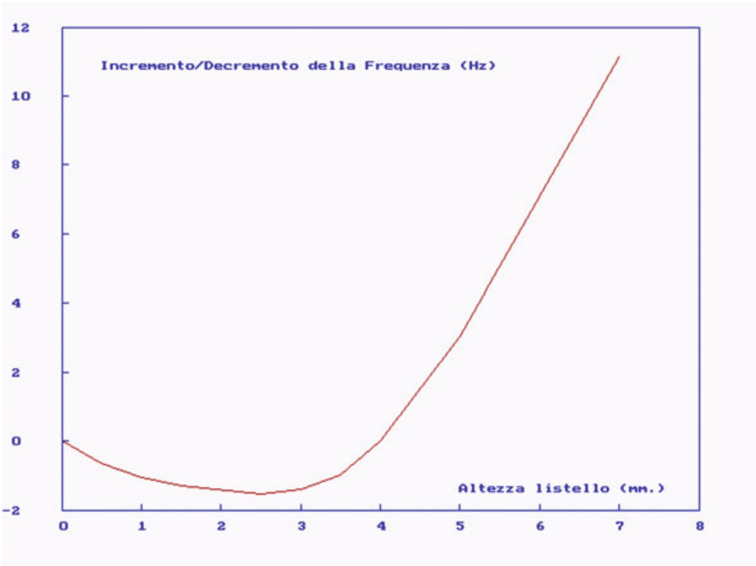
- The strip mass is proportional to height h and thickness s
- Stiffness is proportional to the *cube* of height ($h \times h \times h$, or h^3).

This outcome has a significant practical meaning, i.e. stiffness depends above all on the *height of the strip*. When the strip is fastened to the plate, these two respond with the same oscillation velocity to an external force, and their relative stiffness and mass add together.

The modal frequency in mode ⟨0 0⟩ of this combined oscillator is

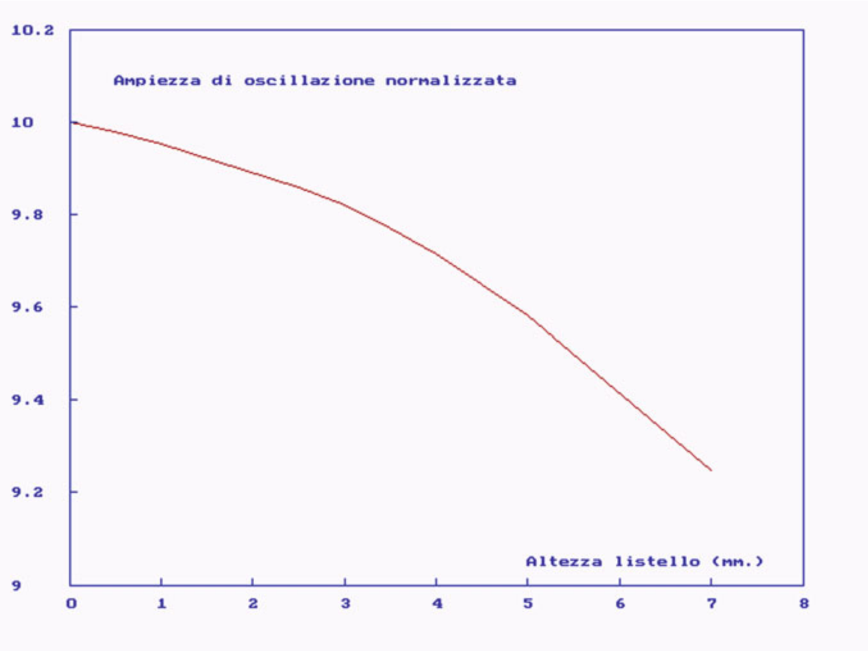
$$f_{plate+strip} = \frac{1}{2\pi}\sqrt{\frac{K_{plate} + K_{strip}}{m_{plate} + m_{strip}}}$$

Modifying the height of the strip, mass and stiffness change. The following figure represents the outcome of FEM analysis for mode ⟨0 0⟩. This test was performed with strip height spanning from 0 to 7 mm. The vertical axis reports positive and negative development of the modal frequency with respect to the frequency of the plate alone, when no strip is added.



With small height, the mass of the strip prevails and the modal frequency diminishes, whereas with adequate height, stiffness prevails and frequency increases.

The following graph reports normalized values of amplitude in mode ⟨0 0⟩ versus the strip height, in a conventional scale referred to the case when the strip is missing. Amplitude decreases because of losses brought about by the strip. These losses are due to the material of the plate and the strip as well as to the vibrating mass, this last increasing with the strip height.



So, for a proper adjustment of the frequency in mode ⟨0 0⟩, the thickness *s* of the strips must be reduced—compatibly with construction requirements—in order to diminish the weight of the bracing and, consequently, losses in the vibrating mass.

Once the thickness has been established, the plate frequency can be regulated by modifying its stiffness, hence the height of the strips. From the foregoing diagram we understand that, beyond a certain limit, variations are problematic since little differences in the height of the strip cause considerable changes in frequency. We will review practical implications later on.

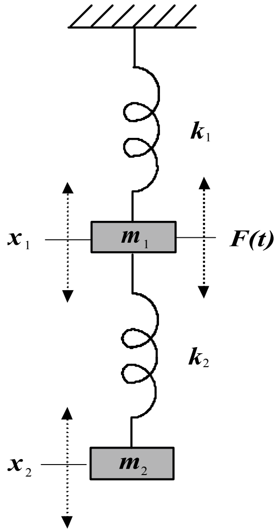
As for upper vibration modes of the plate, the influence of the strip is more complex. By testing we observed that the strip does not behave like a body firmly joined to the plate but undergoes individual elastic movements inside its own structure. To understand this phenomenon, imagine cutting the strip into *layers parallel to the gluing surface* that independently slide over the adjoining ones in the longitudinal sense; at the same time, imagine cutting the strip into *layers perpendicular to the gluing surface* and also capable of sliding over one another.

In mode ⟨0 0⟩ the strip behaves as part of the whole, its mass and stiffness adding up to the mass and stiffness of the plate whose modal frequency consequently diverges. On the other hand, in upper modes the force conveyed by the plate into the strip through their interface causes a sliding movement of the parallel and perpendicular layers of the strip, which in turn modifies the behaviour of the plate oscillation.

This is possible because of the substance of the strip – wood—which is an elastic material.

To define this combined vibration we need a more complex model than the simple mass-spring oscillator applicable to mode ⟨0 0⟩, which involves a model based on the concept of coupled oscillators. This model illustrates the interaction between two oscillating systems that exchange part of their vibrating energy to form a new and different oscillating system, whose response depends both on the characteristic of the original oscillators and on their interaction.

The following is a model of the coupling between table and strip.

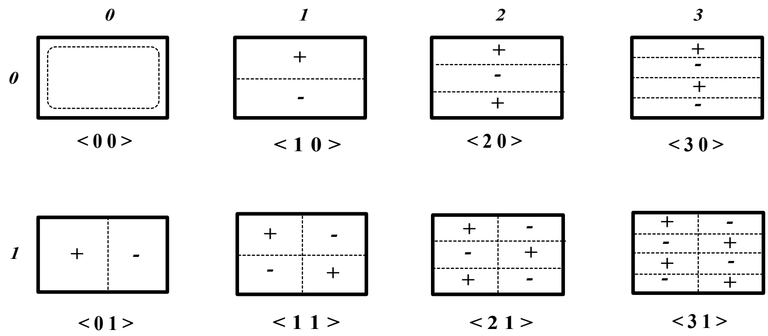


Here the plate is defined by mass $\mathbf{m_1}$ and stiffness $\mathbf{k_1}$, the strip by mass $\mathbf{m_2}$ and elastic components $\mathbf{k_2}$. Apart from *Young's modulus* (or *modulus of elasticity*), the sketched vibration phenomenon also depends on the *shear modulus* and *Poisson's modulus* (both longitudinal and transverse to the grain of the timber). Please refer to specialized publications for detailed description.

The system composed by plate and longitudinal strip behaves in a very complex way and is most interesting for practical purposes. This can be better understood by observing both the previous vibration scheme and the one that follows.

- The strip crossing the centre of an antinodal area (in mode ⟨0 0⟩ or mode ⟨2 0⟩) behaves as an added element with its own mass and stiffness, engendering the same effect described for the simple mass-stiffness model: *with low height mass*

prevails and frequency diminishes, while raising the strip height stiffness prevails and frequency increases.



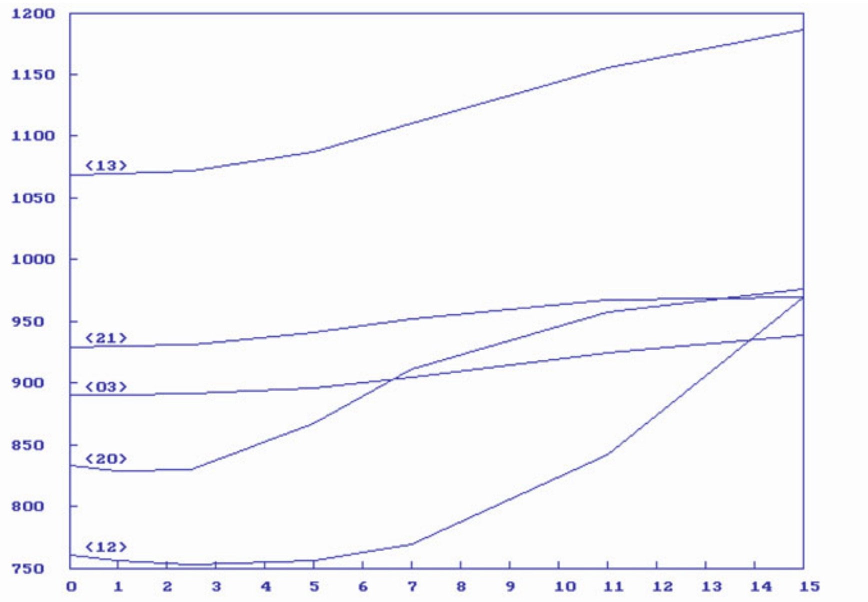
- In addition to mass and stiffness, the influence of a strip that runs along the nodal line of a plate vibration mode (e.g. modes ⟨1 0⟩ and ⟨3 0⟩) depends on the particular movement of the strip sides (upper and lower with respect to the nodal line). The strip behaves as if it were cut along the nodal line and only its longitudinal axis were at rest, while its two faces move in antiphase with each other. The strip opposes the movement received by the plate and so the stiffness, as well as the frequency of the involved vibration modes, increases. Increase is directly proportional to the strip height.
- If the strip crosses a vertical nodal line (as in mode ⟨0 1⟩) the antinodal areas compel the strip to bend along its longitudinal axis according to the plate motion. For instance in mode ⟨0 1⟩ the right end of the strip moves in antiphase to the left end, while the centre is still: the strip undergoes a longitudinal bending. Once again the strip opposes the deformation enforced by the plate with increased stiffness, hence with increased frequency of the vibration modes. Once again, the greater the height of the strip, the greater the increase.
  A strip crossing two vertical nodal lines (as in mode ⟨0 2⟩ which is not present in this figure) would undergo a much greater bending, because three sections would be set into a 'chequer-like' vibration by the plate, while two points would stay still. So, *the greater the number of vertical nodal lines that the strip meets, the greater its relative stiffness.*
- Normally, two or more of the mentioned situations co-exist. For instance, the strip runs along a nodal line or through the centre of an antinodal line, while crossing several vertical nodal lines. Then the overall effect is a mixture of the ones described above.

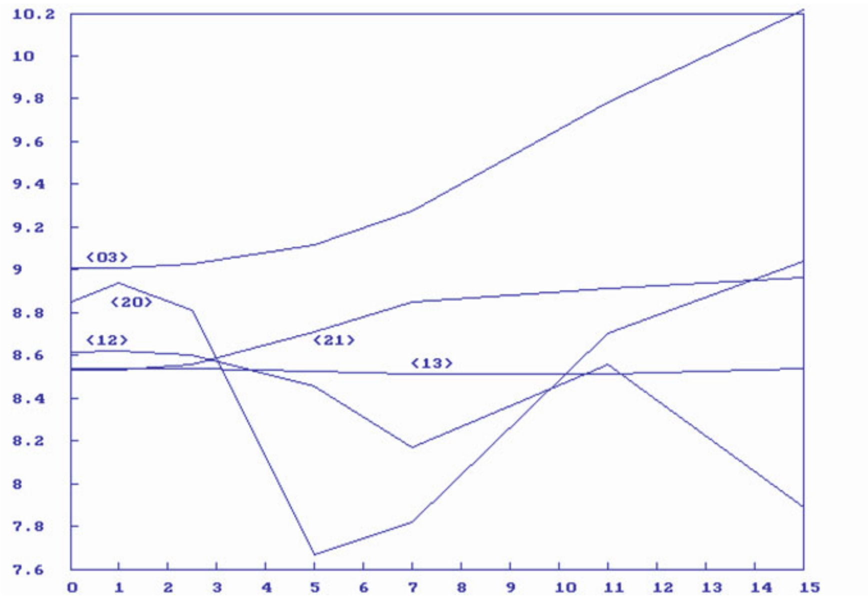
One would expect that the longitudinal strip also affects the vibration amplitude of the plate in its different modes of vibration. Normally, as seen in mode ⟨0 0⟩, the plate vibration amplitude decreases when the height of the strip increases. The decrease in amplitude clearly depends on two factors: losses due to the properties of the materials out of which the plate and the strip are made; amount of the vibrating mass, which is proportional to the strip height.

But sometimes, at certain modal frequencies (or vibration modes) a particular height brings the plate-strip system into resonance, causing

- *Increase in oscillating amplitude* of that mode.
- *Inversion in the oscillation phases* of that mode. Vibrating areas that, before the resonance, moved in a certain direction, after the resonance move in the opposite direction. So, with reference to the previous vibration scheme, areas that were marked with + take the mark −, and vice versa). As we will see subsequently in this chapter, the inversion of phase in the oscillation amplitude is a significant indicator of the resonance.

The following graphs present the frequency (above) and the amplitude (below) of some vibration modes versus the strip height (expressed in arbitrary units).





We tested different strip styles (e.g. vertical, oblique, more strips). These situations can be studied applying the FEM analysis to the structure. Phenomena taking place in upper vibration modes cannot be explained by the simple mass-stiffness model ruling mode ⟨0 0⟩. To examine them we supposed that the strip does not behave as an element firmly joined to the plate, but as an element undergoing individual elastic movements inside its structure. These movements are provoked by the force that the plate transmits to the strip through the interface of the contact surface; this brings about a reciprocal sliding movement of the parallel and perpendicular layers of the strip. These movements (made possible by the elasticity of the strip) in turn reflect onto the plate and modify its frequencies as well as its modal amplitudes. The phenomenon we defined as 'co-vibration' of plate and strip can be understood on the base of the 'coupled oscillators' model, which will be comprehensively discussed in Chap. 5.

The same phenomena come about in much more complex ways between soundboard and bracing. These cases can also be studied through FEM analysis, as will be exemplified later on. The plate-strip model, though very simple, can give the luthier very useful information and help him understand the mechanisms by which the wood strips composing the bracing affect the soundboard vibration modes.

3.1.3 Arching the Table

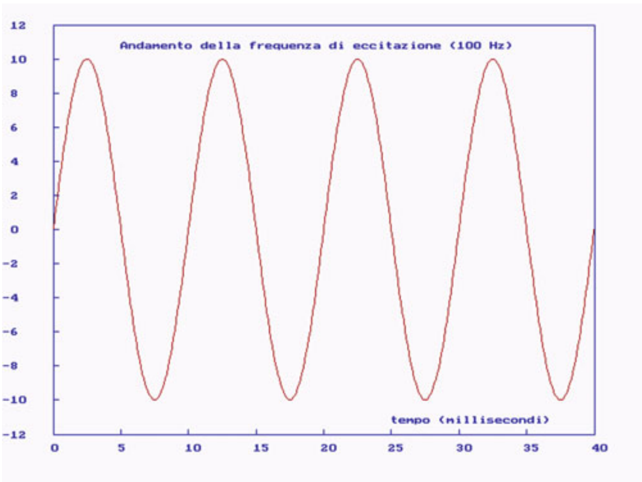
The back and the soundboard of the guitar often have an arched shape, obtained through suitable profile of transverse bars and, sometimes, of sides. A simple test

explains the resulting effect. If we bow a playing card with two fingers and then push the centre, the fingers tend to separate, but if we firmly hold the bowed card we feel a much stronger opposition to the pushing force. This means that the arch diverts the perpendicular forces onto the supporting surfaces and, if the supports are rigid, stiffness also increases on the arched surface opposing deformation.

Tests performed on our plate measuring 300 × 200 × 2.5 mm when rigidly fastened on the longer sides and longitudinally shaped with a 5 mm high arch confirmed that the natural frequency can increase up to 100%. As a result, desired resonant frequencies can be obtained with lesser thickness of board and bracing, hence with reduced mass. Obviously, alternative measures are necessary to grant the board static stability against the pulling force of the strings. We will review this issue afterwards.

3.2 The Resonance Phenomenon

Suppose a mass-spring pendulum undergoing a force whose amplitude periodically fluctuates according to the sinusoidal law shown in the following diagram, where the frequency $\mathbf{f_{ext}}$ (external frequency) is 100 Hz and the arbitrary amplitude value is 10. We can see that positive peaks succeed with a period $\mathbf{T = 1/f_{ext}}$ (10 ms in this case).



Suppose the system has a 200 Hz natural frequency resulting from the combination between its mass and stiffness.

If we apply to the mass-spring system a force whose frequency is $\mathbf{f_{ext}} = 100$ Hz, *after an initial transient period* the system starts oscillating at 100 Hz, regardless of its 200 Hz natural frequency. In fact, any excitation frequency causes a *forced response* of the system at the same frequency value.

Nevertheless, the peak of the force is generally not synchronized with the peak of response. The time between two peaks is usually defined as *phase difference* (between excitation and response). If excitation and response are simultaneous the phase difference is nil; if the response peak occurs when excitation is nil, the phase difference is 180°.

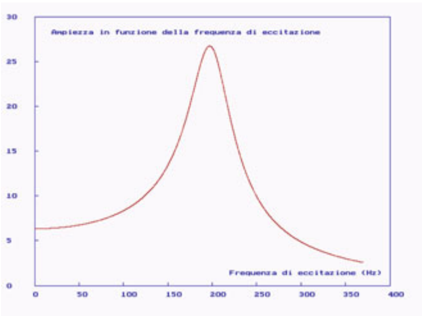
So the response of the mass-spring system is represented by both the *peak amplitude* and the phase difference (with respect to the applied force).

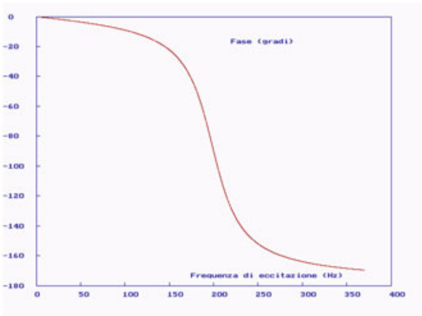
Now we excite the system with sinusoidal forces at different frequencies, spanning from much lower to much higher values than the natural frequency of the mass-spring system. Amplitudes and phase differences are always measured *after extinction of the initial transient period*. The following left diagram reports amplitude responses versus excitation frequencies, the right diagram the phase difference between excitation and response.

The excitation frequency with maximum response is the *resonant frequency* of the system. Differently from what we may expect, the resonant frequency is not equal to the natural frequency of the system but slightly inferior, owing to viscous losses and radiation resistance gathered in the generic parameter **r**.

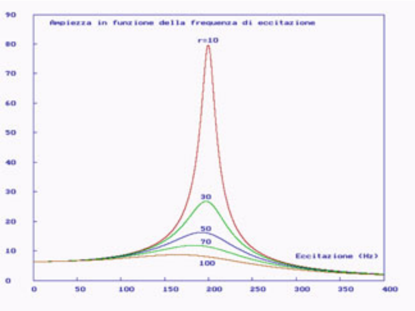
In general, losses are fortunately limited in guitar tables, so we can assume that the resonant frequency matches the natural frequency.

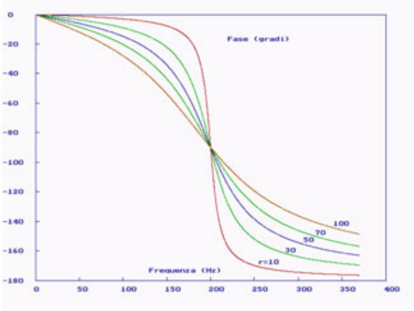
On the right panel the phase difference at natural frequency is exactly 90°: *at resonant frequency the phase response reverses.*





Losses do not affect the resonant frequency but deeply influence amplitude and phase response. The following figures show amplitudes and phase differences in resonance versus the loss coefficient **r**.





The resonance amplitude is inversely proportional to losses. If losses were absent, amplitude would be infinite, and so the *gain in resonance*.

Great resonance amplitudes are represented by pointed curves, where the sharper the point the faster the amplitude decrease. This characteristic is best described by the −3 dB *bandwidth* (see Chap. 1 for definition of dB), corresponding to the interval between two frequencies f_1 and f_2, where amplitude decreases to 0.707 (or 50% of the power) of the value in resonance. The equation for the bandwidth is

$$BW = f_2 - f_1 = \Delta f$$

The bandwidth is the range of frequency where gain is high and close to that in resonance. Out of the band, gain rapidly drops. This way, the frequency development of a system can be included in concise data. As an example, from the −3 dB bandwidth of an audio amplifier (for instance 20 Hz to 20 kHz) we know that the midpoint value of the amplitude response decreases by 3 dB in correspondence with the ends of the band (and power response reduces by 50%).

The resonance amplitude is greater if losses are low, while the bandwidth is larger with high losses (as shown in the figure). Therefore we need a satisfactory compromise—depending once again on the loss coefficient r—between good gain and large bandwidth. The quality factor Q sums up these two aspects of the resonance phenomenon, linking the resonant frequency to the bandwidth as:

$$Q = \frac{f_{resonance}}{BW}$$

The Q factor can be related to both the mechanical parameters of the system and to gain at natural frequency by the equation

$$Q = resonance\,Gain = \frac{Amplitude\,at\,natural\,frequency}{Amplitude\,at\,zero\,frequency} = 2\pi\, f_{natural}\,\frac{m}{r}$$

This implies that, upon certain conditions, a definite frequency can bring a simple oscillator into resonance, where the *acoustic parameters* of the response (amplitude, phase, bandwidth and **Q** factor) are linked to the *mechanical parameters* of the

system (mass **m**, stiffness **k**, loss coefficient **r**). This also applies to more complex oscillators like plates or guitar tables, where any vibration mode can be excited into resonance by a *frequency* at which its *impedance* is equal to zero.

As seen in Chap. 2, impedance Z defines the relation between force applied and vibration velocity of the oscillating system.

$$Z = \frac{Force\,applied}{Vibration\,velocity}$$

Low or zero impedance implies greater 'propensity' (or *mobility)* of the system to oscillate under an external impulse. If instead the displacement velocity is low for any force value, impedance is immensely great; the force cannot produce any significant oscillation of the system and mobility is nil.

This is true *after the attack transient* (or *attack time*) is extinguished, when the response frequency has been enforced to equal the frequency of the external force, and amplitude develops according to the above-illustrated response curve.

3.2.1 The Transient Response

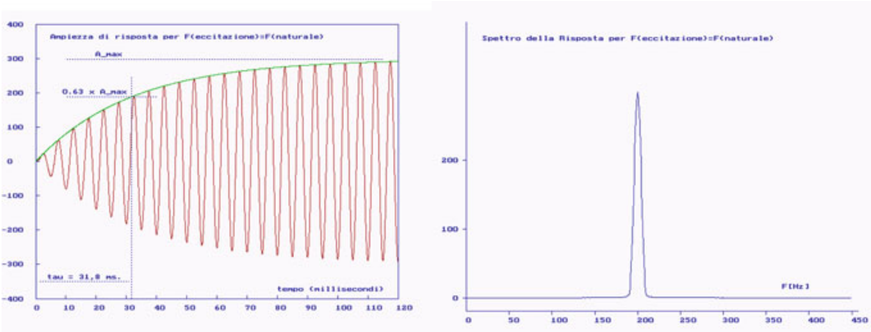
Let us analyse the behaviour of a system in the initial transient when undergoing an abrupt sinusoidal force from the rest position, and in the decay transient when the force is suddenly removed.

Transients, though short-lived, are important since the guitar functioning primarily relies on these. After plucking (see Chap. 2) a wave propagates from the excitation point towards the bridge where it produces a force impulse that sets the resonator components into motion, entering an *attack time*. But once the string is released, the motion progressively damps down because—among other reasons—energy passes into the resonator oscillators that, as a consequence, enter a *decay transient*. After some time both string and oscillators recover their initial rest position. So the force that acts on the bridge is never constant and the motion, differently from the violin or wind instruments, always develops between these two transient periods (attack and decay).

When we abruptly apply a sinusoidal force to our mass-spring system, this undergoes a sort of "shock" and reacts by trying to vibrate at its own natural frequency, while the external excitation mechanism tries to impose its *own* oscillation frequency *F* (*excitation*). During this initial transient both the natural frequency of the oscillating system and the "forcing" frequency of the external generator are present. Finally, the external generator prevails and imposes its own oscillation frequency.

Amplitude and composition of the response signal spectrum actually depend on the relation between the excitation frequency and the system natural frequency.

The simplest case is when the two are equal. The situation is illustrated in the diagrams below, obtained—like subsequent ones—from a mathematical simulation.



As the two frequencies coincide, only one line is present in the response spectrum (right) which corresponds to the natural frequency of the mass-spring system.

However, some time is required for the external generator to bring the oscillator at its maximum oscillation amplitude. This only happens after the generator overcomes the contrasting phenomena of both inertia and losses of the system.

Amplitude grows according to a regular *exponential curve* (the second part of the curve in the left panel). An exponential curve is typical of many ordinary phenomena. If we consider for instance to set a temperature (final value) on our home thermostat, the heating system (external generator) will try to reach that temperature by opposing both the thermal losses and the environment inertia (related to the mass of air and the thermal capacity of the ambience. As a consequence, temperature will rise by an exponential curve.

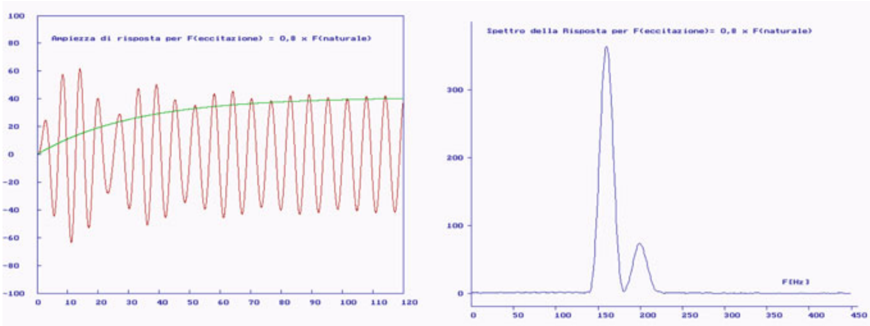
In Appendix 1.3 we have seen the formula of exponential curves (defined by the time constant) that rule growing and decay transients. Having examined the phenomena on which resonance is based, we can get the formula of the time constant τ:

$$\tau = \frac{2m}{r} = \frac{Q}{\pi\ f_{natural}}$$

So the time constant depends on the mechanical parameters of the oscillator, not on the excitation frequency. The time constant is great (and amplitude growth is slow) when the Q factor is great (and the resonance bandwidth BW is narrow). And vice versa.

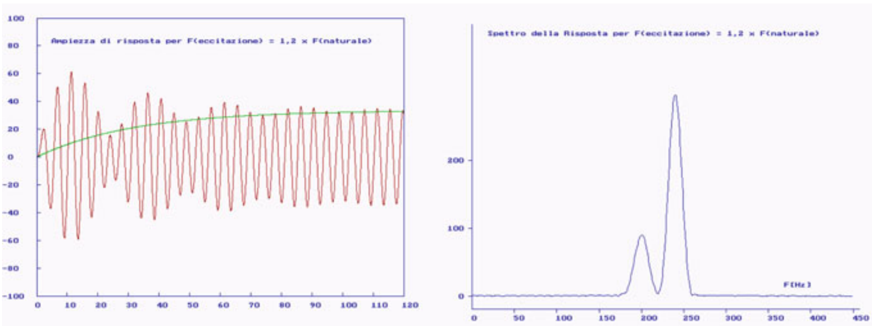
When the frequency of the external generator is different from the natural frequency, during the initial transient the two frequencies clash, as shown in the following diagrams.

The first diagram depicts the response when $f_{excitation} = 0.8 \times f_{natural}$



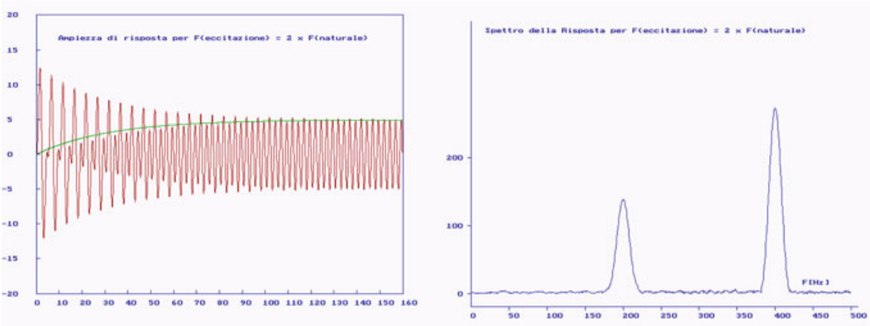
The spectrum of the response signal (right panel) shows two lines, the higher one representing the excitation frequency and the lower one the natural frequency of the oscillatory system. This means that during the transient, the disturbance caused by the intervention of the external frequency (though very different from the natural frequency) is able to excite the resonance of the oscillating system. The conflict between the excitation frequency and the natural frequency produces a phenomenon called *beat* (left panel) which implies exchange of energy between generator and oscillating system, while amplitude grows and decreases at the frequency $f_{natural} - f_{excitation}$. Once again the external frequency tends to prevail in accordance with an exponential law. The transient extinguishes when the external generator prevails and enforces its frequency into the system.

The case when the excitation frequency is higher than the natural frequency, specifically $f_{excitation} = 1.2 \times f_{natural}$ is shown in the following diagrams.



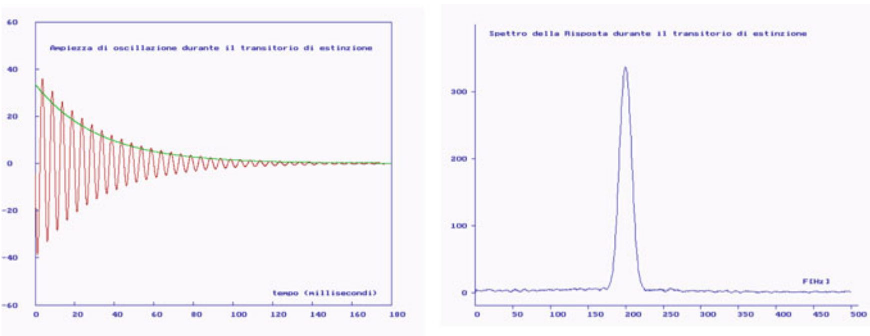
As in the previous example, both mentioned frequencies are present during the transient time where, in the response spectrum, two corresponding lines are present of which the higher one represents the excitation frequency. A beat is also evident, causing amplitude to fluctuate at the frequency $f_{excitation} - f_{natural}$. Again, the excitation frequency tends to prevail and follows an exponential law.

Interestingly, in the following and last case the excitation frequency is twice the natural frequency.



We clearly see that, during the transient (when both the frequencies are present) amplitude at excitation frequency increases according to an *exponential growing curve*, while at natural frequency it diminishes by an *exponential decreasing curve*. In this and all of the cases so far examined, exponential curves have the same time constant τ because, as formerly mentioned, this constant only depends on the mechanical parameters of the oscillating system (not on the frequency of the external generator).

At the end of the attack time the external generator, overcoming resistances, has imposed its vibration frequency on the mass-spring system. If we remove the external excitation, which has sustained the vibration of the system until now, amplitude will progressively extinguish. The following graphs illustrate what happens during this *decay transient*.



The system keeps on vibrating, though *at its natural frequency*. This is evident in the response spectrum (right panel) where only one line is present, corresponding to the natural frequency. The system does not retain the imposed frequency and amplitude progressively decreases (left panel) until the energy that was stored during the attack time is completely dispersed because of inner or radiation losses. Once more the amplitude decay follows an exponential decreasing curve with the same time constant so far observed for transients.

Let us summarize how the analysis reflects on the guitar functioning:

- In the guitar, owing to its excitation/release modus operandi, the resonator and its oscillating components always work within *transient times*: *attack*, when set into motion by the string excitation; *decay*, when the string is released.
- The overall force acting on the resonator is composed of several elementary forces, respectively corresponding to the fundamental and to each of the harmonics present in the string motion recipe. When the string is set into oscillation, not only the resonator modes matching (or nearby) the fundamental and the harmonics of the played tone are excited, but also very distant modes, though showing progressively decreasing amplitude. This means that the sound recipe includes the components of the spectrum excited by the fundamental and by the harmonics of the string, as well as the resonator frequencies excited during the attack transient.
  During this initial transient each of the string motion components tends to prevail over the fundamental frequencies of the resonator by imposing its own frequency. The amplitude of the fundamental and of the string harmonics follows an exponential law whose time constant depends on the $\boldsymbol{Q}$ factor of the resonator resonances. This parameter is only related to the mechanical properties of the resonator. On the whole, the recipe of the sound during the attack time depends on this conflict between the string motion components and the resonances of the resonator.
- When the string is released, the components of the spectrum relative to the string motion recipe—fundamental and harmonics—that determine the excitation force, begin to damp down. At the same time also the resonating components of the resonator response—that were excited during the attack transient—damp down.

These mechanisms are involved in the functioning of all guitars and actually of all kind of instruments, though influenced by their different characteristics, and are crucial for the quality of sound.

In Chap. 1 we have examined the time/frequency representation of an A played on the third string of the guitar, second fret. We have seen the Waterfall chart and the *decay time* graph of each spectral component of the string motion recipe (the tuning note, the fundamental at 220 Hz and the subsequent harmonics). These phenomena can be better understood by considering the above-discussed interaction between the excitation signal (the string) and the resonances of the oscillating systems that compose the guitar resonator. As we discussed in the same chapter, this behaviour deeply affects the sound and timber of a guitar.

After observing the interaction between oscillating systems and external force generators, another phenomenon due to the coupling between string and resonator is worth noticing. In Chap. 2 we wrote that the harmonic components that define the motion of a string are somehow ‘filtered’ by the resonator, according to its characteristics at the bridge: some of them are enhanced, since they lie close to some resonant frequency; others are attenuated, as they correspond to antiresonances of the resonator.

Actually, the harmonic components of the string motion are not only enhanced or attenuated by the coupling with the resonator, but also *transposed in frequency*. When the string fundamental (or one of the harmonics) lies close to a resonance of the connected oscillating system, both the spectral lines (of the resonance and of

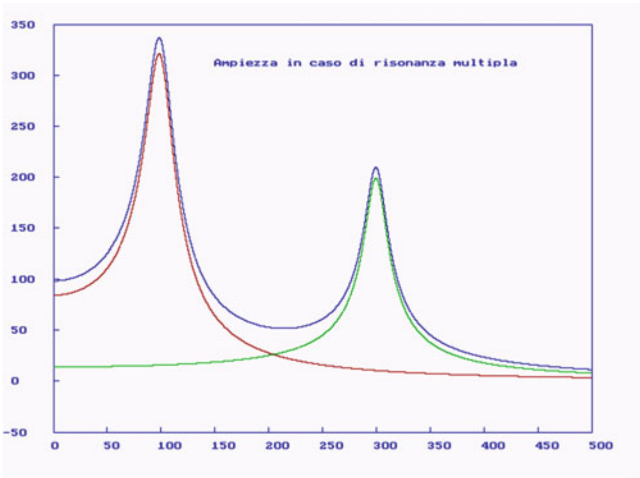
the harmonic) tend to diverge, so that the harmonic is not exactly multiple of the fundamental any more. This is a consequence of the relation between the characteristic resistance of the string and the resonator impedance at the bridge. The effect is more noticeable in the piano, because its lower strings have considerable mass and characteristic resistance. Nevertheless, slightly false notes also result from guitar strings, whose motion recipe components, differently from the general rule, are not all multiple of the fundamental but tend to *disperse*. This can be experimentally proved, or even perceived by a very sensitive ear.

3.3 Multiple Resonances and Antiresonances

An external generator can excite *more than one resonance* in a multidimensional oscillator having many vibration modes (as the plate or the guitar table). This is a natural behaviour for large bandwidth generators like the string, since they comprise a wide range of frequencies. The string generates an excitation signal at the tuning frequency and at all harmonics included in the sound recipe. The excitation signal generated by a pendulum percussing the table is also a typical large bandwidth signal. We use this signal to determine the response of a system.

This phenomenon is illustrated in the following diagram, resulting from a generator that excites two resonances (at 100 Hz and at 300 Hz). The overall response of the system is the product of both the amplitudes of the resonance curves and their relative phase.

In this example, the curves have the same phase between the two resonances and so they add together (cf. the foregoing phase curve in resonance). Instead, above the first and under the second resonance the curves have different phases, and so they subtract from each other.



As a result, at a certain frequency value between the two resonances the response of the system is very low, which means an *antiresonance* takes place.

Though resonances are normally considered most crucial in determining the acoustic properties of oscillating systems like the guitar soundboard, antiresonances are also very important, and this for many reasons. First of all, the sound level at antiresonance frequency is scarce. So, in designing the instrument, we must prevent any tone to coincide with an antiresonance, as this would engender a disagreeable performance, symmetrical with respect to the ‘wolf tone’: a long-lived but weak tone, where the elastic energy that the string transfers to the resonator is hindered and partly dissipated owing to friction of both string and nut.

Another interesting aspect we will examine is that antiresonances generally come about at ‘typical’ frequencies—connected to the nature of coupled oscillators—whose estimation greatly helps optimizing the elements of these kind of systems.

The above-described condition is not the only one possible in multiresonant oscillating systems. Sometimes the curves present equal phases above and under the resonances, and opposite phases between the two resonances. Once again an antiresonance takes place, but with different shape and amplitude.

These behaviours are related to the structure of the oscillating system and to the kind of interaction taking place between its vibrating surfaces. As we will see in the guitar resonator, at the resonant frequency of the air in the body (ca. 100 Hz) the air flow through the hole is contrary (in antiphase) to the direction of the table motion. On the other hand, at the second fundamental resonance (ca. 200 Hz) the air flows through the hole in the same direction (in phase) of the table motion. Between the two resonances there is a marked antiresonance that corresponds to the frequency of the Helmholtz resonator.

In the following chapters we will deeply investigate these aspects. Mentioning multiresonant systems introduces us to a further analysys of the oscillating system we are mostly concerned with: the guitar resonator.

Chapter 4
The Resonator Components

Abstract This chapter presents the three main components forming the resonator of the guitar. The first one is the Helmholtz resonator; additionally, the various cavity air modes and their phase relationship are discussed, presenting a simple method to approach them mathematically and to measure in practice. Some data on modern and historic guitars going back 150 years are given, and the effect of the *tornavoz* is introduced. The modes of the top table and the back are presented and typical values of frequency are given. The modes of the guitar table are modelled using the finite element method, contrasting them with the modes of a simple rectangular plate. The transversal stiffness due to the braces of the back results in the modal shapes obtained via the FEM model.

In the previous Chap. 3 we summarized the phenomena involved in the functioning of the resonator, which in every guitar—though subject to specific characteristics of each instrument—relies upon three main oscillating systems: the soundboard, the back, the air contained in the body, and their interaction (or *coupling*).

Obviously other elements—like the bridge and the sides—also affect the resonator performance. Leaving to further investigation the role of these elements in the quality of sound, we will now focus on the three main oscillating systems, since they rule amplitude and frequency of the fundamental resonances of the instrument. We begin by studying each of them as a standalone system. The next chapter will be concerned with the interaction between them, which will lead us to finally consider the global functioning of the guitar resonator.

We will see through this chapter how the characteristic parameters of each oscillator are referable to the previously observed operation of some elementary systems.

We will also describe tools that enable the luthier to objectively measure these characteristics and, to some extent, to manage them. But we first need to gain an adequate understanding of the individual functioning of these three oscillating systems.

G. Cuzzucoli and M. Garrone, *Classical Guitar Design*,
https://doi.org/10.1007/978-3-030-32992-1_4

4.1 Vibration of the Air in the Guitar Body

The guitar body is a sort of container whose limits are the soundboard, the back, the sides and the surface of the hole (or sound hole). These walls are vibrating surfaces and cooperate in producing sound. The hole itself is a vibranting surface and, as already mentioned, we can recognize this if we cover the hole with cardboard: in the low-mid register the sound becomes ‘shallow’, ‘dull’ and also reduces in amplitude since part of the sound radiation is hampered by the obstruction. This is mostly important for low-mid tones, between the first and the sixth open string.

Let us consider the soundhole operation. Differently from the other wooden parts of the body, the hole is not solid and only encloses a certain volume of air between its surface and thickness. At low-mid tones the air contained in the hole behaves like a rigid piston with mass

$$m_{hole} = \rho \times S \times l_{effective}$$

where ρ is the air density, S is the surface of the hole and $l_{effective}$ is the effective thickness of the hole. Suppose applying a sinusoidal force to a piston with the same characteristics. We will see that, increasing the frequency, both the amplitude and velocity of the piston oscillation diminish. Therefore, when the frequency increases the piston oscillation meets an increasing resistance, and practically comes to a standstill when reaching a given frequency (dependent on the geometrical features of the object). As far as the hole is concerned, this frequency limit is about 370 Hz (corresponding to F# on the first string).

We should not be surprised that the air in the body, just like the back and the soundboard, is an oscillating system on its own. Air is in fact a compressible elastic medium, used for instance in pneumatic suspension systems of some vehicles where air-springs are employed instead of traditional leaf-springs.

To examine the properties of this oscillating system we first of all must separate the behaviour of the air from that of back and soundboard; in other terms we must assume that no interaction exists between the cavity and the surrounding surfaces, supposing that all the surfaces of the body (hole included) are rigid and therefore unable to vibrate. Hereinafter we will call this system a *rigid wall cavity oscillator*. This operates like the mass-spring system examined in the previous chapter with reference to elementary oscillators. The typical attributes of the system are

- Firstly, the cavity encloses an air volume $\boldsymbol{V}$ whose mass $\boldsymbol{m_a}$ is

$$m_a = V \times \rho$$

  where $\boldsymbol{\rho}$ is the air density ($\rho = 1.204$ kg/m^3 at 20 °C). $\boldsymbol{V}$ depends on the table surface and on the average height of the sides. In modern guitars $\boldsymbol{V}$ is about 13 L (13,000 cm^3).
- Secondly, the cavity has a characteristic similar to the parameter $\boldsymbol{k}$ that, as already mentioned, designates the lengthening degree of a spring under a force $\boldsymbol{F}$ (the lengthening is $x = F/k$ and $\boldsymbol{k}$ is the stiffness of the spring). For rigid wall cavities

we must replace the stiffness with the *mechanical compliance* $\boldsymbol{C_m}$, conceptually being the reciprocal of k.

In Appendix 4.1, through a simplified analysis of the phenomena involved in rigid wall cavities, we will define the formula of the mechanical compliance. We will see that the *natural frequency* in this system (just like the mass-spring system) is

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{C_m\, m}}$$

The air contained in more complex cavities (like the guitar body) shows a wave propagation phenomenon in an elastic medium comparable to the previously exposed example of the stone cast into the pond. The stone causes a disturbance of particles that vibrate in every point, going up and down with respect to the water level, while maximum and minimum peaks travel at a velocity that depends on the properties of the medium (water in this case). The phenomenon generates *amplitude* (or displacement) *waves* that propagate from the point of impact of the stone towards the edges of the pond. Similar waves (*pressure waves* in this case) result from an air perturbation in the cavity.

The behaviour of this oscillating system is similar to that of a system we have formerly observed, i.e. the plate. As the plate oscillation, the oscillation of the air in the cavity can be described by different *vibration modes* (depending on the shape and dimensions of the cavity) associated each with a certain *natural frequency*, and the motion of air particles at any instant is the sum of displacements due to different modes. As in the plate, each vibration mode of the air in the cavity can be described by a mass-spring system, where the *modal frequency* associated with each mode of vibration depends on *modal mass* and *modal stiffness* (or *modal compliance*).

Nevertheless, we must point out some significant distinctions between oscillations of a plate and oscillations of the air in a cavity.

- As a consequence of external excitation, *amplitude oscillations* (e.g. brought about by the string) take place in the plate, while *pressure oscillations* take place in the air included in a cavity. As in the plate, not all of the possible vibration modes contribute to the air oscillation; a vibration mode of the cavity is only excited when the air is perturbed nearby an antinodal area with high propensity to large oscillations. Only in this case the frequency associated with this mode joins in the 'recipe' of the air motion. On the contrary, when the air is excited in a nodal area of one or more vibration modes, that mode or those modes are not excited, so they do not affect the motion of the air in the cavity.
- As already mentioned, the plate can be connected to the support in different ways: the plate can be fixed to the support (*clamped plate*) so that any displacement or bending along the edge is prevented; the plate can be simply laid on the support (*simply supported plate*) which allows rotation along the edge; as in the guitar back and soundboard, a condition midway between the clamped and the simply supported plate is normal. In the guitar this condition is due to the elasticity of the sides and linings that allow a certain flexion along the edges.

We have also observed the case of a plate whose edges are totally *free* of bonds, which is not relevant for the back and soundboard of the guitar but is useful to study the behaviour of the air contained in a cavity. *Absence of bonds* is in fact the condition of the air in the body. This is intuitive, considering that air vibration is not subject to any constraint (or bond) along the container's walls which, as a consequence, do not define a nodal line. In this condition, the pressure reaches maximum values in correspondence with certain symmetrical axes along the edges of the container.

- The plate undergoing an external sinusoidal force generates sound pressure (or *radiates sound*) in the surrounding air. The sound pressure is especially high when the excitation frequency matches one of the plate resonant frequencies. Differently, in a rigid wall cavity, air does not radiate sound because the walls (being rigid) cannot vibrate the air outside the cavity.

The foregoing aspects apply to rigid wall cavities of any shape. To understand how the air contained in the resonator body affects the instrument functioning, we first need to identify the main oscillation modes and natural frequencies of a rigid wall cavity with the same shape and dimensions of the guitar body. The behaviour of the cavity at low-mid frequencies, where the hole intervenes, must be described by considering an important phenomenon known as *Helmholtz Resonance*.

The vibration modes of the air in the cavity will be identified by the same classifying method we formerly applied to plates, which will also be used throughout the text. According to this classification, mode $\langle 0\ 1 \rangle$ has no longitudinal nodal lines and just one transverse nodal line; mode $\langle 0\ 2 \rangle$ has no longitudinal nodal lines and two transverse nodal lines. The first numeral indicates longitudinal nodal lines, the second indicates the transverse ones. Naturally, the *Helmholtz Resonance* is associated with mode $\langle 0\ 0 \rangle$, since only one nodal line runs along the perimeter and no longitudinal or transverse nodal lines are present.

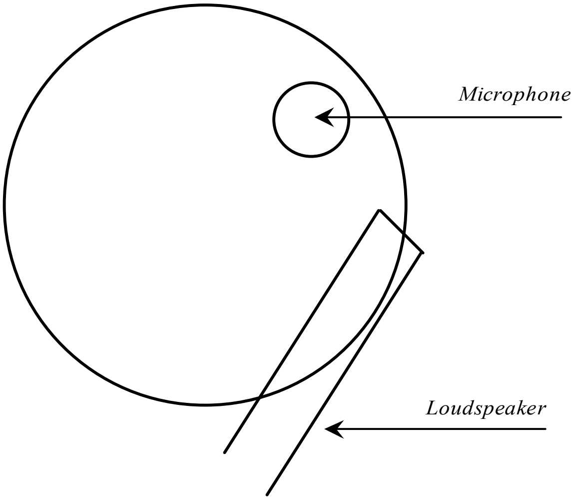
4.1.1 Measurement of Oscillation Modes in the Cavity

The method we are going to describe applies both to a real guitar and to a rigid wall resonator model. With a real instrument we must first of all increase the rigidity of the tables (back and soundboard) in order to replicate most faithfully the *perfectly rigid wall cavity*. For this purpose we lay the guitar body on a thick carpet and distribute some weights on the soundboard, so preventing vibrations of both the tables. Some published researches suggest dipping the guitar body in sand to make the walls of the resonator rigid, but the method we propose is simpler and reliable enough for evaluating the modes of vibration in the cavity.

The measurement set-up (sketched below) is the following.

A little high-quality loudspeaker is placed inside the hole for the purpose of exciting the air. The cone must measure about 3 in. in diameter. An adjustable frequency sinusoidal generator sends a properly amplified signal to the loudspeaker.

A ca. half-inch-diameter microphone is also placed inside the hole. The signal picked up by the microphone is amplified, then sent to a measuring device capable to assess its effective (RMS or Root Mean Square) value—a digital or analogic tester, preferably an oscilloscope.



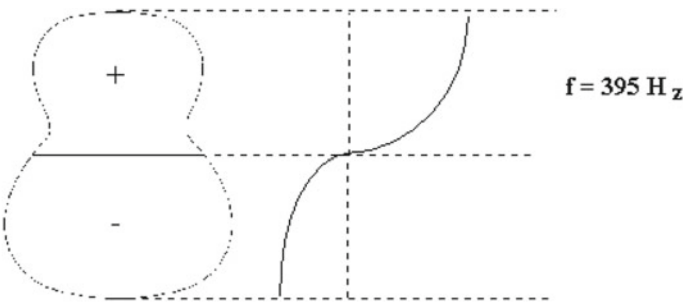
The signal frequency sent to the loudspeaker is progressively varied, for instance between 60 and 1000 Hz. At certain frequencies the signal amplitude detected by the instrument is maximum, while at slightly lower or higher frequencies the amplitude diminishes significantly. Maximum values correspond to resonant frequencies and so, as a first approximation, they also correspond to the oscillation modes of the cavity.

We will show afterwards how, *in correspondence with a certain resonant frequency*, maximum and minimum pressure values are present along the distinctive longitudinal and transverse axes of the cavity. To verify this, the acoustic pressure inside the cavity must be measured. On its own, a microphone placed *inside the hole* cannot reveal these maximum and minimum pressure values; we need a *microphone probe*, composed of a microphone fixed out of the instrument and connected to a flexible thin pipe. In resonance condition we will move the pipe into the body in order to pick up the position of maximum and minimum values. This is a rather approximate but adequate method for the examination of the phenomena we are concerned with.

The devices used during this test (sinusoidal signal generator, amplifier, microphone, loudspeaker) must also be available to the luthier for other measurements that will be later described.

4.1.2 Mode ⟨0 1⟩

This is the first vibration mode of the air in the rigid wall cavity of the guitar, with a single nodal line running along the transverse axis of the instrument (just a little below the waist). In modern guitars this mode starts off between 380 and 400 Hz. In a Garrone guitar tested for reference with the above-described method we found it at 395 Hz.



The image illustrates the peak of the pressure wave that progresses along the longitudinal axis of the cavity. The pressure wave motion recalls one of the string motion modes but—while the string is clamped at the ends (nut and bridge)—the pressure wave in the body can freely take on maximum values at the ends of the longitudinal axis. The two air volumes above and below the nodal line vibrate in antiphase. Because of the asymmetry, the pressure peak is greater above the waist, where the volume is smaller. Transversely, the acoustic pressure is roughly constant.

In Appendix 4.2 we provide a detailed calculation whose outcome is given here as follows.

The natural frequency of the air in the cavity in this vibration mode is:

$$f = \frac{c}{\lambda} = \frac{c}{2l}$$

where $\boldsymbol{c} = 343.3$ m/s is the velocity of sound through air and $\boldsymbol{l}$ is the longitudinal span of the cavity.

In the examined Garrone guitar the *geometrical inner length* of the body is $l =$ 482 mm, which is equal to the external length (486 mm) less the thickness of the sides. The *actual* value of the body length is slightly inferior, owing both to the curved profile and to the reinforcing parts glued inside the body for structural purposes (tail block below and neck block above). Both factors reduce the actual progress of the wave, in contrast with the theoretical one. Establishing the actual length to 90% of the geometrical length, or $l_{actual} = 0.9 \times l$, our formula develops into

$$f = \frac{c}{2 \times l_{actual}} = \frac{c}{2 \times 0.9 \times l}$$

If we put the reference guitar values of velocity **c** and inner length $\boldsymbol{l}$ (c = 343.3 m/s; l = 482 mm) into the formula we obtain f = 396 Hz which basically matches the measured value (f = 395 Hz).

A similar procedure will lead us through the examination of the other modes of the cavity.

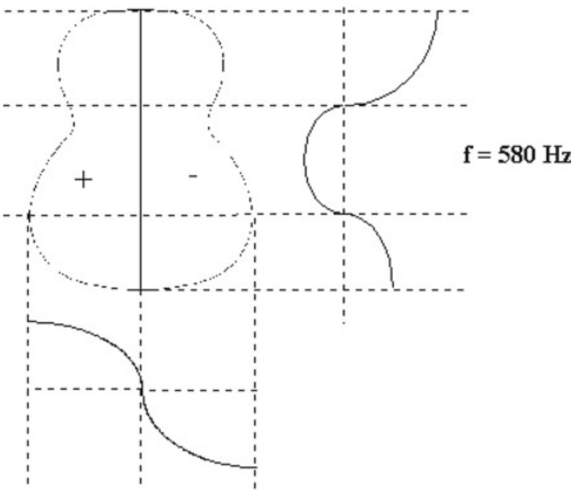
4.1.3 Mode ⟨1 0⟩

In the sequence of vibration modes of the rigid wall cavity, this is the next relevant one. Here a single longitudinal nodal line running along the body axis defines two air volumes (left and right) that vibrate in antiphase. At resonance the inner pressure is comparable to *half a sinusoid* and presents a positive maximum and a negative minimum in correspondence with the edges. This means that a pressure-semi-wave runs along the transverse axis of the body and determines a *transverse half-wave resonance*.

Pressure values show two nodes in the longitudinal sense (one through the hole and one near the upper bout) and maximum levels at the upper and lower end of the cavity. Because of the asymmetry in the cavity, the maximum upper level is greater than the lower one.

In modern guitars the resonance for this mode is comprised between 550 and 600 Hz. In the reference Garrone guitar, as confirmed by the above-mentioned method, this resonance arises at 580 Hz.

The following figure shows the nodal line and the pressure wave along the transverse and the longitudinal axis.



The same calculation criterion used for mode ⟨0 1⟩ can be applied to this resonance, whose frequency is

$$f = \frac{c}{2 \times l_{actual\,transverse}}$$

Now we put the *actual transverse width* into the formula. This value depends on the three transverse dimensions of the guitar body, which in the reference guitar are:

- Upper bout width l_a (280 mm)
- Waist width l_b (246 mm)
- Lower bout width l_c (373 mm).

The actual transverse width that determines the resonant frequency for mode ⟨1 0⟩ is the average of the external transverse dimensions, each deducted of the thickness of the sides, so:

$$l_{actual\,transverse} = \frac{280 + 246 + 373 - 12}{3} = 295.7\,[\mathrm{mm}]$$

Setting this value into the preceding equation of the resonant frequency we obtain:

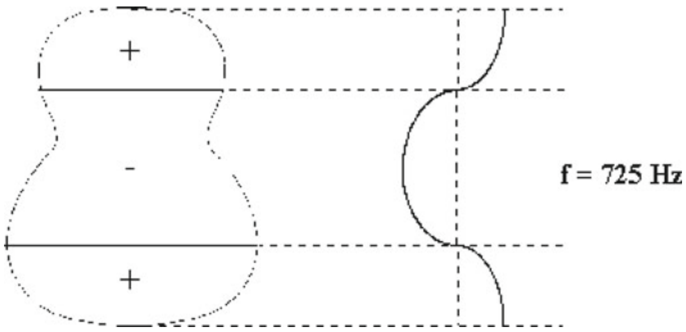
$$f = \frac{c}{2 \times l_{actual\,transverse}} = 580\,\mathrm{Hz}$$

This outcome equals the measurement, proving that our calculation method, though approximate, gives a fairly accurate reading of the development of this vibration mode of the air in the cavity, dependent on transverse dimensions of the body (upper bout, waist, lower bout). So, given equal length, a narrow body (Torres style) yields a higher resonant frequency than a body with wider bouts (Simplicio style and above all acoustic guitars).

4.1.4 Mode ⟨0 2⟩

Two more relevant modes follow in the sequence of the air vibration modes in the cavity. Though resulting from different mechanisms, they lie very close in frequency. The first one is mode ⟨0 2⟩, with two transverse nodal lines that, in this case, separate three air volumes that vibrate in alternate phases. At resonance the air pressure follows a *full sinusoid* with maximum values at the longitudinal ends of the cavity. This means that the pressure wavelength λ equals the actual length of the cavity.

In the transverse sense the air pressure is essentially constant. The following illustration shows the development of the pressure wave along the longitudinal axis and the two transverse nodal lines.



We introduce these data into our formula

$$f = \frac{c}{\lambda} = \frac{c}{l_{actual}}$$

The actual length $l_{actual} = 482$ mm is the same we used for the calculation of the frequency in mode $\langle 0\ 1 \rangle$, that is the external length (486 mm) deducted of the thickness of the sides. Setting this value into the formula the resulting frequency is

$$f = \frac{c}{l_{actual}} = 712\,\text{Hz}$$

This little divergence from the measured value (725 Hz) is due to the curvature of the sides and to the reinforcing elements inside the body, which slightly reduce the calculated length introduced into the formula.

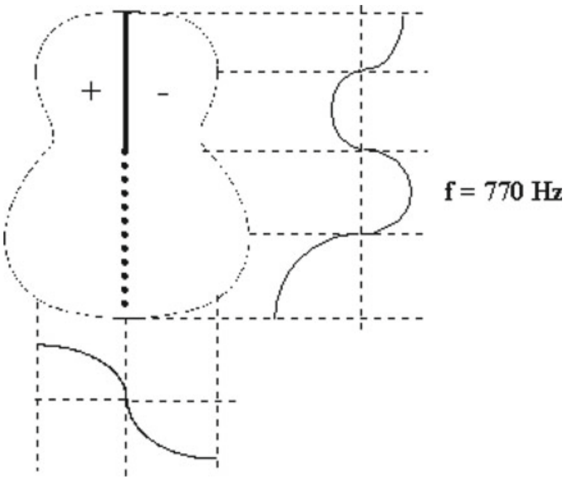
4.1.5 Mode ⟨1 0⟩ in the Upper Volume

The frequency in this mode (770 Hz for the reference guitar) is close to the one of the previous mode. This, in contrast with the previous ones, is an irregular mode, since it only takes place in the upper volume of the cavity, between the waist and the upper end, where the energy of the oscillation concentrates, while the modes we have examined so far involve the whole cavity. The air contained in the lower bout which, as formerly observed, should resonate at 580 Hz in mode $\langle 1\ 0 \rangle$, is 'forced' into a 770 Hz oscillation, therefore beyond its own resonance (cf. Chap. 3 for a survey of the *forced oscillation* phenomenon).

The oscillation in the upper volume develops as a *longitudinal half-wave mode*, hence with a nodal line that runs along the longitudinal axis from the waist to the upper end of the cavity. We will call it *mode* $\langle 1\ 0 \rangle$ *of the upper volume*.

Also in the volume below the hole a longitudinal nodal line is present but here, as already mentioned, it is due to a forced oscillation (in mode $\langle 1\ 0 \rangle$) with the same characteristics of the enforcing oscillation.

There are four points along the longitudinal axis where pressure takes maximum values: one at each end of the cavity, one between the waist and the middle of the upper bout, and one between the waist and the middle of the lower bout. There are also three minimum pressure points, approximately at the waist, at the lower and at the upper bout. All of these points are shown in the following illustration.



This 'anomalous' mode exemplifies a situation that sometimes occurs in oscillating systems, both in the air in the cavity we study here as in the soundboard. In such 'anomalous' modes the oscillating energy concentrates exclusively in one part of the system, while the other part is forced into a vibration outside its own resonance. This kind of coupling between vibration modes is only possible when both their respective frequencies and nodal lines are fairly similar, that is when they have the same pattern of oscillation phases.

The formerly observed mode ⟨0 1⟩ representation of pressure in the cavity shows one full sinusoid, whereas in mode ⟨1 0⟩ we have two full sinusoids. Reasonably, the frequency of the upper volume in mode ⟨1 0⟩ should be about twice that of mode ⟨0 1⟩ (or an octave above). There is actually a little difference (ca. 20 Hz) because the progress of the pressure along the longitudinal axis is not perfectly sinusoidal, its lower end peak being wider than the upper one.

In addition to the ones we observed so far, there are other vibration modes of the air in the body, some of which being very conspicuous. In Appendix 4.3 we report the values of modal frequencies that generate outstanding resonances in the cavity and the associated modes of vibration; in some cases the resonance is evident but cannot be assigned to a specific mode.

The luthier generally fixes the shape and dimensions of the guitar body once and for all in his production. Depending on these dimensions, air resonances cannot be modified but, on the other hand, both back and soundboard can (and must be) properly designed in order to optimize their coupling with the cavity. In Chaps. 5

and 6 we will see what conditions are favourable for this coupling and for the arising of sound-quality supportive resonances.

Deferring exhaustive study, we summarize here these issues:

- Supporting soundboard modes with frequency close to that of the air in the cavity.
- Optimizing the distribution of oscillation phases in each of these modes, so that the table can effectively excite the air in the cavity.
- Supporting, also in the backboard, proper modes for a good coupling with the modes of the soundboard and of the air in the cavity.

4.1.6 Helmholtz Resonance (Mode $\langle 0\ 0 \rangle$)

In 19th century Hermann von Helmholtz studied the acoustic characteristics of a particular oscillating system consisting in a spherical glass or brass bowl with two opposite openings. The larger one (the resonator neck) was placed in front of a sound source and the little one against the researcher's ear. The system had a specific natural frequency dependent on its main geometrical features. Due to resonance, a component of the sound source at that same frequency—when present—came out enhanced from the little opening.

Differently sized resonators were produced, each of them 'tuned' at a specific frequency value of the musical scale. This way the spectral composition of sounds—for instance those emitted by a violin—could be determined.

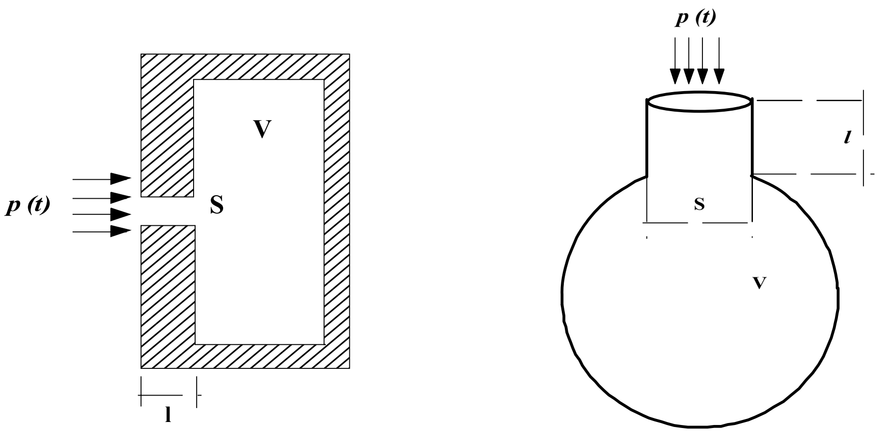
Today the harmonic analysis of a signal is performed by means of mathematical algorithms (like the Fast Fourier Transform that we use) which require a computer. Nevertheless, the Helmholtz Resonator is still useful, being at the heart of many common acoustical systems like, for instance, bass reflex speakers. For our purposes, we will see that, at low frequencies, the guitar body can be referred to a Helmholtz Resonator.

A Helmholtz Resonator is a rigid wall cavity that encloses an air volume **V**. The cavity communicates with the exterior through a *neck* of length **l** and surface **S**, in the course of which the air in the body is set into vibration by an acoustic pressure ***p***. The following figure shows two possible models of the system. In the left image the neck is integrated into one of the walls, while in the right one it is external and gives the system a bottle-like shape.

Quite interestingly, this object behaves like the previously examined mass-spring system. Its natural frequency is

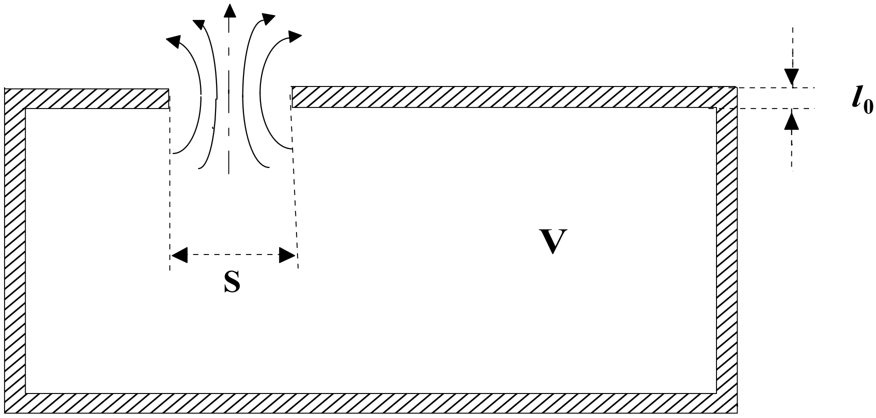
$$f_H = \frac{c}{2\pi}\sqrt{\frac{S}{lV}}$$

In Appendix 4.4 we report the mathematical details of this formula.



Since the natural frequency does not depend on the shape of the volume, this formula can also be applied to more complex forms like the guitar body.

The guitar resonator is comparable to a thin-walled rectangular case (shown in the next figure) with a circular opening of given surface **S** on one of the major sides.



The case has *nearly* all the characteristics of a Helmholtz resonator. An inner air volume **V** communicates with the exterior through a neck of surface **S** and length $\mathbf{l_0}$. The difference in the guitar is the very short neck (the thickness of the soundhole) with respect to the surface of the hole. As illustrated in the sketch above, the air streams run parallel into the hole forming a sort of *virtual neck* whose effective length is different from the actual physical length l_0. Without going into calculation details, the resonant frequency of the *Helmholtz Resonator of the guitar* is formulated as

$$f_H(guitar) = \frac{c}{2\pi}\sqrt{\frac{\pi r^2}{V\left(l_0 + \frac{\pi}{2}r\right)}}$$

where the effective length of the neck is:

$$l_{effective} = l_0 + \frac{\pi}{2}r$$

Let us apply this formula to a reference guitar (Garrone) with the following features.

- The surface of the table (hole excluded) is 1443.7 cm^2. We obtained this value from the table profile through the method of circle sectors that will be later illustrated.
- The sides are 9.1 cm high at the neck foot and 9.7 at the tail block, and the *equivalent average height* is 9.6 cm.
- The volume of the body is 13,800 cm^3. This is the product of the table surface and the average height of the sides less the volume of the internal elements (neck foot, tail block, etc.).
- The geometrical thickness of the hole l_0 is the sum of the thickness of the table and the thickness of the reinforcement along the hole, here corresponding to 43 mm.
- The radius r of the hole is 42 cm.

Setting these values into the formula we obtain $f_H = 130.6\,\mathrm{Hz}$ which fairly matches the measurement executed on the instrument (129 Hz) with the previously described method.

The formula we have given for the natural frequency of the Helmholtz resonator does not apply to instruments with non-circular hole, or when the hole is placed in a higher position, near the table periphery. For instance in the two-hole Garrone guitar, where the hole is not round and is very distant from the centre of the body, even if the volume is still the same—the Helmholtz resonator occurs at 121 Hz.

As we will see in Chap. 5, the Helmholtz resonance is one of the parameters that contribute to the first basic resonances of the instrument, i.e. the air resonance and the table resonance.

The development of the guitar shape in XIX and XX century is distinguished by a gradual increase of the body volume, by different thicknesses of the soundboard and different diameters of the hole. It is interesting how this development affects the value of the Helmholtz resonance in these instruments. The following box reports the values measured on some ancient and modern instruments.

Instrument	Helmholtz resonance (Hz)
Guadagnini (1821)	176
Italian guitar (ca. 1850)	176
Lacôte style guitar	174
Torres style guitar	140
Guitar of the early 1900s	138
Estudiantina (early 1960s)	141
Novelli (1994)	129.5
Garrone (2003)	129
Japanese guitar (early 1970s)	127
Kohno (1985)	127
Gallinotti (1956)	129
Ramirez (1970)	121

Notice that in the most ancient instruments, where the sides were low and the volume of air was scarce, the Helmholtz resonance occurred at about 175 Hz, dropping to 140 Hz in Torres style guitars and down to 130 Hz in modern guitars. In one of these guitars (Ramirez) the Helmholtz resonance drops to 121 Hz. The body in this instrument has a bigger volume compared with the other instruments mentioned in the table and, among other things, a scale length of 66.5 cm.

In former times some luthiers (Torres, Simplicio) added a *tornavoz* into the guitar case. The tornavoz is a cylindrical component made out of metal or timber, connected to the hole and with the same diameter, but disconnected from the back. According to testimonies, this enhanced the instrument lower tones. Our interpretation, based on the previous reasoning, is that by the tornavoz the length of the

Helmholtz resonator neck could be regulated, so allowing to control its frequency. That way these luthiers were able to control (possibly lowering) the fundamental resonances of the instrument—mainly the *tuning note*—by simply modifying the length of the tornavoz.

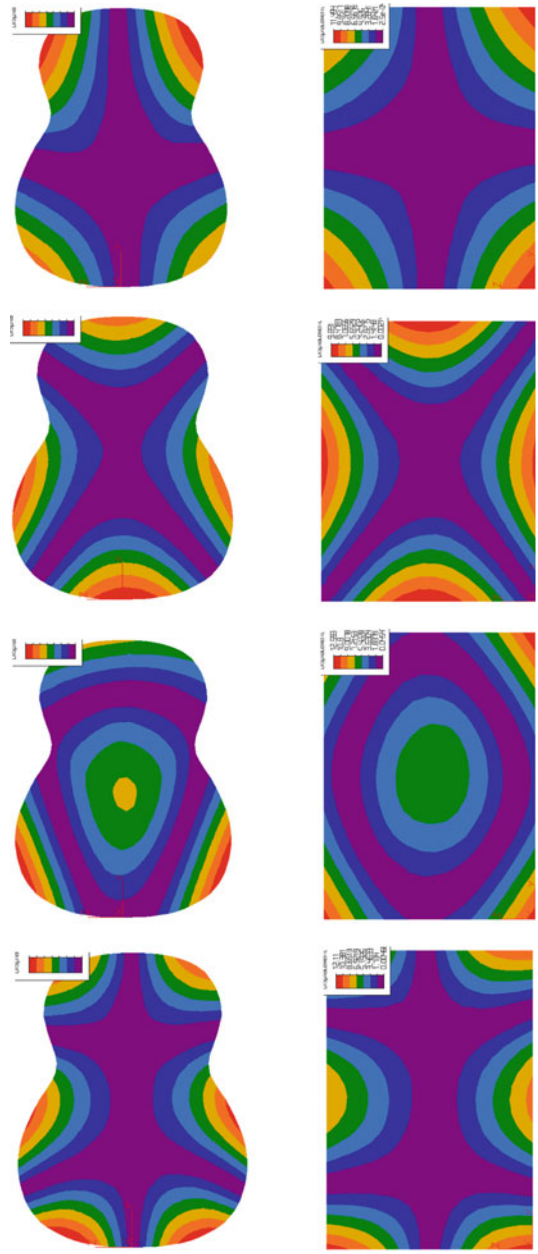
4.2 Fundamental Vibration Modes of the Soundboard

The guitar soundboard presents many modal frequencies, therefore many vibration modes which, at lowest frequencies, resemble those of the *rectangular plate oscillator* seen in the previous chapter. But while the natural frequencies of the plate can be calculated—at least with in certain boundary conditions—by means of mathematical formulae, a mathematical assessment of these frequencies is not possible for the back and the soundboard because of their irregular shape. Two methods are used instead:

- Experimental methods, that will be discussed in detail later on, allow measurement of frequencies and vibration modes of a specific table.
- Modelling, or building an abstract representation of the table (the *model*) whose characteristic parameters can be varied: elastic parameters, density, grain direction and, obviously, shape. The model is processed by software that provides natural frequencies and their amplitude as well as the shape of the vibration modes.

The advantage in modelling is that parameters can be directly changed to evaluate their effect on frequencies and modes. The advantage in experimental methods is to handle results regarding the specific table we are working upon while trying to optimize it. On the other hand we must be aware that no model can carefully reflect the characteristics of a specific timber table. So these two methods are not alternative but complementary. It is advisable for the luthier to know both of them and be able to apply them according to individual experience and requirements. In this chapter we will employ the FEM (Finite Elements Method) to describe the main natural frequencies of the soundboard and of the back. Afterwards instead, when dealing with optimization of real tables, we will only use results acquired through experimental methods.

The following figures represent some vibration modes of a guitar table not connected to the sides, in contrast with a rectangular plate with free edges. Comparison shows the analogy between the two kind of tables.



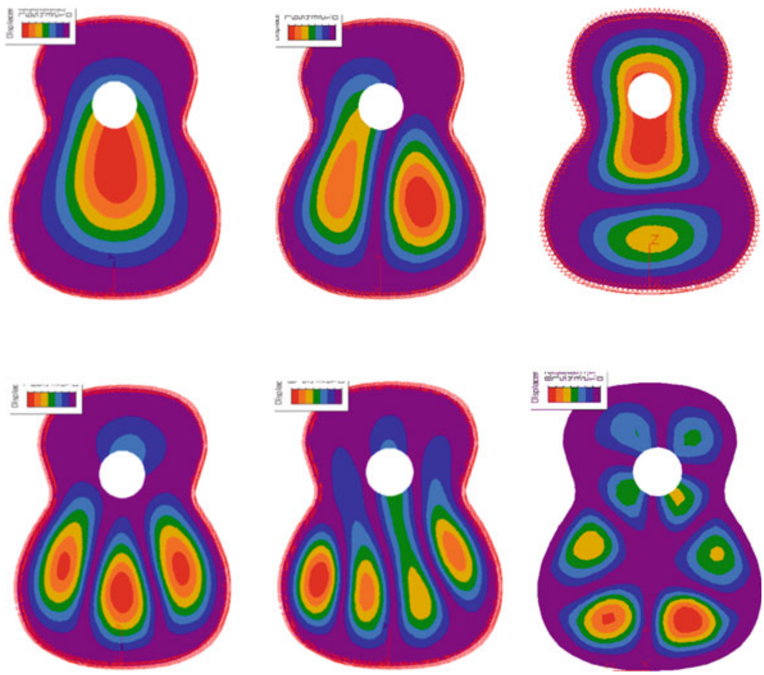
enlargethispage-12ptThe guitar soundboard is glued to the sides that act as a bond, hindering its movement along the perimeter. As mentioned in Sect. 3.1 (*Natural Frequencies in Oscillating Systems*) the bond between the table and the sides is an intermediate condition between the clamped and the simply supported edge, since the elasticity of the sides allows the table a certain degree of movement along its

perimeter. In fact the bond between table and sides is affected by construction parameters such as the kind of sides (solid or laminated) and linings (kerfed or unbroken) and by the shape and number of transverse reinforcement pillars along the sides.

The following figures resulted from a FEM modelling of the fastened table.

4.2.1 Vibration Modes and Natural Frequencies of the Fastened Table

The natural frequencies and the form of vibrating areas associated with the modes of a fastened table depend both on the physical parameters of the table itself (thickness, specific weight, elastic parameters) and on the characteristics of the bracing.



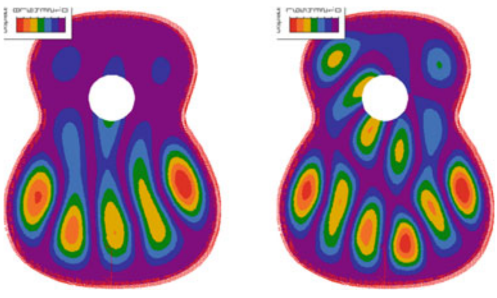
We will see how, to some extent, we can optimize both the natural frequencies and the actual shape of vibration modes by varying the arrangement and dimensions of the braces. We are now going to know these vibration modes of the guitar soundboard, deferring to following chapters a deeper investigation of the interaction between modes of the table, modes of the air in the body and modes of the back.

- *Mode* $\langle 0\ 0 \rangle$. This is the main vibration mode of the guitar soundboard. It is illustrated in the first of the previous patterns, where the nodal line runs along the table border and the oscillation reaches even the surface over the hole. For this reason it is also called '*ring mode*'. This natural resonance of the table plays a

crucial role in determining the basic resonances of the guitar, coupled with the main air resonance (Helmholtz resonance).

- *Mode* ⟨1 0⟩. A longitudinal nodal line distinguishes this mode, whose frequency is about 300 Hz. This mode can efficiently contribute to sound radiation from the table, provided that the bracing offers an asymmetry in the surface/mass ratio of the two areas that vibrate in antiphase and form a vibration dipole.
- *Mode* ⟨0 1⟩. Only one nodal line running across is present. The typical natural frequency associated with this mode takes place between 380 and 420 Hz. The arising of the resonance associated with this mode of vibration is primarily connected to the presence of transverse stiffening elements (bridge, under-bridge bar, crosswise braces). As we will see, this mode is apt to couple with mode ⟨0 1⟩ of the air and with modes ⟨0 1⟩ and ⟨0 2⟩ of the back, yielding a strong instrument resonance around 400 Hz.
- Mode ⟨2 0⟩. The natural frequency of the table in this mode arises between 550 and 600 Hz. As pointed out in the previous paragraph, coupling with mode ⟨1 0⟩ of the air is possible, provided that the three vibrating surfaces exert a net positive effect on the two lateral volumes of air that vibrate in antiphase to one another.
- *Mode* ⟨3 0⟩. The natural frequency arises in this mode above 600 Hz. The coupling with air and back is generally weak, so the sound radiation in this resonance only originates from the table.
- *Mode* ⟨1 1⟩ *in the lower bout*. The main vibrating areas concentrate in the lower bout, divided by a longitudinal nodal line that runs from the hole to the tail block and by a transverse nodal line. These lines define a *low* ⟨1 1⟩ mode where the oscillation phases are arranged in a typical grid-like manner. Another 'daisy' is the typical arrangement of vibrating areas around the hole. We will see that this vibration mode of the table can favourably interact with mode ⟨0 2⟩ of the air.

As the frequency increases, the vibrating surface splits into progressively smaller and more articulate areas, and the oscillating energy mainly concentrates in the lower bout, from the waist down. The following figure shows two more typical modes of the table that take place at the highest frequencies.



- *Mode* ⟨4 0⟩. The five vibrating areas extend up to the hole, separate by four almost longitudinal nodal lines. The oscillation phases of the vibrating areas alternate according to the usual chequered scheme (adjacent areas vibrate in antiphase). We

will see later on that a good coupling between this vibration mode of the table and mode ⟨0 2⟩ of the air is possible at 725 Hz.

- *Mode* ⟨5 0⟩. This mode primarily develops in the lower bout, though reaching the surface around the hole, and shows the typical alternation in phases of the vibrating areas. Once again we will see that conditions are appropriate for a beneficial coupling between this mode of the table and the formerly observed *mode* ⟨1 0⟩ *of the air in the upper volume.*

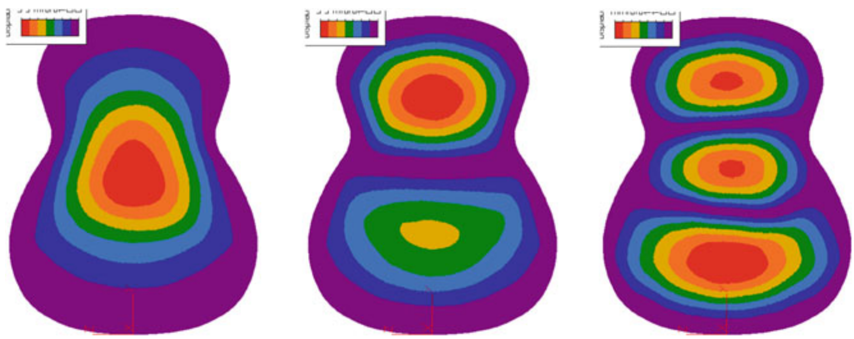
These modes are comprised in the range of frequencies up to 800 Hz and above. At low-mid frequencies the resonances are clearly defined but, when the frequency increases, they tend to gather—being produced by the excitation of two or more vibration modes close to one another—and generate a sort of *continuous resonance area*.

4.3 Fundamental Vibration Modes of the Back

From the point of view of acoustics, the soundboard and the back are similar. Both are flexible surfaces that couple with the vibration modes of the air in the body, primarily the main mode (the Helmholtz resonance). Obviously, different elastic parameters and different arrangements of the braces lead to very different natural frequencies of both the back and the soundboard, and to different interaction between these tables and the air in the body.

In a free-edged back the vibration modes are very similar to those of the free-edged table and so to those of the free-edged rectangular plate seen above in this chapter but, clearly, at different frequencies.

The bracing of the back is generally made up of three (or more) *transverse* braces that determine its vibration modes and natural frequencies. Later on we will thoroughly discuss the function of the back and how to optimize the bracing in order to promote its interaction with the soundboard and the air. For now, we briefly present the first vibration modes of a back constrained in the mould. Shown below are the results of the FEM simulation of a back where three braces are applied.

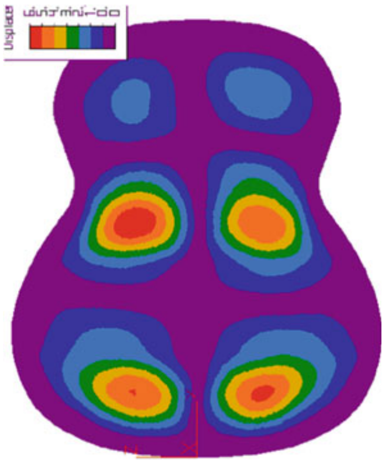


The first three vibration modes of a fastened and braced back are the most important ones for its coupling with the soundboard and the air in the body, at least in the low-mid frequency register. These modes *do not display longitudinal nodal lines*, since the stiffness of the transverse bars prevent any bending (i.e. oscillation) of the back along its longitudinal axis. We can easily prove that if we try to bend a braced back along its longitudinal axle while holding it at the waist: it is simply impossible.

- *Mode* ⟨0 0⟩. In this mode (the first shown in the illustration) the nodal line forms a ring along the table border. The antinodal area—where the oscillation energy concentrates—covers almost the entire table.
- *Mode* ⟨0 1⟩. As shown in the figure, this mode features a single transverse nodal line at the waist, and the natural frequency arises above that of mode ⟨0 0⟩.
- *Mode* ⟨0 2⟩. Here the three *maximum amplitude* values roughly correspond to the position of the three braces. The natural frequency is higher than that of mode ⟨0 1⟩.

In Chap. 5 we will see how these resonances of the back contribute to the instrument response in the middle register between 200 and 400 Hz. We will also observe that the back interacts with the soundboard and the air in the body and, accordingly, it influences the instrument fundamental resonances.

While the braces prevent longitudinal bending at least to 350 Hz, at higher frequencies they 'acquiesce' to bend transversely, because their *longitudinal impedance* diminishes. As a consequence, also modes of vibration displaying longitudinal nodal lines arise. This is exemplified in the following illustration.



Two nodal lines (longitudinal and transverse) are visible under the waist, forming a *low* ⟨1 1⟩ *mode*. We find here the typically chequered distribution of phases that always results from intersecting horizontal and vertical nodal lines. In the fastened back this mode develops just a little above 600 Hz and, as we will see, conditions are suitable for a good interaction between table, air, and back, which generates a significant resonance of the instrument.

What we formerly observed for the soundboard is also true for the back: when the frequency increases the vibrating surface splits into progressively smaller and more articulate areas, while resonances tend to gather and generate a sort of continuous resonance band.

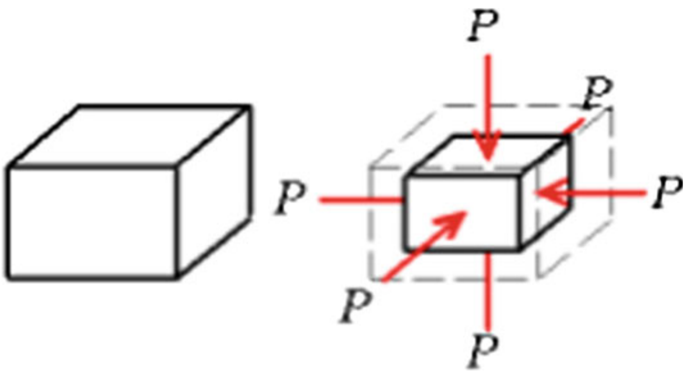
Appendices

Appendix 4.1: Elasticity of the Air

The air contained in the body is an elastic and compressible medium.

Consider a rigid-wall container (or *cavity*, as we called it) where the shape is irrelevant. The cavity encloses a volume of air $\mathbf{V_0}$, initially at standard ambient pressure. Then suppose we can apply a force from outside into the cavity, that by compressing the air causes a rise in pressure and a consequent contraction of the air volume.

The situation is sketched below.



A parameter **K** (also called *bulk modulus*, which measures a substance resistance to uniform compression) tells us how the volume ΔV changes with respect to the initial volume V_0 under a pressure variation ΔP (with respect to standard ambient pressure):

$$\Delta p = -K\frac{\Delta V}{V_0}$$

The negative mark indicates that volume diminishes when pressure grows.

The bulk modulus **K** recalls the elastic modulus **E** (or Young's modulus) whose physical definition was given in Chap. 3 regarding the plate oscillator. Both the **K** and the **E** modulus describe the elastic properties of a substance, with some important differences.

The elastic modulus **E** indicates how much a substance (like aluminium or wood) elongates when undergoing a pulling force **F**. Orthotropic materials (like wood) go through different deformation depending on the sense of pulling (along or across the grain); that is why we have defined two elastic moduli for orthotropic plates (E_x along the grain and E_y in the perpendicular sense).

On the other hand, the bulk modulus **K** tells us how much an air volume diminishes when undergoing an increase in pressure ΔP. Since air is a homogeneous substance, the **K** modulus does not depend on the direction of the force that modifies both pressure and volume; instead it is constant and corresponds to the product of the density of the air $\boldsymbol{\rho}$ and the square of the sound speed **c**, that is

$$K = c^2 \rho$$

From this formula we can obtain the value of the bulk modulus.

Being $c = 343.3$ m/s and $\rho = 1.204$ kg/m^3, the result in standard MKS units is

$$K = 1.42 \times 10^5$$

The air contained in the rigid wall cavity of volume $\mathbf{V_0}$ behaves like a spring under a compressing (or expanding) force. As formerly observed, the lengthening of a spring under a force **F** applied at one end is $x = F/k$ and **K** is the stiffness of the spring; the higher the stiffness **K** the minor the lengthening. The equivalent parameter applied to a cavity enclosing a volume of air is the mechanical compliance $\mathbf{C_m}$ which is the reciprocal of the stiffness and is defined as

$$C_m = \frac{V}{KS^2} = \frac{V}{c^2 \rho S^2}$$

where S is the surface on which the force is applied. So this is the parameter we need to evaluate the characteristics of the oscillation in the cavity.

While—as seen in Chap. 3—the natural frequency of a mass-spring system is

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

the natural frequency of a cavity is

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{C_m m}}$$

where m in this case is the mass of the air volume, therefore $m = V \times \rho$.

Appendix 4.2: Air Frequency in Mode ⟨0 1⟩

In mode ⟨0 1⟩ the pressure wave in a guitar-shaped cavity develops along the longitudinal axis and propagates at velocity **c** of the sound in the air ($c = 343.3\,\mathrm{m/s}$)). The wave travels as a half sinusoid through the length **l** of the body. In other words this is a *longitudinal half-wave mode of vibration*. So in this mode the wave traces a full sinusoid over a distance *twice* the length **l** of the body. This span (between two consecutive maximum points of the wave) is called *wavelegth* **λ**, which in this vibration mode is written as

$$\lambda = 2l$$

The wave inside the cavity covers the distance between two consecutive maximum or minimum points in a time **T**, and the relation between the three units involved (propagation velocity c, covered distance **λ** and time **T**) is plainly

$$velocity = \frac{travel\,length}{time} \quad \text{therefore} \quad c = \frac{\lambda}{T} = \frac{2l}{T}$$

But the time **T** required for the wave to trace a full sinusoid *at the natural frequency associated with this particular vibration mode* (mode ⟨0 1⟩) is also the period of this frequency, or $T = 1/f$.

By combining these correlations we can determine the theoretical formula for the natural frequency of the cavity in mode ⟨0 1⟩:

$$f = \frac{c}{\lambda} = \frac{c}{2l}$$

The formula $f = \frac{c}{\lambda}$ is valid for all of the vibration modes we are concerned with in this text, provided that, case by case, the wavelength **λ** is duly associated with one of the instrument dimensions. This connection will be reviewed in each of the modes we are going to deal with.

Appendix 4.3: Air Frequencies Measured in a Reference Guitar

Frequency (Hz)	Mode
129	⟨0 0⟩ (*Helmholtz resonance*)
395	⟨0 1⟩
580	⟨1 0⟩
725	⟨0 2⟩

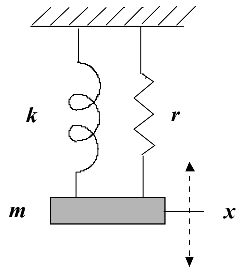
(continued)

(continued)

Frequency (Hz)	Mode
770	⟨1 0⟩ (*in the upper bout*)
954	?
1009	⟨2 0⟩
1179	⟨3 0⟩
1270	?

Appendix 4.4: Helmholtz Resonance

The Helmholtz resonator behaves like an oscillating mass-spring system whose operation model is:



As we have observed, the natural frequency in this system is

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

where **k** is the stiffness of the spring, **m** is the mass and **r** is the parameter that sums up the energy losses in the system.

In the Helmholtz resonator

- Mass corresponds to the weight of the air in the neck, so
  Mass $=$ *length of the neck* $\times$ *surface* $\times$ *density of the air* $(m = l\, S\, \rho)$
- The elastic component is derived from the volume of the air in the body. In Appendix 4.1 we provided the mathematical expression of the compliance of the air volume (which is the reciprocal of the stiffness):

$$C_m = \frac{V}{K S^2} = \frac{V}{c^2 \rho S^2}$$

where S is the surface of the opening. Introducing the expressions of mass and stiffness into the frequency general formula of a mass-spring system we get the

traditional expression of the Helmholtz resonator natural frequency:

$$f_H = \frac{c}{2\pi}\sqrt{\frac{S}{lV}}$$

For thin-walled resonators or for those where the openings have a particular shape (like the *ff* in the violin) the parameter l must be replaced by the parameter $l_{effective}$ which considers how the air flows into the opening forming a sort of virtual channel.

In the case of a circular opening the effective length of the resonator neck was calculated as follows:

(Rayleigh—**Theory of Sound**—1878):

$$l_{actual} = l_0 + \frac{\pi}{2}r$$

Chapter 5
The Resonator as a Global System

Abstract In this chapter the individual components presented heretofore are tied together, and the interactions of string, top, back and body are analysed. The interaction string-resonator is presented introducing the concepts of string impedance and reflection factor. The global resonant system is studied as a three degree of freedom represented as an analog circuit in Appendix, and the important subject of the coupling between oscillators is discussed as the basis for the formation of the fundamental resonances of the guitar through the interaction between air in the cavity and the front and back plates. The equations of the model allow determining several parameters like vibrating mass, stiffness, effective areas of plates, coupling coefficient. The parameters can be determined from a few basic resonances measurable in a finished instrument. Conversely, given a set of basic resonances, the model helps selecting the natural mode of the top and back and the Helmholtz resonance to support the design phase. Typical values of the parameters are given as an example. Special attention is dedicated to the influence of the back and its coupling to top and air through the contribution of its modes.

In the last chapter we stated that the functioning of the guitar resonator is based on three fundamental oscillating systems: soundboard, back, and the air contained in the resonator. The latter is comparable to a *Helmholtz resonator*—at least in its fundamental oscillation mode—while back and soundboard can be compared to elementary systems like *plates*, which we have examined discussing the generic properties of oscillating systems.

As already observed, other oscillating systems exist, like sides and neck, that play a minor role in the resonator functioning. The bridge itself, which is a partially rigid element, deeply influences the soundboard properties by its weight and shape.

When back and soundboard are glued to the sides, so enclosing the instrument body, the resulting object (the body connected to the neck) assumes new specific characteristics (new resonances, new amplitudes, new damping progress). These turn it into a new system, whose *global* behaviour depends on the *individual* behaviour of its components, though not corresponding to any of them. Just so when a number of persons join in a group, each maintaining their own personality: the group assumes a distinctive global identity which—generally—is not identified with any of the

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members, even though the parlance of the group is the product of the single characters, of the *interchange* and *interaction* between them.

Based on this analogy, we deduce that the global behaviour of the new object created by gluing soundboard and back to the sides, so enclosing the instrument body, depends on the interaction between the three fundamental oscillating systems.

Having examined the fundamental characteristics of elementary systems, and those that are specific of soundboard, back and air in the body, all of the required elements are now available for an evaluation of these exchanges and interactions, and so their effect on the guitar sound.

5.1 Fundamental Resonances

The resonator behaviour in the mid-low frequency register is defined by two single and very evident resonances. The lower one usually takes place between 90 and 110 Hz. The upper one arises typically between 180 and 220 Hz—generally an octave apart from the lower one. The two resonances *always* appear in every guitar, independent of the quality and characteristics of the instrument, since they are connected to the resonator structure, which is basically unchangeable (two flexible tables enclosing an air volume that communicates with the exterior through an opening—the soundhole—in one of them). Other stringed instruments—as those belonging to the violin family—have a resonator structure similar to that of the guitar; so they, too, feature two main resonances, though at different frequencies.

But the mere manifestation of two resonances in the mid-low register does not account for equal sound quality in different guitars. As we will see afterwards, the quality of sound does not merely depend on the presence of these resonances but, mainly, on their specific characteristics, that is to say on frequency, amplitude, and damping behaviour. Of course, sound quality also depends on the characteristics of other resonances we will review later on.

Hereinafter we will call these either *first basic resonance* or *air resonance* and *second basic resonance* or *soundboard resonance*.

Air resonance (occurring between 90 and 110 Hz) is often and incorrectly called Helmholtz resonance. In fact the two must not be confused with one another, since air resonance also depends—but not exclusively—on the Helmholtz resonance already discussed in the previous chapter.

Similarly, the second basic resonance (occurring between 180 and 220 Hz) must not be confused with the soundboard natural frequency in mode $\langle 0\,0 \rangle$ that, as seen in the previous chapter, is one of its essential characteristics, and can be measured by fastening the soundboard to a rigid frame. So the second basic resonance depends on the natural frequency of the soundboard but does not coincide with it.

In the guitar, the first and sixth open strings are respectively tuned at 329.6 and 82.4 Hz. The tones comprised in this interval (pertaining to the mid-low register) are particularly accentuated by the basic resonances: the closer a partial tone to a resonance peak, the louder the radiated sound. Yet if the tone is very close or coincides

with a resonance, the decay time of the tone is particularly brief: a high volume of the sound goes with a low sustain, so the sound assumes a percussive, not very agreeable character. This is because, in this condition, the resonator absorbs very quickly the oscillating energy of the string. We have already detailed this phenomenon in the chapters concerning the string (Chap. 2) and the elementary oscillators (Chap. 3). For the reader's convenience, we will recall concepts expounded there, but this time considering the resonator as a global system.

A distinctive attribute of the string is the *Characteristic Resistance* R_c.

$$R_c = \frac{Longitudinal\ Tension}{Wave\ propagation\ speed} = \sqrt{\mu T}$$

that is the relation between force applied (the tension **T**) and propagation velocity of the waves that travel along the string, independent both of the fundamental the string is tuned at, and its upper harmonics. In fact it only depends on tension **T** and mass per unit length $\boldsymbol{\mu}$.

At the clamping point of the string with the bridge, the resonator as well presents an impedance Z_r, defined by the ratio between force applied and displacement velocity. In this *excitation* point, impedance is very low at resonant frequencies, and the resonator is more 'inclined' to exploit the string energy and propagate it through the air as sound pressure. At frequencies where impedance in the excitation point is great (and velocity low—the *antiresonances*—) the resonator is 'reluctant' to give back the energy of the string as sound pressure.

This *propensity* of the resonator to exploit the string energy is expressed by the *reflection coefficient*

$$\rho = \frac{R_c - Z_r}{R_c + Z_r}$$

At antiresonances (where the resonator impedance Z_r is great) the *reflection coefficient* is -1, meaning that the incoming wave reflects almost completely, without energy absorption by the resonator; the negative sign indicates that the incoming wave reverses specularly at the bridge and changes its polarity. On the contrary, at resonant frequencies (where impedance is small) the incoming wave is mostly absorbed by the resonator; the reflection is too scarce to support the oscillation of the string which, as a result, undergoes a quick damping, while all of the available energy is rendered as sound pressure through the oscillation of the surfaces that define the resonator (soundboard, back, soundhole). Therefore, this mechanism is connected to the impedance *seen* in the point where the string is fastened to the resonator (the excitation point, namely the bridge). This impedance is a complex function of the resonator response in terms of frequency, where resonance peaks alternate with antiresonances.

In order to achieve a well-balanced sound in the mid-low register it is necessary and—within limits—possible to regulate the position of the resonances, so that they do not exactly match any tone of the scale but lie between two semitones. Further on

in this chapter we will investigate the relationships between the resonator impedance at the bridge and the values of parameters (mass, stiffness, damping, vibrating area) that distinguish its elementary components. We will also study the kinds of interaction between these elementary components, that is to say how they interchange share the energy drawn from the bridge, and—finally—how important this interaction is in the development of resonances and antiresonances. We will then see how to influence the basic resonances. Finally, we will be pushing off into an almost unexplored sea, which is essentially dominated by the resonances of the back.

Now what is the consequence of a semitone lying close to, but not coincident with a basic resonance?

The string, plucked and released at the moment of maximum displacement, enters a free-vibration stage: upon release a travelling wave starts off, running from the excitation point towards the bridge (cfr. Chap. 2 for a deeper analysis). Here, before turning back to the nut, the wave generates a *force impulse* at the bridge; this impulse sums up in its shape, duration, and amplitude, what we have called the string motion 'recipe', therefore the frequencies and amplitudes of the fundamental and of the whole sequence of related harmonics, which in turn are connected to the way and place of plucking. Part of the energy associated with this force impulse is exploited by the resonator, while the rest reflects back to support the contrary motion of the standing wave, from the bridge to the nut. The reflection mechanism of the wave at the bridge is governed by the previously-mentioned *reflection coefficient*.

The *unreflected* part of the force impulse brings about a sort of shock in the resonator, which reacts by trying to vibrate *at its own natural frequencies* (primarily those of the basic resonances). At the same time the excitation mechanism (the string) strives to impose *its* oscillation frequencies—the fundamental and the harmonics. Therefore, in the time following release (the *attack time*) both the natural frequencies of the resonator and the 'enforcing' frequencies of the string are present; so the sound spectrum includes both the fundamental of the string (the played tone) with its harmonics and the resonator modes excited by the force impulse at the bridge. As seen in Sect. 3.2, the interference between the excitation frequency and the natural frequency brings about a *beat* (phenomenon), due to the exchange of energy between the generator (the string) and the oscillating system (the resonator), while the amplitude increases and diminishes with a frequency $f_{natural} - f_{excitation}$.

At the end of the attack transient the string (*if no damping has yet occurred*) has succeeded in imposing its own vibration frequency to the oscillating system.

Once the external excitation has been removed, the oscillating system keeps on vibrating, but *at its natural frequency*: the oscillator holds no 'memory' of having been previously forced by the string into a different frequency than its natural one. Amplitude gradually diminishes by a decreasing exponential law, till the available elastic energy is completely dissipated through losses due to inner friction or radiation. During the *decay transient*, a single line associated with the natural frequency of the oscillating system would be displayed by the spectrum, those of the excitation signal being expired.

We wish to remind that the guitar only 'works' in transient conditions. The string excitation (attack time) is immediately followed by a decay time, differently from other (bowed or wind) instruments, where the excitation is normally sustained.

During transients, the amplitude of the sound pressure grows or decreases by an exponential law, of which we have given the mathematical definition in Chap. 1 (Appendix 1.3). This exponential law is defined by the *time constant* **τ**, depending on the mechanical parameters of the oscillating system (mass **m** and friction coefficient **r**). See also Sect. 3.2.

$$\tau = \frac{2m}{r} = \frac{Q}{\pi f_{natural}}$$

The *time constant* **τ** in turn depends on the quality factor **Q** we introduced previously, which at resonance corresponds to

$$Q = 2\pi f_{natural} \frac{m}{r}$$

During the attack time, in order to impose its own frequencies, the string must essentially prevail over the inertia of the resonator (which behaves like a flywheel getting into motion). On the contrary, during decay, the gradual amplitude decrease is basically due to losses brought about by viscous friction or radiation. In other terms, the time constant **τ** is very different in the two transients. An instance of this has been provided in Chap. 1, when dealing with the tuning fork sound.

In Chap. 1, studying the guitar sound, we considered the sound radiation delivered by pressing the A on the second fret of the third string. We observed that the sound pressure signal (picked up by a microphone and sent to a computer for further processing) can be examined from three points of view:

– The <u>waveform</u> (*time domain* representation).
  The chart illustrates the development over time of the sound pressure amplitude, during attack and decay transients.
– The <u>spectrum</u> (*frequency domain* representation).
  The chart shows the amplitude of the sound components due to the force impulse transferred by the string to the resonator, the fundamental, and the associated harmonics. It also shows lines that, though not belonging to the string motion recipe, correspond to resonances excited by the force impulse; most relevant among these are the air resonance (the *tuning note*) and the soundboard resonance.
– The <u>spectrogram</u>, that describes the development over time of the spectrum (*time/frequency* representation).
  The Waterfall Chart—presented in Chap. 1—offers a perspective of the variations occurring over time in the amplitude of spectral lines, and provides the amplitude decrease rate of each harmonic that is present in the sound recipe once the string has been released.

We notice on the graph that some of the harmonics rapidly die out, while others tend to last longer, according to losses in the string, losses in the resonator (radiation resistance included) and impedance that each harmonic sees looking to the resonator.

The *sound recipe* dynamically changes over time. This is a fundamental aspect for determining the character and sound quality of an instrument.

These are complementary, not alternative representations. Each of them can contribute to understand the resonator reactions to the stimulus it receives from the string and, accordingly, how the sound pressure we perceive is generated.

We pointed out that

- The harmonics of the string motion that are close to a resonance are enhanced.
- The harmonics that are close to an antiresonance are damped down.
- The string can even excite distant resonances, for instance the air resonance and the soundboard resonance.

Generally, *the air in the body is easily driven, even if the excitation frequency is much higher*. On the contrary, *the soundboard resonance is not so easily driven*, even when lying closer to the excitation frequency. In fact, air is more elastic than wood, and more inclined to be set into vibration by an external excitation through the mechanism we have explained. This mechanism is defined by the reflection coefficient, hence by the resonator impedance at basic resonances.

In Chap. 1 we presented the overall diagram of decay times, drawn from the spectrogram of an exemplar sound (the A on the second fret of the third string). Looking at this graph we can see that the spectral line relative to the resonance of the air shows a rather long decay time, comparable to the segments relative to the string. As for the soundboard basic resonance, the case is different: string partials that lie somewhat apart from it can excite the soundboard, but with limited amplitude. Furthermore, its decay within the sound texture is pretty fast, as shown by the example in Chap. 1. We will see more clearly afterwards that the soundboard resonance has generally a great amplitude but a smaller bandwidth than the air, so its influence is great on the string partials comprised in this bandwidth but scarce on the others.

The described mechanisms are common to *all* guitars, but they combine with the characteristic features of *each* instrument, and constitute a fundamental aspect for determining the sound character of the single tones. Please refer to Sect. 1.6, where we examined the connection between these physical parameters and the quality of sound in a guitar.

From all of these aspects, the correct positioning of resonances takes on a crucial meaning for the design and construction of the guitar. It is necessary to achieve a comprehensive understanding of the factors that contribute to these resonances, and how the luthier can act on them. This will be the topic of the following paragraphs.

While the two basic resonances typically arise between 90 and 220 Hz, in the register just above (between 220 and 400 Hz) we find the resonances due to the back (one of the three fundamental oscillating systems of the guitar resonator) and the '*third*' resonance due to the soundboard.

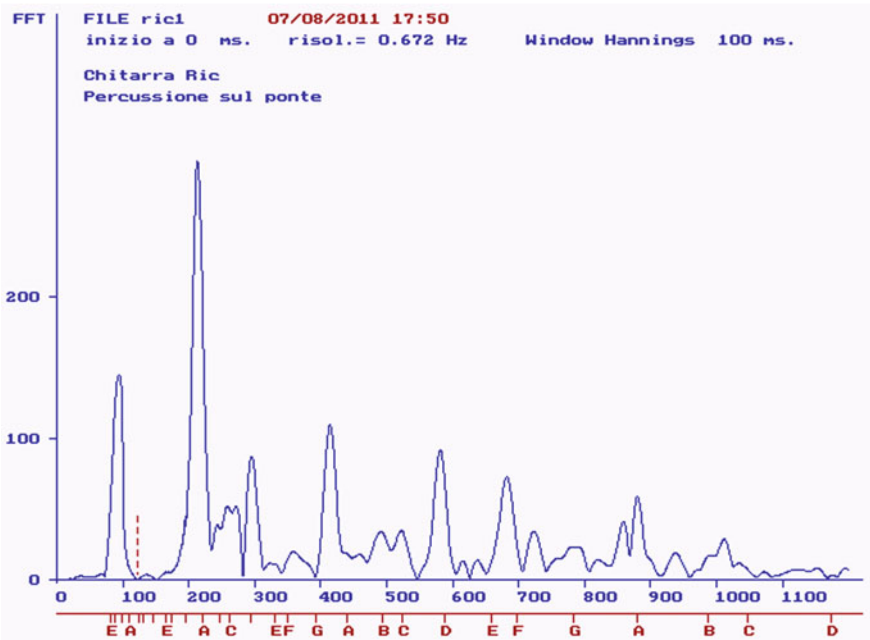
In summary, according to the classification we will use in this chapter, from 90 to 400 Hz the guitar resonator features the following resonances: *first basic resonance* (or *air resonance*); *second basic resonance* (or *soundboard resonance*); *resonances of the back*; *third resonance* of the resonator. Beyond 400 Hz we find other resonances that, owing to specific characteristics, affect the quality of sound; among them the fourth resonance is most important. These resonances will be discussed in Chap. 6.

Further on in this chapter we will investigate the determining factors of resonances and antiresonances in the mid-low register (between 90 and 220 Hz). We will begin by identifying the basic resonances, then the upper ones, just like an explorer begins by ‘naming’ the most outstanding peaks before familiarizing with minor elevations.

To understand the origin of the basic resonances we put forward a series of tests performed on a significant Garrone guitar. Luthiers can perform these tests on their own guitars by using method and devices that will be fully described later on. Of course they will get different results from a quantitative point of view—since every instrument holds its own ‘character’—but similar in general aspects. Following the path we suggest, they will be able to evaluate and interpret their results, getting useful indications for further improvement.

5.2 Nature of the Fundamental Resonances

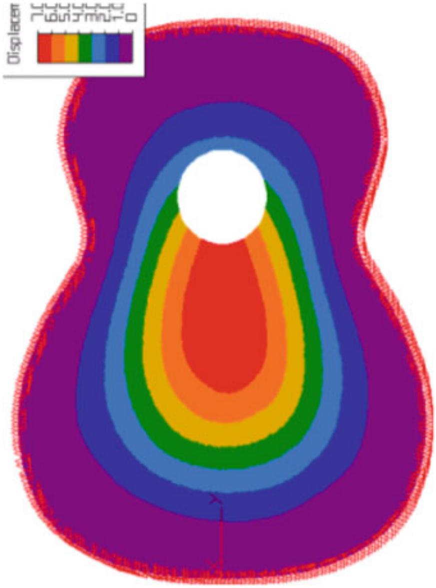
First of all we must acquire the global response of the instrument (in terms of sound emission) in a band of frequency reaching at least 1200 Hz. This band, represented in the diagram below, covers the fundamentals and the partials of tones in the mid-low register, as well as the fundamentals up to the highest *D* on the first string. All this is reported on the graph.



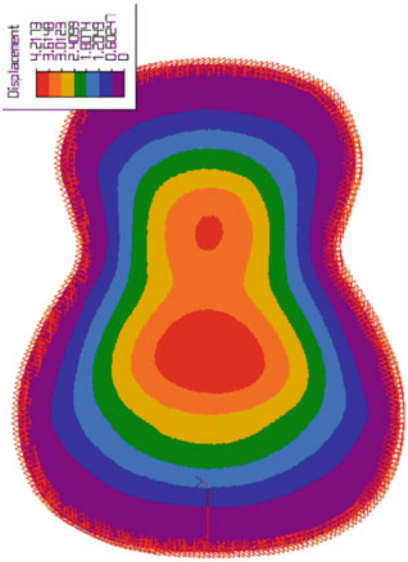
The first two peaks represent the basic resonances, at 93 Hz and at about 213 Hz. We remind that the resonance of the soundboard fastened to the mould in mode ⟨0 0⟩ was about 155 Hz (corresponding to the *natural frequency of the soundboard in mode* ⟨0 0⟩) and the Helmholtz resonance in a Garrone guitar typically appears at 129 Hz, as the analysis executed separately on the elementary components of the resonator revealed.

When the fastened soundboard vibrates at its main frequency (mode ⟨0 0⟩ or *ring mode*) the oscillating energy mainly concentrates under the soundhole, and the nodal line runs along the soundboard periphery—at least when the braces are correctly sized. The two following illustrations respectively show the FEM simulation and the Chladni pattern of a soundboard that vibrates in this mode.

In this situation the natural frequency typically arises between 140 and 180 Hz. So the natural resonance of our soundboard (155 Hz) fits well in this interval.



For this instrument we measured the resonance of the back fastened to the mould, that is the natural resonance of the detached back in mode ⟨0 0⟩, found at 215 Hz. We remind that when the fastened back vibrates at its main frequency (mode ⟨0 0⟩) the oscillation almost involves the whole surface and—when braces are properly sized—the nodal line runs entirely along the soundboard periphery.

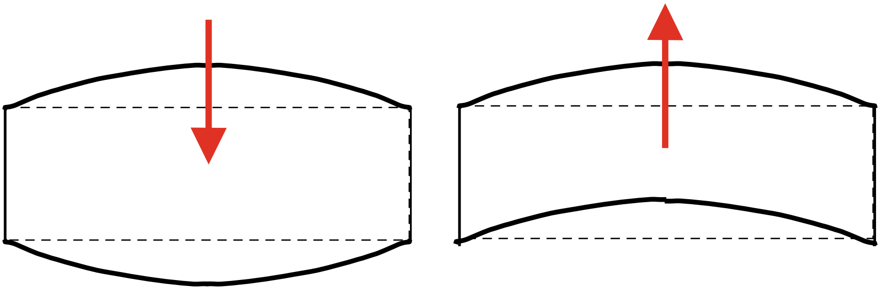


In summary, the two basic resonances (at 93 and 213 Hz) are far from similar to the typical frequencies of the three oscillating systems that compose the guitar resonator: the soundboard (155 Hz), the back (215 Hz), and the Helmholtz resonator (129 Hz)! We reason out this fact by first providing a qualitative explanation and, further on, a deeper quantitative clarification.

When the soundboard vibrates free in the air—detached from the body though fastened to the mould that blocks its periphery—the oscillating motion meets an almost nil resistance (except the *radiation resistance*) and its own frequency depends exclusively on the stiffness and mass of the soundboard (cfr. Chap. 3 on elementary oscillating systems). When, on the contrary, the soundboard is fastened to a body enclosed by sides and back, the oscillating motion tends to compress (or expand) the air in the body; so the compliance of the air comes into play and, owing to the elastic nature of the air, counteracts the soundboard oscillation. As a result of this coupling soundboard/air in the body, the vibration frequency of the soundboard increases, shifting in our case from 155 to 213 Hz (the *second basic resonance*).

At the same time, during the oscillation of back and soundboard, the body walls are not rigid as they were when we defined and measured the Helmholtz resonance (see Chap. 4 on elementary components). The air volume shrinks or expands following the deformation of the flexible walls. This means that the *compliance* of the air in the body rises, so the natural frequency of the cavity (or body) diminishes, shifting from 129 Hz (the Helmholtz resonator frequency) to 93 Hz (*the first basic resonance*).

To better understand these notions we are going to see how the back and soundboard move in connection with the basic resonances. As shown in the following figure, at the first air resonance, back and soundboard move in *antiphase*: as the soundboard moves upward, the back moves downward; consequently, the volume of the body increases and the air flows into the body (and vice versa). So the air flows through the soundhole in the opposite direction (in antiphase) with respect to the soundboard motion. In this situation, the movement of the walls diminishes the stiffness of the air in the body, and so its resonant frequency—shifting in this instrument from the Helmholtz frequency (129 Hz) to the first basic resonance (at 93 Hz).



F = 93 Hz ***F = 213 Hz***

The antiphase movement between the air in the soundhole and the soundboard brings into being a *vibration dipole*: if we consider the soundhole and the soundboard like two vibrating pistons, the two pressure waves they generate oppose each other with a tendency to mutual extinction—at least beyond a certain distance from the soundboard, or in the *far-field*. This dipole is fortunately asymmetric, since the surface of the soundhole is much smaller than the vibrating surface of the soundboard (the ratio being normally of 8–9). Further on we will better define the 'vibrating surface' of the soundboard, which does not correspond to its geometrical surface.

In order to support emission at the first basic resonance the vibration dipole must be asymmetric, which can be obtained by different expedients, for instance

(a) Increasing the vibrating surface of the soundboard by modifying the bracing.
(b) Displacing the soundhole upward like, for example, the two-hole Garrone guitar, where this brings about an extension of the vibrating surface.

Anyway we leave the luthier, who has understood the mechanism of sound production at the first basic resonance, the pleasure of finding alternative solutions.

At the first basic resonance instead, back and soundboard move *in phase*: the back follows the soundboard motion and the soundhole behaves like a rigid piston considered as a built-in part of the soundboard. So the air flows now through the soundhole in the same direction (in phase) as the soundboard motion. No vibration dipole is brought into being, because now the two sources of sound pressure act in

concert; therefore, at the second soundboard resonance a powerful sound emission is released.

The motion of back and soundboard is antagonized by the elasticity of the air that acts like a spring. Accordingly, the soundboard resonance rises, shifting from 155 Hz (when the soundboard is fastened but moving in free air) to 213 Hz (when fastened to the body).

This tells us that the air in the soundhole, with its alternate motion, acts like an *oscillating piston* characterized by mass (of the air in the soundhole), stiffness (of the same air), and loss coefficient (caused primarily by *radiation resistance*). So the soundhole, apart from being a fundamental component of the Helmholtz resonator, by means of the inflow and outflow of the air also contributes to the acoustic response of the instrument and to sound radiation.

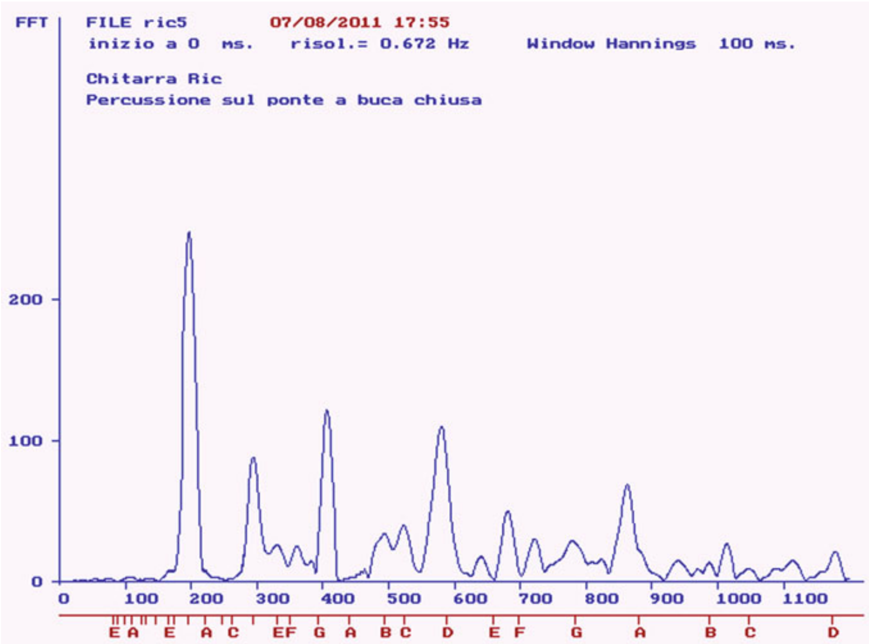
As already stated dealing with elementary oscillators, this system too presents a characteristic impedance. As the vibration frequency rises, the *impedance of the soundhole* also rises, since the airflow meets an ever growing opposition, till total extinction; from now on the soundhole behaves as if it were closed.

Between these extreme conditions due to the two basic resonances a third condition exists, that is when the soundboard velocity is equal but opposite (in antiphase) to the air velocity. In this condition, both velocity and displacement amplitude of the soundboard are nil. This occurs at the Helmholtz resonance, where sound emission is virtually nil, and can be observed on the previous graph as a *prominent antiresonance between the basic resonances*.

To familiarize with the elevations of our 'mountain range', we must now observe the peaks from different perspectives. That is to say we must perform additional measurements on the guitar resonator, and deepen our knowledge of its operation by a comparative reading of results acquired from different situations.

The next diagram depicts the 'covered soundhole' response. We covered the soundhole with a disc of expanded polystyrene exactly sized as the soundhole and fixed to the soundboard by adhesive paper strips. The advantage in polystyrene is lightness, but any soundproof material is just as good. This method suppresses the airflow through the soundhole without encumbering the soundboard.

The first thing we point out is a quite evident resonance at about 196 Hz. Secondly, the first basic resonance of the air (which with open soundhole came out at 93 Hz) has completely disappeared. The reason is rather intuitive: by covering the soundhole we have actually 'killed' the Helmholtz resonator, turning the resonator body into an enclosed volume subject to the oscillation of back and soundboard. These last oppose by their movement the elasticity of the air in the enclosed body, cutting off the inward and outward gate (the soundhole) for the air.



Under this condition a ‘covered soundhole’ resonance of the soundboard comes into being (visible on the foregoing diagram); in this guitar it takes place at 196 Hz, halfway between the value of the natural soundboard resonance measured on the mould (155 Hz) and the basic ‘open soundhole’ resonance (213 Hz).

In summary, we have so far identified and shown the evaluation procedure on the instrument of *five* important resonances:

- The *first basic resonance of the air* $\mathbf{F_1}$ (in this guitar at 93 Hz).
- The *second basic resonance of the soundboard* $\mathbf{F_2}$ (in this guitar at 213 Hz).
- The *Helmholtz resonance* $\mathbf{F_h}$ (in this guitar at 129 Hz).
- The *natural resonance of the soundboard* $\mathbf{F_{p0}}$ (in this guitar at 155 Hz).
- The *covered soundhole resonance* $\mathbf{F_p}$ (in this guitar at196 Hz).

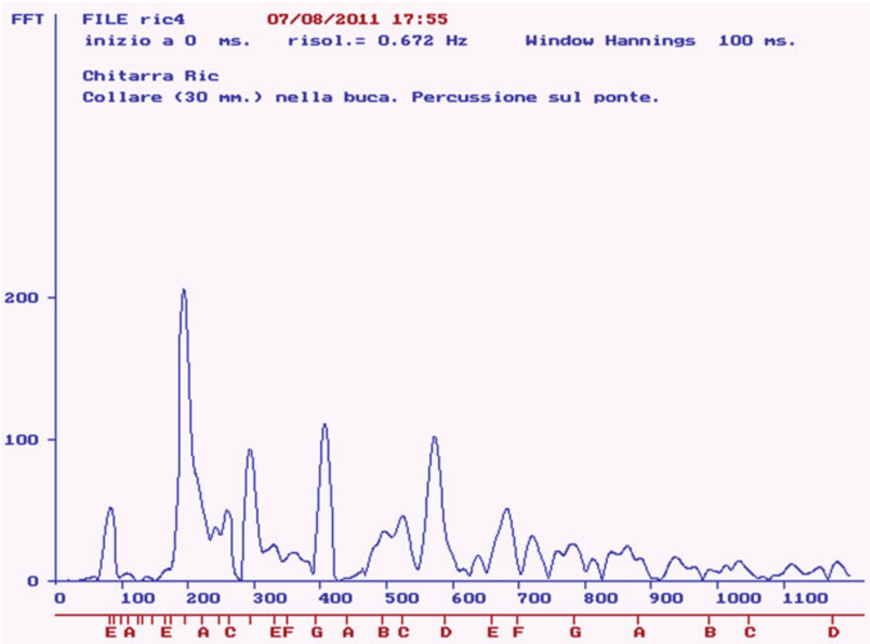
We will see that the covered soundhole resonance $\mathbf{F_p}$ plays a very significant role in the coupling phenomenon between soundboard, back, and air.

We are now going to propose a measurement performed after inserting a *collar* into the soundhole, so as to evaluate the new basic resonances that result from this altered resonator geometry. The collar consists of a 30-mm-high cylinder with the same diameter as the soundhole, made out of a veneer wood strip 0.5 mm thick.

We fit the cylinder into the body, bordering on the soundboard surface, and fix it provisionally with some adhesive tape strips.

It is not our purpose to reproduce the *tornavoz*, which was a structure used in former times—incorporated into the guitar resonator—by some famous luthiers like Torres, Simplicio, and others. Our collar is just an expedient for us to investigate some of the resonator properties. Nonetheless, as we will see, what comes out of this experiment allows us to find out some attributes of the real tornavoz, and guess its functioning in terms of sound quality.

The following graph represents the sound response with the 30-mm-high collar inserted into the soundhole.



The collar brings down the Helmholtz resonance, since it extends the length of the neck. The natural frequency of the Helmholtz resonator, as mentioned in Chap. 4, can be formulated as

$$F_h = \frac{c}{2\pi}\sqrt{\frac{S}{lV}}$$

Therefore, with an increased length l of the neck, the frequency F_h of the Helmholtz resonator must accordingly drop down.

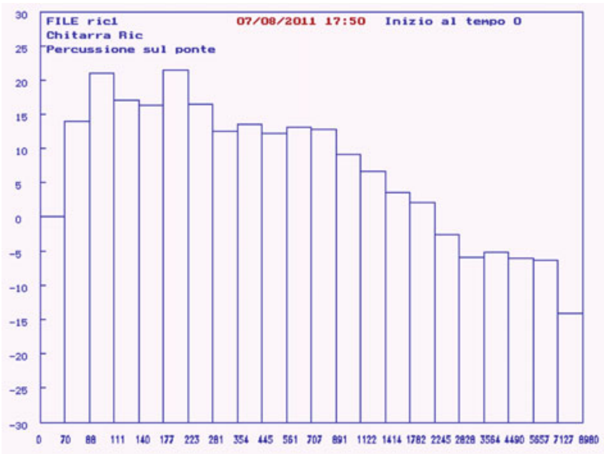
The effects of this frequency decrease are evident on the graph: the first air resonance shifts from 93 to about 83 Hz, and the second basic resonance (or soundboard resonance) also drops down from 213 to about 194 Hz.

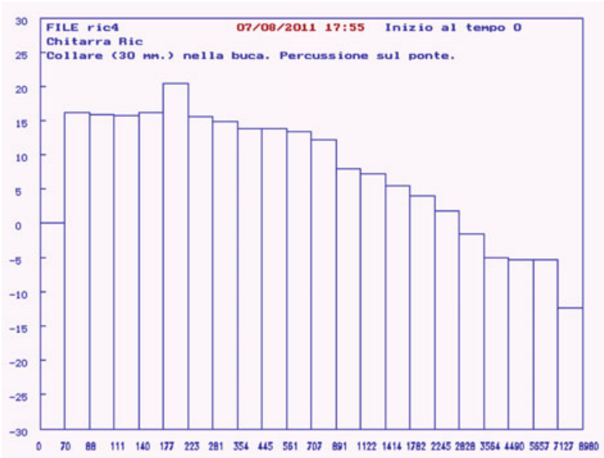
Also the amplitude of the air resonance is considerably reduced, going down from about 134 (cfr. the response diagram *Ric1*) to about 52 (in the diagram above, *Ric4*). This is due to the action of the collar as an obstacle to the airflow, which causes an increase of the *soundhole impedance*.

The collar has a much slighter effect in reducing the amplitude of the second basic resonance (or soundboard resonance), which goes from about 230 (response graph Ric1) to about 206 (response graph Ric4). This is because, at this frequency, amplitude is more influenced by other parameters (for instance friction losses in the soundboard) that are not significantly modified by the presence of the collar.

It is important to notice that the observed differences which, at low frequency, distinguish the response graph Ric1 (standard situation) from Ric4 (collar applied), diminish considerably beyond a certain frequency (about 300 Hz), where the two graphs become very similar with regard to both position and amplitude of the resonances. This is when, by increasing frequency, the air is no more able to flow through the soundhole; its impedance grows so much that the soundhole behaves as if it were covered. Obviously, under these conditions, the collar plays no significant role on the instrument response.

The following illustration puts side by side the response under standard conditions (file Ric1) to the response when the collar is present (file Ric4). This is a *third of octave* representation, and it shows that when the collar is inserted the response at mid-low frequencies is generally more flat, which implies a more balanced sound output. This occurs in the frequency range where the collar affects the impedance of the soundhole (that is, as observed, more or less below 300 Hz), leaning towards an equalization of the sound response.





Beyond this limit the response in the two conditions (with and without collar) is relatively similar, but we notice that in the highest frequency range (beyond 900 Hz) the response with the collar is improved, and contributes to a feeling of better balance in the whole frequency field.

From this reasoning we can draw some suppositions about the functioning of the real *tornavoz*. According to our interpretation, this object consented to control the length of the Helmholtz resonator neck, and so its frequency. We have seen that, in fact, a collar inserted into the soundhole diminishes the natural frequency of the Helmholtz resonator, and brings down both the first resonance of the air and the second resonance of the soundboard. In keeping with this analysis, the tornavoz allowed to bring down the tuning note of the instrument providing a darker, lower-toned sound.

But we have also observed that the collar tends to equalize the sound response in the mid-low frequency register and to enhance the response in the high frequency register, so producing a more balanced response in a quite large frequency range. We can presume that the real tornavoz had a similar effect, i.e. leading to a more balanced sound response, though at the cost of a *lower sound power* due to reduced radiation from the soundhole.

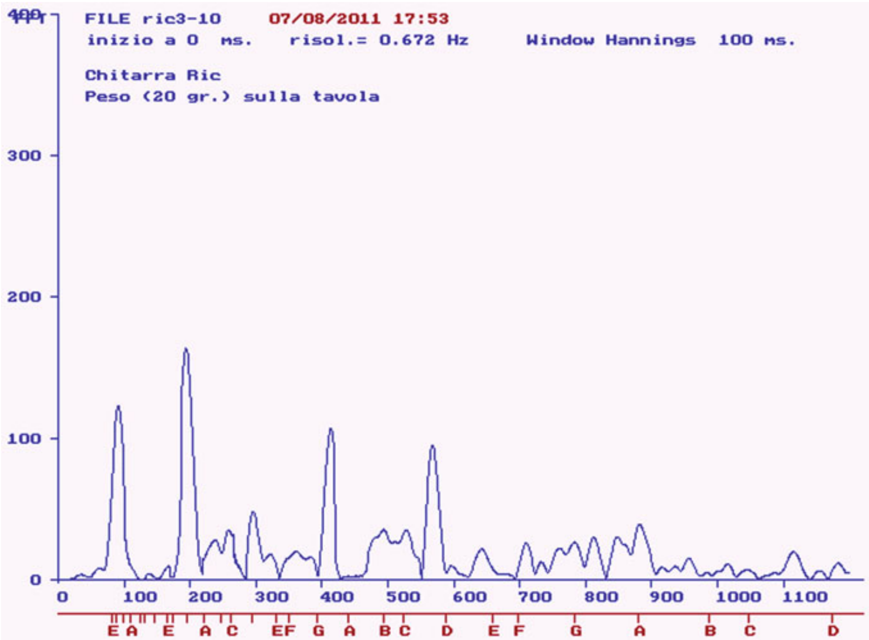
The last test we propose consists in measuring the sound response when the soundboard is loaded with a 20 g weight. The weight is provisionally fixed to the soundboard in the middle of the bridge.

The following graph illustrates the response under this condition. We can see that the first basic resonance of the air drops from 93 to 91 Hz, and the second basic resonance of the soundboard from 213 to 194 Hz.

Loading the soundboard with a weight, its natural frequency goes down from about 155 to about 137 Hz. In Chap. 3 we observed that the natural frequency of the soundboard in mode ⟨0 0⟩ depends on stiffness and mass, as defined by the formula

$$F_{natural} = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$$

where K is the stiffness of the soundboard (which is not affected by the load applied). Accordingly, it is logical that, increasing the mass on the soundboard, its natural frequency declines. It is yet important to notice that the decrease in the natural frequency of the soundboard from 155 to 137 Hz causes a *decrease in frequency of both the basic resonances*, and the additional mass causes a *decrease in amplitude of both the two basic resonances*.



Now, from the outcomes of the foregoing measurements we can draw some preliminary conclusions:

- The two basic resonances (at 93 and 213 Hz) diverge from the natural frequencies of the three oscillating systems that compose the guitar resonator: the soundboard (at 155 Hz), the back (at 215 Hz) and the Helmholtz resonator (at 129 Hz).
- The soundhole, apart from being a component of the Helmholtz resonator, is one of the resonator structural elements, and contributes to the acoustic response of the instrument by means of the air flowing in and out of it, at least until its impedance is low (up to around 300 Hz). If we cover the soundhole, both the basic resonances are altered: the first basic resonance of the air disappears altogether, the second drops from 213 to about 196 Hz.
- If we alter the attributes of a *single* resonator component, *both* the basic resonances are affected. First of all, in our measurements, we diminished the frequency of the

Helmholtz resonator by inserting a collar into the soundhole. As a consequence, the first resonance of the air drops from 93 to about 83 Hz, and the second soundboard resonance drops from 213 to about 194 Hz. Secondly, we altered the natural frequency of the soundboard by loading it with a 20 g weight. So the natural frequency shifts from 155 to about 137 Hz, resulting in a frequency decrease of both the basic resonances.

- On account of all these observations we are led to the conclusion that, *by acting on a single resonator component, we modify both the basic resonances*.

A global quantitative description of these phenomena must take into account that the elementary oscillators that constitute the single resonator components interact with each other, interchange energy, and mutually affect their frequency and vibration amplitude. In other words, they are coupled oscillators.

It is important to understand this interaction mechanism between the resonator components, and how this affects the basic resonances.

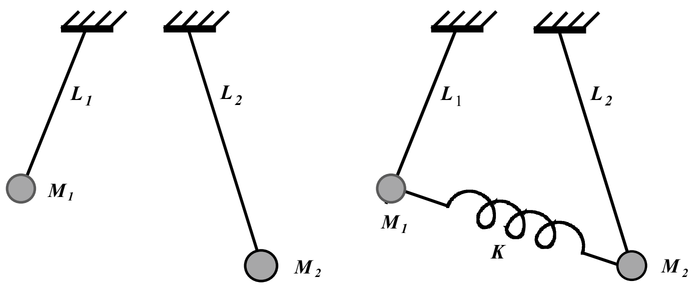
This will enable the luthier to optimize the position of the basic resonances and to adjust the instrument design.

5.3 Coupled Oscillators

The following left figure represents two pendulums that can independently oscillate at their own natural frequencies:

$$f_a = \frac{1}{2\pi}\sqrt{\frac{g}{l_1}} \quad f_b = \frac{1}{2\pi}\sqrt{\frac{g}{l_2}}$$

In this example, because of its greater length, the frequency f_b of the second pendulum is lower than the frequency f_a of the first pendulum. To be accurate, we must suppose the poles of both the pendulums to be rigid, in order to avoid longitudinal oscillations. The two pendulums oscillate each at their own natural frequency and amplitude which, because of the friction, tends to damp down progressively until the pendulums (at different times) recover their rest position. So this system is composed of two *independent* or *uncoupled oscillators* that do not interact with each other.



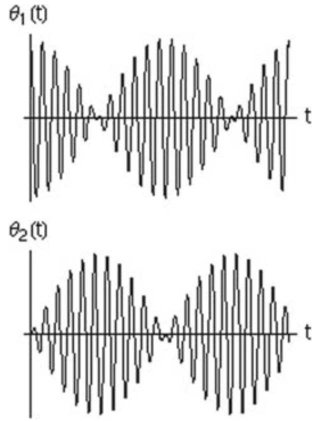
Suppose now connecting the two pendulums by an elastic medium like, for instance, a spring with stiffness **K** (right-hand figure).

Next, suppose displacing one of the two pendulums from its rest position, then releasing (no matter which, we will call this *pendulum A*). Now pendulum A starts oscillating from maximum amplitude, but, being fastened to *pendulum B* (initially still) by an elastic medium, it transfers to the latter a quantity of its own energy, setting it in motion. On each subsequent oscillation cycle, pendulum *A* transfers part of the energy (and motion) to pendulum *B*; consequently, its oscillation amplitude diminishes until pendulum *A* practically comes to a standstill. At that moment pendulum *B* holds all the remaining energy (initial minus losses), and the process reverses: now, in turn, pendulum *B* restarts pendulum *A* through the elastic coupling element. This energy exchange goes on until the initial energy is completely dissipated in losses due to friction with the surrounding medium (air in this case).

The following image illustrates the oscillation amplitude of the two pendulums. We can see an oscillation progress like the one that, in acoustics and electronics, is due to the *beat* phenomenon between nearby frequencies. Here θ_1 and θ_2 respectively represent the oscillation amplitude of pendulum *A* and pendulum *B*.

The exchange of energy between the two pendulums takes place at a frequency f_c (the *coupling frequency*), which is lower than f_a and f_b and depends on the stiffness **K** of the coupling spring. Intuitively, if the stiffness is low the coupling is *loose*, so the frequencies of the two coupled oscillators will be very close to the values of the original pendulums, and the exchange of energy will be minimal. If instead the coupling is *stiff* (that is if the spring is very rigid), the motion of the first pendulum will be almost entirely transferred to the second. Consequently, the two pendulums oscillate at a nearly equal frequency *f*, yet different from their original frequencies f_a and f_b.

Between these two extremes, countless in-between situations exist, that are expressed by a *coupling coefficient* related to both the stiffness **K** of the spring and the parameters of the system when uncoupled. We will see ahead how to assess the coupling coefficient with practical reference to the guitar resonator.

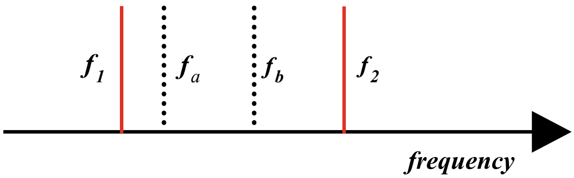


Therefore, because of the elastic coupling, the two pendulums oscillate now respectively at frequency f_1 and f_2, different from the original frequencies f_a and f_b.

We started from a system composed by two *uncoupled oscillators*, thereby building a new object (a system of *coupled oscillators*) wherein the two original components lose their initial nature to become part of a *global system*, whose response (frequency and amplitude) depends both on the attributes of the original components and their interaction.

This system has two natural frequencies f_1 and f_2 (not corresponding to the natural frequencies f_a and f_b of the two original oscillators) and a coupling frequency f_c.

Regardless of mathematical details in this system, we just point out that, on the frequency scale as drafted below, the two original frequencies *tend to diverge*, while the natural frequencies f_1 and f_2 of the coupled oscillators lie respectively left and right of the two original frequencies f_a and f_b.



Now we have found five characteristic frequencies in our system:

- The two frequencies f_a and f_b of the original oscillators.
- The two frequencies f_1 and f_2 of the coupled oscillators.
- The coupling frequency f_c.

There is an important relationship among these parameters, that is valid for <u>all</u> the coupled oscillators, hence for the guitar resonator, too:

$$(f_1^2 + f_2^2) = (f_a^2 + f_b^2)$$

The sum of the square of resonant frequencies in the coupled system equals the sum of the square of resonant frequencies in the uncoupled system.

Yet this 'theorem' does not enable to obtain the resonant frequencies of the coupled system (f_1 and f_2) from given frequencies of the uncoupled system (f_a and f_b). To do that, we should know and introduce into the equation the *coupling coefficient*. We will see later on how to calculate the coupling coefficient, and settle the equation with practical reference to the guitar resonator.

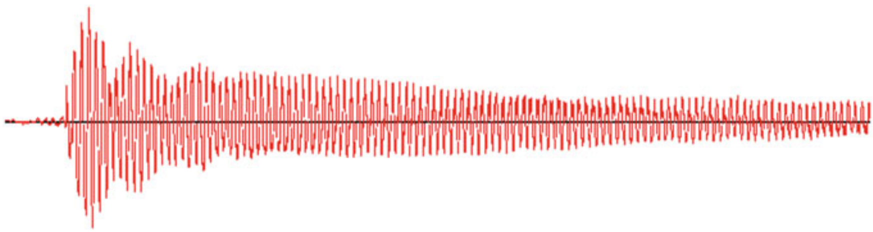
Now the question is: *why do we go so far into details about coupled oscillators*? The answer is twofold: we must do it in order to understand both the physical behaviour of the resonator and the effect these interactions produce on the guitar sound. These are actually complementary, not alternative aspects, of the same issue. In summary, the mechanism of coupled oscillators is crucial in guitar acoustics. We can assert that, in the guitar, whichever oscillating system couples with other oscillating systems with whom it exchanges energy, and the global response is influenced by the interaction between each system and the ones it cooperates with. In the previous

paragraph we observed that *by individual action on one of the resonator components both the basic resonances are modified*. This can only be accounted for by an interaction between the elementary oscillators that constitute the guitar resonator (soundboard—air—back) in keeping with the mechanism of coupled oscillators.

Another example is the *string* oscillator coupled with the resonator. The string does not merely transfer energy periodically to the resonator: as already observed, part of the energy stored in the string is absorbed by the resonator, while another part is reflected back to the string, proportionate to the value of the characteristic resistance of the string itself with respect to the resonator impedance. This energy exchange phenomenon between two oscillating systems (string and resonator) is typical of coupled systems. When the coupling between string and resonator is very stiff (i.e. when the fundamental frequency of the string is very close to one of the resonator characteristic frequencies) the *beat* phenomenon illustrated in the preceding figure can be clearly perceived: the sound amplitude rises and diminishes periodically, resulting in the phenomenon identified as *wolf* (*tone*).

Aside from this 'pathological' condition, in Chap. 1, dealing with the guitar sound, we observed that the string/resonator interaction is affected by the coupling between the resonator components (soundboard—back—air in the body): once the string has been released, during the decay transient the sound radiation amplitude is modulated at a relatively high frequency, as a result of the coupling between the fundamental of the tone fingered on the string and the fundamental resonances of the resonator.

We go back now, for the sake of argument, to the example provided in Chap. 1. The image depicts, in a 500-ms. window, the microphone recording of an **A** (220 Hz) played on the third string.



We can see how the amplitude progression closely resembles the previous illustration of the oscillation amplitudes relative to the two coupled pendulums.

In concluding, as a last instance, in Chap. 3 about oscillating systems we saw that the co-vibration of a plate with an attached wood strip can be explained by the mechanism of coupled oscillators.

The measurements we carried out on an instrument and the considerations about coupled oscillators enable us now to look deeper into the guitar resonator, and build a model of it that will help us to operate on the physical parameters, aiming to optimize the design.

5.4 The Resonator Model

The next scheme outlines the guitar resonator: a cavity with rigid side walls. From the upper face two structures face the surrounding ambient, comparable to two *vibrating pistons* respectively representing the soundboard and the resonator soundhole. These vibrating pistons are respectively defined by surface S_p and mass m_p (the soundboard) and by surface S_b and mass m_b (the soundhole). In the same way, from the lower face a third vibrating piston, representing the resonator back, faces the air. Each vibrating piston is a system that oscillates, as already observed, at a natural frequency dependent on its mass, stiffness and loss coefficient; on the sketch these parameters are labelled $m_p - K_p$—r_p for the soundboard, and $m_f - K_f$—r_f for the back. Based on a formula we have already seen, the *natural soundboard resonance* F_{p0} is

$$F_{p0} = \frac{1}{2\pi}\sqrt{\frac{K_p}{m_p}}$$

while the natural resonance of the back is

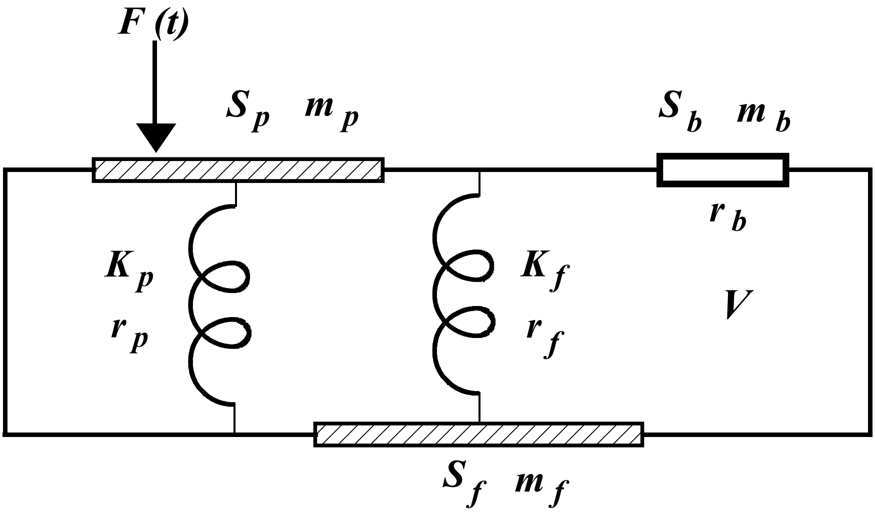
$$F_f = \frac{1}{2\pi}\sqrt{\frac{K_f}{m_f}}$$

The third oscillating system is composed by the air in the soundhole (mass m_b and surface S_b) and the elasticity of the air volume $\boldsymbol{V}$. These parameters define the *Helmholtz resonator* and its natural frequency, according to the formula presented in Sect. 4.1.

$$F_h = \frac{c}{2\pi}\sqrt{\frac{S}{lV}}$$

where **c** is the speed of sound through air (343.3 m/s).

We would remind that amplitude damps down in an oscillator because of losses, which we have condensed into three factors, respectively r_p for the soundboard, r_f for the back, and r_b for the soundhole. As for the oscillating systems that compose the guitar resonator, losses are due to two primary reasons: *friction in the substance*—wood or air (*viscous friction*)—and *radiation resistance*, which every surface meets in emitting sound radiation while vibrating in free air.



This scheme is a *model of the guitar resonator*, or a depiction of the elementary components (soundboard, back, Helmholtz resonator) and their interaction.

Whatever model is a simplified and partial representation of what happens in the concrete. So our model is:

- *Simplified*, in that the oscillation phenomena involved in the resonator are related to simple elementary oscillators and their interaction.
- *Partial*, because this model represents the resonator behaviour in the mid-low register, where the basic resonances and the main resonances of the back stand out. This register spans up to about 400 Hz, covering the fundamentals of the tones up to about *G#* on the thirst open string. As already seen, from just above 300 Hz the air cannot oscillate any more in the soundhole which, from now on, behaves in practice as if it were closed. In higher registers, the resonator operates differently, and the study of its behaviour requires a different approach.

To be useful, a model must not be a plain description of its components. The previous figure provides just some partial information about the single components. The structure must be thoroughly described *as a whole*, which involves a mathematical formulation to establish the global response of the system, in keeping with the relations between the single components.

In Appendix 5.1 we report the criteria whereby we built our model. We do not expound the equations relative to the model, deeming instead worthwhile to provide their outcomes, and illustrate how these can help in optimizing an instrument under construction, as well as finding out the construction parameters of a finished instrument.

5.4.1 Fundamental Resonator Parameters

In the first part of this chapter we reported the outcomes of measurements made while percussing the bridge under normal condition (open soundhole) and with covered soundhole. Besides, we gave the Helmholtz frequency of this Garrone instrument, obtained through the procedure described in the previous chapter. The experimental data are:

- $\mathbf{F_1}$ (air resonance) = 93 Hz
- $\mathbf{F_2}$ (soundboard resonance) = 213 Hz
- $\mathbf{F_h}$ (Helmholtz resonance) = 129 Hz
- $\mathbf{F_p}$ (covered soundhole resonance) = 196 Hz.

In this instrument, the natural frequency value $\mathbf{F_{p0}}$ of the soundboard under construction was directly measured:

- $\mathbf{F_{p0}}$ (soundboard natural frequency) = 155 Hz

We would remind that the natural frequency $\mathbf{F_{p0}}$ is the one we measure on the soundboard fastened to the mould, before being glued to the frame. This frequency cannot be always measured directly, for two reasons:

– Obviously, the natural frequency of the soundboard cannot be ascertained in a finished instrument.
– Prior to fixing the back, the soundboard glued to the sides can plausibly undergo further adjustments, which may modify the natural frequency measured beforehand.

In that case, nothing wrong: in Appendix 5.2 we provide a formula that allows to *estimate* the natural frequency of the soundboard when a direct measure is not possible, provided that we know the other frequencies (still measurable on a finished instrument):

$$f_{p0}(estimated) = \frac{f_1 f_2}{f_h}$$

We can compare the value measured on this reference instrument with the estimated value: the measured value is 155 Hz, while the value estimated by means of the formula is 153.6 Hz, which is a pretty good match.

The 'covered soundhole' frequency can also be obtained from the model, using the formula for the calculation of f_p (*estimated*) presented in Appendix 5.2. Actually, the 'covered soundhole' frequency can be easily measured on a finished instrument, so this estimation is not really necessary. We introduced the formula and its outcome in order to assess the discrepancy between f_p (*measured*) and f_p (*estimated*), resulting to be 2.7 Hz.

Based on these verifications we can conclude that the simplified representation of the resonator we suggest here is reliable enough for our purpose, that is to optimize the instrument construction and assess the quality parameters of a finished instrument.

In summary: the luthier willing to optimize an instrument under construction, or to investigate the characteristics of a finished instrument, must measure the two basic resonances (f_1 and f_2) and the Helmholtz resonance f_h. In addition to that, the soundboard natural frequency must be known, whether by means of a direct measure on an instrument under construction or by an estimate through the first equation given in Appendix 5.2.

Now, to proceed in the study of the guitar resonator, we need to introduce other concepts, primarily the *vibrating surface* and *vibrating mass*, and the *coupling coefficient*.

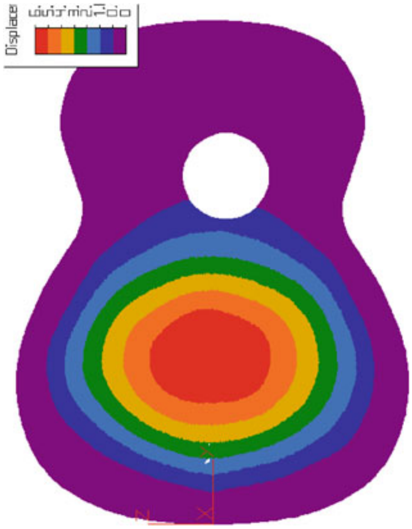
5.4.2 The Vibrating Surface

The overall surface of the guitar soundboard clearly depends on its perimeter outline (the *plantilla*, after the traditional Spanish term).

A typical value of the overall soundboard surface in a modern guitar is about 1450 cm^2. Nevertheless, not all of this surface can vibrate efficiently in mode $\langle 0\ 0 \rangle$. This for two reasons:

- The soundboard is glued to the sides along the edge: this rigid bond (or *semi-rigid*, since both sides and linings are somewhat flexible) hampers the soundboard motion along its perimeter, so that the vibrating area of the soundboard is to some extent reduced with respect to the outline defined by the sides.
- The brace under the soundhole confines the vibrating surface to the region of the soundboard between the soundhole and the back, posing an additional limit to the operating surface.

The next image depicts the situation. It represents the FEM simulation of the vibration amplitude (in mode $\langle 0\ 0 \rangle$) of a soundboard presenting a brace under the soundhole, one under the bridge, a reinforcement to the soundhole and a simplified bracing.



Here the soundboard, rather than like a *piston* (as in the previous simplified resonator scheme) behaves like a sort of *vibrating bell* that expands and contracts. The oscillation amplitude is maximum at the centre and minimum at the periphery—along the border of the sides—and under the soundhole. Nonetheless, the oscillation of this bell is comparable to that of an *equivalent piston* like the one described in the model, whose surface is able to *move an equal air volume as the real vibrating bell does*. This way the piston and the bell are essentially equivalent with respect to both the outward sound radiation into the surrounding environment, and the inward interaction with the air in the body.

The previous figure serves as an example. In fact, without prejudice to the validity of general concepts, the vibrating surface of a real instrument can be enlarged acting on the bracing and, possibly, on the shape and position of the soundhole. We will see hereinafter the importance and the advantage in extending the vibrating surface even beyond the soundhole, and some ways to reach that goal.

However, to define the vibrating surface in quantitative terms we also need to consider another key parameter: *the vibrating mass*.

5.4.3 The Vibrating Mass

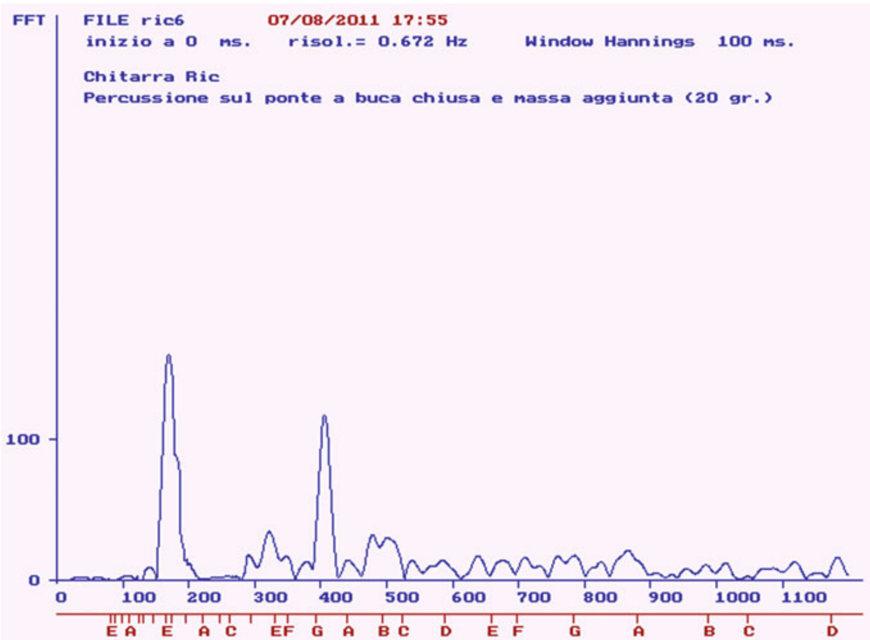
The weight of a 2.2 mm thick soundboard with no bracing and a surface of about 1390 cm^2—*soundhole excluded*—is about 122 g if the density of the timber (spruce) is about 400 kg/m^3. We must add the weight of bridge, rosette and bracing—which depends on the number of wood strips, their geometry and, to a certain extent, their arrangement. The weight of the ‘finished’ soundboard will be no less than 150 g. These are obviously indicative data.

Not the whole mass participates in the soundboard vibration in mode ⟨0 0⟩, for similar reasons as the vibrating surface is smaller than the geometrical surface. Therefore, we will call *vibrating mass* that part of the mass which contributes to the natural resonance of the soundboard. As we will see, this parameter is very important, since it also influences sound radiation and the attributes of the coupling between the soundboard and the air in the resonator body. First of all we point out that—because of differences among various kinds of bracing—there is no direct relation between vibrating mass and vibrating surface, and so these two parameters need to be separately and independently evaluated.

For the assessment of the vibrating mass we suggest a direct measurement, which consists in repeating the above-described 'covered soundhole' measurement, but this time after fixing provisionally—with double-sided tape—a little additional mass just under the percussion point (the saddle).

The following is the chart of the instrument response with a 20 g additional mass. We notice a remarkable resonance at the frequency (we will call this $\mathbf{F_{pm}}$) of 169.5 Hz. Without the 20 g weight the frequency $\mathbf{F_p}$ with the soundhole covered was 196 Hz. This decrease is due to the greater mass involved in the soundboard vibration. Amplitude as well is smaller in this resonance, both because the additional mass absorbs part of the energy provided by the excitation, and because sound radiation is diminished by the mass.

The value of the additional mass is not crucial, but the suggested 20 g is a good compromise that offers an easy outcome reading on the graph.



After obtaining the value $\mathbf{F_p}$ of the covered soundhole resonant frequency *without* additional mass ($\mathbf{F_p}$ = 196 Hz in this reference guitar) and the value $\mathbf{F_{pm}}$ of the covered soundhole resonant frequency *with* additional mass ($\mathbf{F_{pm}}$ = 169.5 in this guitar), we are able to figure out the value of the vibrating mass by the formula:

$$vibrating\ mass = \frac{additional\ mass}{\left(\frac{F_p}{F_{pm}}\right)^2 - 1}$$

As a result, with the values obtained ($\mathbf{F_p}$ = 196 Hz and $\mathbf{F_{pm}}$ = 169.5) and an additional mass equal to 20 g, the formula gives a *vibrating mass of about 59 g* for the reference guitar. This is an optimal value: much higher vibrating masses (above 65 g) indicate that the soundboard or the bracing is excessively heavy, whereas much lower values (below 50 g) indicate that the soundboard is *weak*, that is to say unable to efficiently transfer the force received by the string to the entire operating surface. In this situation the interaction between the soundboard and the air in the body at low frequencies is scarce, as well as the soundboard efficiency at high frequencies. Now, from the vibrating mass, we can get to the value of the average stiffness of the soundboard.

5.4.4 Soundboard Mean Stiffness

We would remind that any vibration mode of a plate can be compared to the motion of a simple oscillator like the one we have been observing in the previous chapters. For the reader's convenience we recall the formula that connects the *modal frequency* associated with a given vibration mode to the *modal mass* and *modal stiffness*.

$$f_{\text{mod}} = \frac{1}{2\pi}\sqrt{\frac{k_{\text{mod}}}{m_{\text{mod}}}}$$

As we have seen, at basic resonances the soundboard essentially oscillates and behaves, in mode $\langle 0\ 0 \rangle$, like a simple oscillator. Its natural frequency can be calculated by the preceding formula, provided that we take into account what follows: now *the modal mass equals the vibrating mass*, the modal stiffness corresponds to the mean stiffness of the soundboard and *the modal frequency corresponds to the soundboard natural frequency in mode* $\langle 0\ 0 \rangle$ (which we have called $\mathbf{F_{p0}}$). So, as far as the soundboard oscillation in mode $\langle 0\ 0 \rangle$ is concerned, the mean stiffness is

$$K_{mean} = \left(2\pi F_{p0}\right)^2 \times vibrating\ mass$$

Setting the above-mentioned values ($\mathbf{F_{p0}}$ = 153.6 Hz, vibrating mass = 59 g) into the formula, we get $K_{mean} \cong 55{,}000$ N/m.

This value represents the *global* (or mean) *stiffness* that determines the natural frequency of the soundboard. However, in specific areas, the soundboard may present lower stiffness values. For instance, the luthier often lightens the soundboard along the lower bout, by modifying the profile of braces at the joint with the linings or by slightly reducing the soundboard thickness. This way the local stiffness is reduced, allowing the soundboard to 'breathe' more efficiently when oscillating in mode ⟨0 0⟩. This brings the nodal line to a nearly exact coincidence with the soundboard edge, which involves an increase of the vibrating surface. Another expedient can be implemented, consisting in reducing the local stiffness of the soundboard in the area of the soundhole (which is yet rather stiff because of the rosette).

For the moment, we confine ourselves to these general observations. The wide-spreading topic of soundboard optimization will be thoroughly investigated in the second part of the book.

We appraised values of the K_{mean} comprised between 50,000 and 60,000 N/m in various instruments. In one of the instruments we examined, the mean stiffness was notably high (about 88,000), and the vibrating mass of the soundboard was rather great as well (65 g). The natural frequency of the soundboard resulted to be 185 Hz, the vibrating surface 476 cm^2, and the coupling coefficient 0.85. The quality of the instrument was low.

Now we have all of the required elements to establish the vibrating surface of the soundboard in mode ⟨0 0⟩.

5.4.5 Assessment of the Vibrating Surface

This is the formula for the vibrating surface:

$$vibrating\ Surface = 2\pi \sqrt{\frac{vibrating\ Mass \times Volume \times \left(F_p^2 - F_{p0}^2\right)}{14,2000}}$$

This formula requires to know the volume **V** of the air in the resonator body. In Sect. 4.1 we observed that this volume can be estimated by the product of the soundboard geometrical surface (soundhole included) and the average height of the sides. In our reference guitar the volume is about 14 litres (14,000 cm^3).

The constant in the formula (142,000) is the *bulk modulus*, already mentioned when dealing with the elastic attributes of an air volume (being under all aspects a sort of *air spring*—cfr. Appendix 4.1). The other parameters in the formula are already known: *vibrating mass* (59 g), *air volume in the body* (14,000 cm^3), calculated *covered-soundhole frequency* (193.3 Hz), *soundboard natural frequency* (153.6 Hz). The resulting *vibrating surface* is 563 cm^2.

Given invariable geometrical surface, in other high quality guitars we measured a vibrating surface of about 500 cm^2, whereas in lower quality instruments the vibrating surface resulted to be 450 cm^2 (or less). For a better understanding of the relations

between overall surface and vibrating surface, we point out that the soundboard surface in the lower bout is about 920 cm^2, while we remind that the overall surface is around 1450 cm^2. Therefore, even in the best conditions, only part of the soundboard vibrates efficiently in mode $\langle 0\ 0 \rangle$.

The lack in vibrating surface of certain instruments can be traced back to different causes. For instance, the bracing could prevent the soundboard from vibrating beyond the brace under the soundhole, or the soundboard motion could be hampered along the perimeter because of excessive weight or stiffness. For now we are not going further in these general concerns. The role and significance of this resonator parameter will be discussed when dealing with sound radiation and coupling coefficient, while the optimization of the soundboard design (this aspect included) will be thoroughly investigated in the second part.

5.4.6 The Coupling Coefficient

This is the last but not least resonator parameter we are concerned with. To understand its physical and practical significance, we go back to the reasoning about coupled oscillators. If two pendulums, oscillating at frequencies f_a and f_b, do not interact with each other, they make up a system of *independent* or *uncoupled oscillators*. If we connect them through an elastic medium (like a spring with stiffness **K**) they lose their original character and form a new global system, whose response depends on their original attributes as well as their interaction.

This 'new' system presents two natural frequencies f_1 e f_2, that do not correspond to the frequencies f_a e f_b of the two original oscillators. We have also observed that, by their motion, the two pendulums exchange energy at a frequency f_c (*coupling frequency*). If the stiffness of the link between the two pendulums is low, the coupling is loose and, vice versa, the coupling is firm when the stiffness of the link is high.

These same phenomena occur in the guitar resonator, where the two primary oscillators are

- The soundboard enclosing the air volume in the body (whose oscillation frequency is the 'covered soundhole' $\mathbf{F_p}$)
- The Helmholtz oscillator (whose oscillation frequency is $\mathbf{F_h}$)

The coupling medium between these two structures is the air in the resonator body. The coupling frequency (we will call this $\mathbf{F_{ph}}$) is given by the formula

$$F_{ph}^2 = \sqrt{F_p^2 F_h^2 - F_1^2 F_2^2}$$

Introducing in the formula our calculated values, we get $\mathbf{F_{ph}} = 123$ Hz. This is the frequency at which the two oscillating systems (air and soundboard) exchange the available energy. We will make use of the coupling frequency to figure out the *coupling coefficient*: this parameter is crucial in designing the instrument, because it deeply influences the global quality. We define the coupling coefficient as

$$Coupling\ coefficient = \frac{F_{ph}}{F_h}$$

This coefficient varies between 0 (no coupling) and 1 (stiff coupling). Between these two extremes we find the actual coupling coefficient, specific of each instrument.

In our reference guitar, where $\mathbf{F_{ph}} = 123$ Hz and $\mathbf{F_h} = 129$ Hz, the coupling coefficient is 0.95 (we remind that $\mathbf{F_h}$ is the Helmholtz resonator frequency). In other first-class instruments the coupling coefficient resulted to be around 0.9 while, in less valuable instruments, it was 0.85 or lower.

Recalling what has been stated in Sect. 5.3, we point out that the two original frequencies f_h and f_p *tend to diverge* because of the interaction, and the two 'new' natural frequencies f_1 and f_2 of the coupled oscillators fall respectively left and right of the two original frequencies. This divergence grows as the coupling coefficient grows. This means that *the greater the coupling coefficient, the lower turns out to be* $\boldsymbol{F_1}$ *(basic air resonance), and the higher* $\boldsymbol{F_2}$ *(soundboard basic resonance).*

Another formulation exists, that connects more directly the coupling coefficient with the resonator parameters manageable by the luthier. Without going into calculation details, we write

$$Coupling\ coefficient \equiv \frac{(vibrating\ Surface)^2}{vibrating\ Mass}$$

The symbol $\equiv$ indicates that the left part depends on the right one, but is not equivalent. This expression tells us that the coupling coefficient rises along with the *square* of the vibrating surface and drops along with mass. This is somehow intuitive: as the vibrating surface increases, the area of the soundboard that can efficiently interact with the air in the body accordingly increases; on the other hand, if the vibrating mass grows, the force applied by the soundboard on the air diminishes.

5.4.7 Sound Pressure Level

To get a qualitative evaluation of the sound pressure generated by the guitar in the far-field (at least 1 m from the instrument), we remind that we represented the resonator as a rigid-sidewall cavity. From the upper side two *vibrating pistons* look outside (the soundboard and the soundhole), while from the lower side a third vibrating piston corresponding to the resonator back looks outside. We neglect for the moment the contribution of the back, which will be considered later on.

By their motion, the two upper pistons—soundhole and soundboard—displace a certain volume of air and behave as *elementary sources*, each of them conveying a sound pressure through the surrounding environment. The *sound pressure level* or *SPL*—defined in Chap. 1—depends on the overall volume of air displaced by the two sources together (soundhole and soundboard), and is maximum at the two

basic resonances (of air and soundboard) where the motion velocity of the sources is higher.

As stated in the previous Sect. 5.2, below the Helmholtz frequency the two pistons oscillate in *antiphase*, so the two pressures generated by the soundhole and the soundboard tend to subtract, i.e. to oppose each other. On the contrary, above the Helmholtz frequency, the two pistons oscillate *in phase*, and pressures tend to add, i.e. to reinforce each other.

Therefore, in the range of mid-low frequencies we are dealing with, the overall sound pressure depends on the contribution of soundhole and soundboard. The soundhole plays a major role at lower frequencies, and the soundboard prevails at higher frequencies. This is confirmed by the chart of the covered soundhole response, where we clearly see that, when the soundhole is covered, the sound radiation at low frequencies disappears altogether.

The presence of a Helmholtz resonance below the soundboard natural frequency enables to expand considerably the guitar output next to the low frequency range. This mechanism, as it has been often observed about the violin family, recalls the functioning of bass-reflex loudspeakers.

We have said that, as the frequency grows, the airflow through the soundhole encounters more and more opposition or, differently stated, the impedance of the soundhole rises until the airflow stops (along with sound radiation from the soundhole itself). Hereafter, the soundhole behaves as if virtually covered, so the soundboard only is still able to release sound pressure in the environment. The contribution of the soundhole diminishes progressively until total extinction just above 300 Hz.

Regardless of the interaction with the soundhole, the sound pressure level yielded by the soundboard is proportional to the piston's operating surface divided by its mass; so, in accordance with the above definitions, we write

$$Sound\ pressure\ Level \equiv \frac{Vibrating\ surface}{Vibrating\ mass}$$

This rapport is valid for whatever elementary source, back and soundhole included (where mass and surface are those specific of these resonator components).

Since, as seen above, the coupling coefficient rises with the square of the vibrating surface and decreases with the mass, we conclude that *a high coupling coefficient goes with a high sound radiation level*. This proves that high quality instruments are characterized by high coupling coefficient value.

But we must not forget that, in this chapter, we are only dealing with the mid-low register response of the guitar (ruled by the resonances of air, soundboard and—as we will see—back). In fact, the overall sound quality is also notably affected by the high-frequency response.

5.4.8 Overview of the Resonator Parameters

The reader who has patiently followed our reasoning so far might now be in great confusion, risking to 'get lost' among all the formulae and parameters we have presented. So, at the cost of repeating some of the formerly expounded concepts, we deem it worthwhile to provide an overview for the reader willing to use this model to better understand and, above all, optimize the resonator performance. As for the instrument design, we confine ourselves now to generic suggestions, deferring a comprehensive discussion of the topic to the second part of the book.

First of all, luthiers need to know the fundamental characteristics of their own model, that is the volume **V** of the air in the body and the frequency $\mathbf{F_h}$ of the Helmholtz resonator. In this chapter we have given indications on how to evaluate these parameters: basic measurements that can be executed once and for all upon a finished instrument—or a mould reproducing its geometry—that will be valid so far as the luthier uses that model.

Now, to optimize an instrument under construction, or to assess the characteristics of a finished instrument, we need to accomplish a number of measurements:

1. First, we must get the response of the instrument excited under standard conditions, that is at the bridge. Our method to excite the instrument at the bridge and get the related response will be analysed in the second part.
   By the response graph we obtain the values of the basic resonances $\mathbf{F_1}$ and $\mathbf{F_2}$.
   In an instrument under construction it is advisable to measure the natural frequency of the soundboard $\mathbf{F_{p0}}$ as well, whereas in a finished instrument it can be calculated, as a fair approximation, through the formula (presented in Appendix 5.2):

$$F_{p0} = \frac{F_1 F_2}{F_h}$$

2. Now we must *cover the soundhole* of the guitar to execute two measurements.
   We will perform the first measurement without any additional mass on the soundboard, extracting the 'covered soundhole' frequency value $\mathbf{F_p}$ from the response chart.
   We will then repeat the measurement after fixing provisionally a little additional mass just under the percussion point, getting the value of $\mathbf{F_{pm}}$ (covered soundhole frequency with additional mass). Though the value of the mass is not crucial, we suggest a small 20 g weight, this being a proper compromise.

From these three measurements (four if we include that of the soundboard natural frequency $\mathbf{F_{p0}}$) we can derive the data required for the calculation of the resonator parameters: the basic frequencies $\mathbf{F_1}$ and $\mathbf{F_2}$, the 'covered-soundhole' frequency $\mathbf{F_p}$ without the weight, and the 'covered-soundhole' frequency with additional mass $\mathbf{F_{pm}}$, plus the volume **V** and the Helmholtz resonance $\mathbf{F_h}$, evaluated once and for all.

We do not reiterate here the previously provided formulae, which allow a calculation of all the parameters that are crucial to understand the resonator functioning and, above all, to optimize the design. These parameters are: *vibrating mass*, *soundboard*

mean stiffness, *vibrating surface*, *coupling coefficient*. We have given the values concerning these parameters that were obtained from a high quality Garrone guitar, then we have compared them with values obtained from a number of good quality guitars and from less valuable instruments.

These comparisons establish a set of optimal values for each of the relevant parameters. However, we must consider the set of parameters as a whole, rather than acting *singly* upon one of them. Some examples will clarify this concept.

We said that high quality instruments are distinguished by a high value of the ratio *vibrating surface/vibrating mass*, in that this ratio is positive for both sound radiation and coupling between soundboard and air in the body. This may lead one to make a thin soundboard and a light bracing; this way, the vibrating mass would diminish, and the *vibrating surface/vibrating mass* ratio would increase. But, if the vibrating mass is too scarce, the soundboard results to be ‘flabby’, that is incapable of storing efficiently the motion energy received by the string, and properly transfer it so far as the soundboard periphery. The consequence would be a lack of efficiency, mostly at high frequencies. In the reference guitar the vibrating surface/vibrating mass ratio is 9.5 cm^2/g, a very similar value to what we find in other high quality instruments. In other, poor quality guitars, this ratio can be as low as 7 cm^2/g.

Measurement on an instrument under construction resulted in a vibrating surface/vibrating mass ratio of about 7.7 cm^2/g. After some adjustments of the braces, the vibrating surface rose and the vibrating mass diminished, the final value of the ratio growing to 9.6 cm^2/g.

Therefore, the surface/mass ratio is a very useful and significant indicator, even during instrument building. Statistically, considering the results obtained from several instruments, we can conclude that the optimal values for this ratio span from 8.5 to 11 cm^2/g. Lower values may result from scarce vibrating surface or excessive mass, while higher values definitely indicate a too small mass.

We consider now the real case of an instrument being optimized under construction. Initial values of soundboard natural frequency (153 Hz), vibrating mass (56 g) and soundboard stiffness (51,200 N/m) were regular. Basic resonance frequencies (respectively 97 and 198 Hz) were also passable.

Yet, further analysis performed through the Chladni method (thoroughly described afterwards) showed that the nodal line at soundboard basic resonance did not reach the perimeter along the sides, which could be a symptom of the soundboard scarce propensity to vibrate at its maximum extension. The evaluation of the resonator parameters confirmed that: the vibrating surface resulted to be just 430 cm^2, and the coupling coefficient was low (0.86) in comparison with the standards of the best instruments, precisely because the area of the soundboard acting on the air was limited. Consequently, the surface/mass ratio was scarce (7.7 cm^2/g). At this point the luthier resolved to lighten the soundboard along the edge, especially the transverse braces at the joint with the linings. As a result, the vibrating surface rose from 430 to 520 cm^2; the surface/mass ratio shifted from 7.7 to 9.2 cm^2/g and, accordingly, the coupling coefficient also increased from 0.86 to 0.95. After this optimization the resonator parameters fall within the standards of the best instruments.

These examples bring us to the conclusion that it is not correct to evaluate an instrument (whether finished or under construction) only in light of the position of basic resonances. A soundboard might have a proper natural frequency even though the stiffness is too high but the mass is low or, on the contrary, if the stiffness is low but the mass is excessive. In both cases the acoustic performance of the instrument would be poor. The resonator parameters must be considered in their entirety, and in progress interventions must ensure that all parameters as a whole fall within the standards of the best instruments.

When applied to a finished instrument, this analysis of the resonator parameters can help us in the evaluation of its acoustic properties, but can also shed light on construction features involved in the making of that instrument. In this sense, the parameter analysis is not alternative but complementary to the traditional evaluation that consists in assessing thicknesses, timber attributes, and geometry of the braces.

5.4.9 From Soundboard Natural Frequency to Basic Resonances

Thus far we have been concerned with determining the resonator parameters when the basic resonances are known. Now the question is: if we know the natural frequency $\mathbf{F_{p0}}$ of a soundboard under construction, and the frequency $\mathbf{F_h}$ of the Helmholtz resonator, what basic resonances $\mathbf{F_1}$ and $\mathbf{F_2}$ can we expect once the soundboard is fixed to the body? Or else, what shall be the frequency $\mathbf{F_{p0}}$ in order to get the basic resonances at frequencies $\mathbf{F_1}$ and $\mathbf{F_2}$?

This is an inverse procedure with respect to the formerly observed one. We have seen previously how to track down the resonator parameters starting from the basic resonances and the Helmholtz resonance, whereas now we wish to discern the basic resonances $\mathbf{F_1}$ and $\mathbf{F_2}$ knowing $\mathbf{F_{p0}}$ and $\mathbf{F_h}$. As far as construction procedures are concerned, this is a rightful question: the luthier might wonder how to dimension the soundboard in order to obtain the desired air and table resonances $\mathbf{F_1}$ and $\mathbf{F_2}$.

Unfortunately, a one and only answer does not exist. When the soundboard is coupled with the body, the resonator global response will not only depend on the natural frequencies of the soundboard and of the Helmholtz resonator, but also on the characteristics of the elastic system that 'connects' these elements.

We refer the reader to the previous Sect. 5.3, when discussing the mechanism of coupled oscillators. Interaction between the oscillators depends on the *coupling coefficient* we measured and got acquainted with about the guitar resonator. If we only know the soundboard natural frequency $\mathbf{F_{p0}}$ and the frequency $\mathbf{F_h}$ of the Helmholtz resonator, we are not able to figure out the coupling coefficient. This means we are not able to ascertain how the soundboard will behave once it is fixed to the resonator body. This is intuitive: we have seen that the coupling coefficient depends on the square of the surface and on the vibrating mass; we are therefore unable—simply

relying on the measured natural frequency of the soundboard—to assess how much of the soundboard mass and surface will be really going into vibration.

To break this deadlock we can only *estimate* (or rather *impose*) the coupling coefficient, but we shall correct the design if the results diverge from the estimation.

The next chart answers the problem. All frequencies are referred to the frequency $\mathbf{F_h}$: this way we keep one of the variables out of the problem, and we render the graph independent of the Helmholtz resonator frequency. The graph shall be used as follows:

- We impose a coupling coefficient value (usually 0.9–0.95).
- Starting from the known $\mathbf{F_{p0}}$, we calculate the ratio $\mathbf{F_{p0}/F_h}$.
- By this value we enter the horizontal axis, and find the two curves (respectively on the upper and lower panel) that correspond to the imposed coupling coefficient.
- We read the two values $\mathbf{F_1/F_h}$ and $\mathbf{F_2/F_h}$ which, multiplied by $\mathbf{F_h}$, provide the values $\mathbf{F_1}$ and $\mathbf{F_2}$ of the two basic resonances.

Some examples will clarify the procedure.

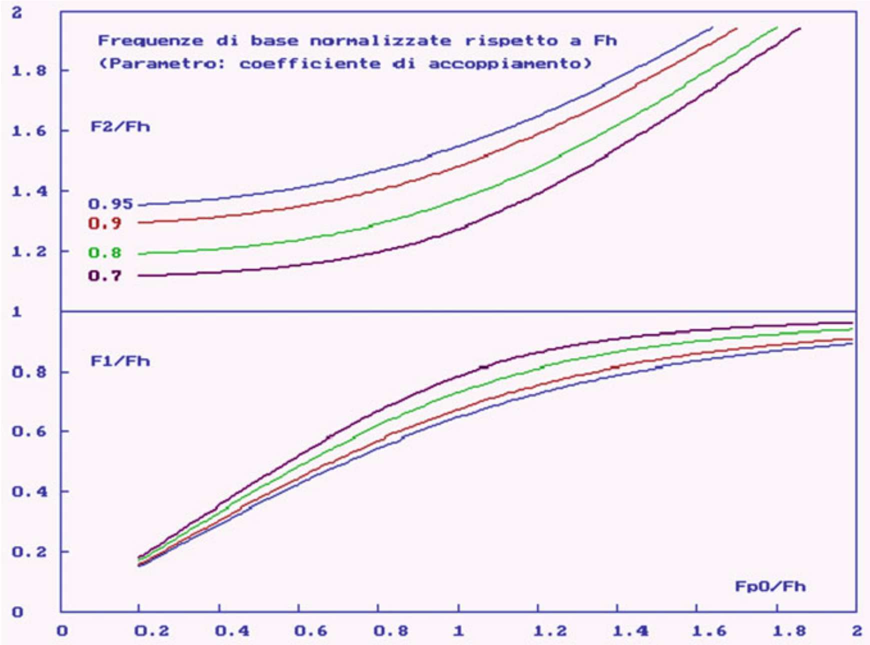
Suppose a soundboard natural frequency $\mathbf{F_{p0}} = 160$ Hz, and a Helmholtz resonance $\mathbf{F_h} = 129$ Hz. We impose the coupling coefficient 0.95 and set in the diagram the value $\mathbf{F_{p0}/F_h} = 160/129 = 1.24$.

The two curves we pinpoint in the two panels report values (0.74 and 1.67) which, multiplied by the Helmholtz frequency, provide the *expected* values of the two basic resonances, respectively $\mathbf{F_1} = 96$ Hz and $\mathbf{F_2} = 216$ Hz.

Now the luthier might consider the two resonances (especially the second) to be too high in frequency, and so decide to lower the soundboard natural frequency.

With $\mathbf{F_{p0}} = 150$ Hz and the same coupling coefficient, the values found on the curves are 0.713 and 1.63, corresponding to the resonances $\mathbf{F_1} = 92$ Hz and $\mathbf{F_2} =$ 210 Hz. These can be considered tolerable values.

Now the (provisionally) finished resonator may manifest different resonances from those expected. This can be due to scarce vibrating surface or incorrect design of the soundboard (excessively rigid or heavy). We refer back now to the previously summarized optimization process, which will be investigated in detail afterwards: evaluation of the resonator parameters, and correction of the far from optimal values. Once this has been accomplished, the finished instrument will comply with the best qualitative standards.



5.5 The Back (Unveiling a Mystery)

All luthiers have a personal way of making the back, and all, according to their individual approach, have an idea (perhaps empirical) of what will be the effect of the back on the global quality of the finished instrument. Diverging experiences and opinions exist, that require at least a brief analysis, in order to understand what reasons lead to certain construction choices.

Many luthiers, based on their experience and, mostly, on well-established traditions, build the back according to a standard design: the table used for the back has always the same thickness, even if timbers are different, and the braces (generally three) have always the same positions, thicknesses, heights, and curvatures. Such a back, once coupled with the soundboard, provides a *fairly* satisfactory performance. But if we could examine the entire production of a luthier (or a significant part of it) we would find that, in some cases, the attributes of this standardized back do not properly couple with the soundboard characteristics. This is logical, as back and soundboard show different behaviours in different instruments. This means that, sometimes, the resonances of the back interfere negatively with those of the soundboard, or they do not offer a worthy contribute to the instrument response.

In many cases, especially regarding factory instruments, the back is willingly *semi-rigid*, to prevent detrimental interferences with the soundboard resonances.

This is achieved by increasing the height of the braces or the thickness of the back, in order to push its resonances up and far from the soundboard basic resonance.

Some manufacturers consciously adopt a completely rigid back that, as a consequence, is unable to vibrate in the range of mid-low frequencies. This is the best option if one believes that the instrument response must only depend on the soundboard; if the back is unable to vibrate, the entire force that the string transfers to the bridge is available to set the soundboard and the air in the body into motion, while the back (being rigid) is not excited. Obviously, in this situation, the back cannot contribute to sound emission, nor to the quality of sound, and the instrument response only depends on the soundboard and its coupling with the air in the body.

Others, on the contrary, believe that the instrument quality can be positively influenced by the back as well, provided that it is suitably sized, in order for its main resonances to properly merge with the soundboard resonances.

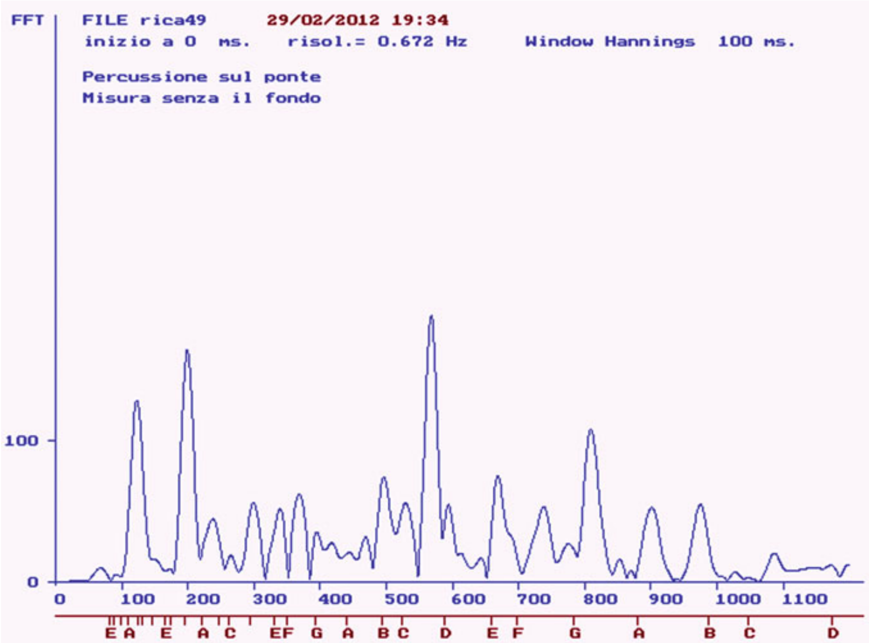
This is the approach we are going to comply with. We will begin by studying the different functions of the back, and how its fundamental resonances affect the overall response of the instrument. Then, in the second part of the book, we will see how to correctly size the braces, so as to optimize the contribution of the back to the instrument quality. We must point out that this fine-tuning demands an additional effort on the part of the luthier, since each soundboard must be combined with its back, rather than just going with a standardized back. But, in the end, it is well worth the trouble!

5.5.1 The Back as a Helmholtz Resonator Wall

Firstly, the back is one of the Helmholtz resonator boundary walls, as observed in the scheme representing the guitar resonator in Sect. 5.4 and in Appendix 5.1.

This obvious statement would not be worth mentioning, except that one may wonder how a hypothetical instrument under construction would behave *without the back*.

The next is the response diagram of the reference instrument without the back when the soundboard is excited on the saddle.



First of all we notice here that, without the back, both the first basic resonance of the air (at 93 Hz—in this instrument- with the back) and the second basic resonance of the soundboard (at 213 Hz with the back) are missing. This is easy to understand: leaving out the back, we practically nullified the effect of the Helmholtz resonator that, through the coupling mechanism between air and soundboard in the body, determines the development of the basic resonances. We would remind that—in Garrone instruments—the Helmholtz resonance typically takes place at 129 Hz.

Secondly, we see two strong resonances on the graph, the first at 123 Hz and the second at 199 Hz. In the situation we are considering here, part of the force applied to the soundboard excites the soundboard itself and part, through the junction (the linings) excites the sides. The characteristics of this junction largely depend on the nature of the linings, which can be more or less pliable, according to their construction features. Because of this mechanism, when the back is absent, the sides waver along the lower perimeter where, normally, they would be blocked by the back.

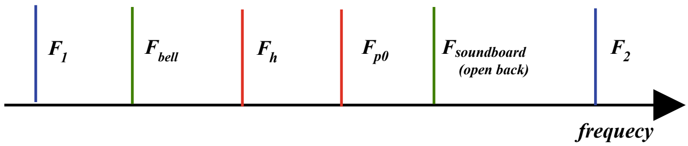
It feels natural to call *open back bell resonance* this vibration mode of soundboard and sides which, on the foregoing graph, appears at 123 Hz. With respect to fundamental resonances, this frequency is comprised between the first resonance of the air with the back (at 93 Hz) and the Helmholtz resonance (at 129 Hz).

In the instruments we have examined this is a recurring situation: for instance, in a ‘two-hole’ Garrone instrument, the bell resonance manifests at 112 Hz, higher than the air resonance with the back (arising in this instrument at 80 Hz) but lower than the Helmholtz resonance (at 119 Hz).

Underlying this bell resonance phenomenon is the fact that the oscillation of the lower edge of the sides along its perimeter is opposed by the elasticity of a layer of air (defined by the perimeter itself) which behaves like a sort of *virtual back*.

The other important resonance we notice on the previous graph arises at 199 Hz. At this frequency the force applied excites *primarily* the soundboard which, in this situation, is subject to a *partially* rigid bond along its perimeter, because of the elasticity of the sides that are not blocked by the back. The consequence is what we will call *soundboard open back resonance*. This resonance typically takes place at a value in between the natural frequency of the soundboard (in this instrument $\mathbf{F_{p0}} =$ 155 Hz) and the first resonance of the air with the back, that is $\mathbf{F_2}$ at 213 Hz.

The following scheme shows the relative position of the mentioned frequencies.



- The red lines represent the natural resonant frequencies of the two resonator components: the Helmholtz resonator (at frequency $\mathbf{F_h}$) and the soundboard (at frequency $\mathbf{F_{p0}}$).
- The green lines represent the bell resonance ($\mathbf{F_{bell}}$) and the soundboard open back resonance ($\mathbf{F_{soundboard-open\ back}}$).
- The blue lines represent the basic resonances with the back, namely the air resonance ($\mathbf{F_1}$) and the soundboard basic resonance ($\mathbf{F_2}$).

But what is the practical meaning of this open back measurement? The issue will be fully developed in the second part. For now, we just point out that, firstly, the measure can be useful to verify that the two open back resonances (those of the bell and the soundboard) fall in the indicated relative positions. Too large gaps can be a symptom of something wrong in the soundboard design. Secondly, the open back response is a very significant indicator of what the finished instrument response will be, mainly in the upper-mid register where the instrument quality of sound is primarily affected by the soundboard.

Obviously, with the open back, the interior of the soundboard (included, but not exclusively, the braces) can be easily reached and worked upon. Hereinafter we will better see how useful it is, once the open back response has been optimized, to apply a provisional back and evaluate quality parameters according to the previously described procedure. This in fact allows us, if necessary, to execute further corrections.

5.5.2 Influence of the Back on Basic Resonances

We have seen that the back is one of the resonator walls. However, being an elastic wall, it influences the global response of the instrument. Firstly, the elasticity of the back affects the position of the basic resonances. Secondly, it can give access to new resonances in an intermediate frequency register (between 200 and 400 Hz), which positively contribute to the instrument quality, but would be absent if the back were rigid or not properly tuned to the soundboard.

Although both the effect on basic resonances and on the response in the intermediate register up to 400 Hz are connected to the vibration modes of the back, we prefer to describe them separately, so as to offer a clearer explanation and, in particular, because the mechanism whereby the back contributes to the instrument response is very different in the two cases.

We begin by considering how the back influences the basic resonances, and we remind what has been said in this chapter about the development of these resonances.

At low frequencies (where the air resonance takes place) back and soundboard oscillate in *antiphase*: while the soundboard moves upward, the back moves downward. As an overall effect, the air in the body expands or contracts because of the oscillating motion of back and soundboard, whose combined action, as a consequence, increases the *air compliance* and makes the cavity own frequency lower than it would be if the cavity walls were rigid (we remind that the air enclosed in a cavity—as the resonator body—behaves like an air spring with characteristic compliance depending on its dimension parameters).

The final deduction is that *the frequency of the air resonance diminishes as a result of the elasticity of the back: the more elastic the back (i.e. the lower its natural frequency) the lower the basic frequency of the air with the back closed.*

At higher frequencies, where the second basic resonance takes place, back and soundboard move *in phase*, hence up or down together. Accordingly, the back *follows* the oscillating motion of the soundboard. With respect to the previous situation, the variation in volume of the air in the cavity is smaller and, above all, the force opposed to the soundboard oscillation is smaller. As a result, *because of the elasticity of the back, the frequency of the soundboard basic resonance is lower than it would be if the back were rigid*. Again, *the more elastic the back (i.e. the lower its natural frequency) the lower the basic frequency of the air with the back closed.*

In summary: basic resonances (of the air $\mathbf{F_1}$ and of the soundboard $\mathbf{F_2}$) are influenced by the elasticity of the back. Both diminish, compared with the value they would have if the back were perfectly rigid. However, different phenomena contribute to the effect of the back on basic resonances: at low frequencies (where the air resonance prevails) the elasticity of the back brings about a growth in compliance for the air in the body, whereas at higher frequencies (where the soundboard resonance prevails) the elastic back—moving in phase with the soundboard—reduces the effort of the soundboard in opposing the elasticity of the air. In both cases, the effect on frequencies $\mathbf{F_1}$ and $\mathbf{F_2}$ of the basic resonances depends on how much flexible the back is, hence on its natural frequency in mode $\langle 0\ 0 \rangle$. As an extreme condition, if the back were perfectly rigid, we would have no influence on the basic resonances.

In practice, as we will see, the natural frequency of the back is normally calibrated between 190 and 230 Hz, much higher than the soundboard natural frequency. In this case, the influence of the back on basic resonances is limited and, without going into details, we can say that the air frequency diminishes by about 2–3 Hz, while the soundboard frequency diminishes just a little more (about 3–4 Hz). Such a back, as a first approximation, behaves as if *quasi rigid with respect to the basic resonances*. If, during construction, we close the instrument with an *absolutely* rigid back, we can ignore the contribution of the back and evaluate the resonator quality parameters according to the previously introduced criteria. Anyway, it is necessary that the luthiers be aware of these phenomena, and know what to expect when they apply a flexible back to their instruments.

5.5.3 The Behaviour of the Back When Fastened to a Rigid Frame

Even though the influence of the back on basic resonances is limited, its contribution to the global instrument response is important in the field of intermediate frequencies, from the soundboard basic resonance (about 200 Hz) up to about 400 Hz; beyond this limit, different phenomena come about in the resonator functioning. But, in order for this contribution to be effective, the back must be so calibrated as to correctly couple with the soundboard. To explain the effects on the instrument response we need to go back to some concepts we expounded in the previous chapter about the back as a resonator component, and see how it behaves when not yet glued to the sides, but simply fastened to a rigid frame blocking its perimeter (we will call this condition 'free back').

The next illustrations show the nodal lines of the first three vibration modes in a free back (the one of our reference guitar) obtained through the Chladni method and, by comparison, the results acquired through the FEM simulation. We can observe, in these photos, the rigid frame that blocks the border.

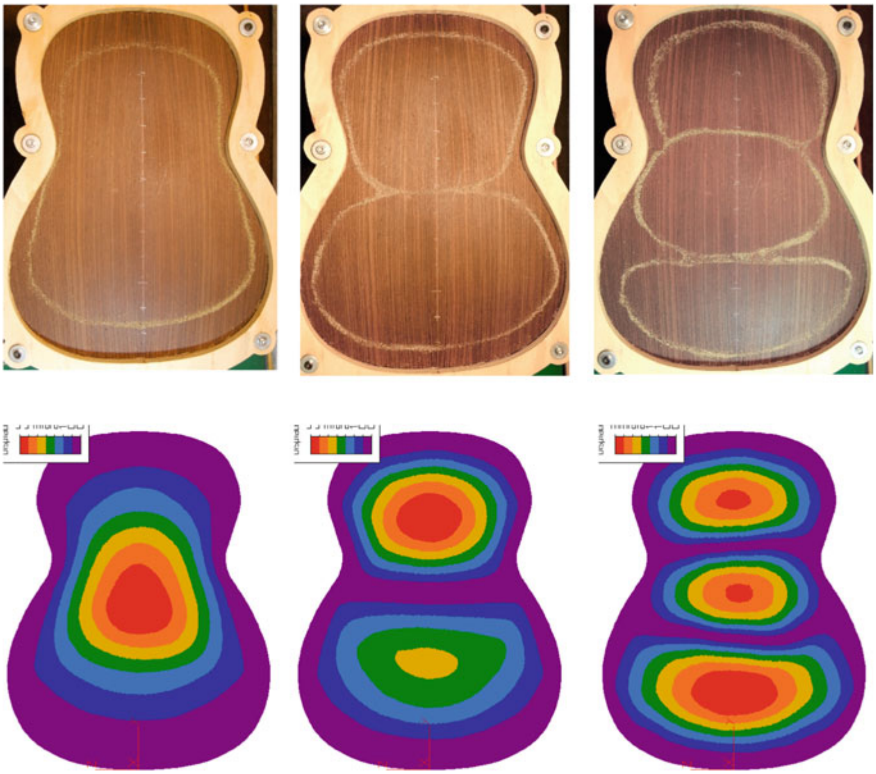
These vibration pictures are typical of a traditional back (three transverse braces, whose semicircular arching determines the curvature of the back itself, and influences the modal frequencies as well). The braces have a static purpose (strengthening the soundboard) and a dynamic one (influencing the vibration modes). Other bracing patterns are possible: specially interesting is, for instance, the one with four transverse braces.

We point out that the first three modes (respectively $\langle 0\ 0 \rangle$, $\langle 0\ 1 \rangle$, and $\langle 0\ 2 \rangle$) do not present longitudinal nodal lines: the transverse braces—by their stiffness—prevent the back from any bending (therefore any oscillation) along the longitudinal axis. This is not the case at higher frequencies, where longitudinal nodal lines are present as well. The first three vibration modes of the free back are the most important ones for the coupling with the soundboard: we will see that, if the back is correctly calibrated with respect to the soundboard, a good coupling is provided through the air in the body, so the back can effectively contribute to the overall response of the instrument.

The left picture (and the FEM simulation) shows the nodal line in *mode* ⟨0 0⟩. The nodal line forms a ring along the edge of the soundboard, and the antinodal area, where the oscillating energy concentrates, covers almost the entire soundboard. *In the reference guitar, the frequency of the back in this mode is 215 Hz.*

The middle picture and the related simulation show the nodal lines in *mode* ⟨0 1⟩. In this mode a single transverse nodal line runs below the waist. The two antinodal surfaces (above and under the nodal line) vibrate in antiphase, like two pistons moving the one up, the other down at the same time. The resulting effect depends on the difference in surface and vibration amplitude between the two pistons. The frequency of this mode is higher than that of mode ⟨0 0⟩. *In the reference guitar, the frequency of the back in this mode is 240 Hz.*

The picture on the right and the related simulation show the two nodal lines of *mode* ⟨0 2⟩, respectively located above the waist and at the middle of the lower bout. The two nodal lines delimit three vibrating pistons: the upper and the lower one vibrate in phase with each other, while the middle one vibrates in antiphase with the other two. *In the back of the reference guitar this mode takes place at 335 Hz, a higher frequency than the two former modes*.



The Chladni method enables a precise evaluation of both the frequency of each vibration mode and the development of the related nodal lines in a *specific* back, but does not consent to know the distribution of oscillation amplitudes.

The FEM model allows to investigate the response of a free back with regard to geometrical parameters (dimensions of braces and basic table) and to elastic parameters introduced in the simulation, but is flawed by a number of limitations. First, it is hard to reproduce the exact geometry of the back with its curvature and the smoothed edges of the braces, and these are details that do affect results. Second, the vibration amplitude is roughly measured according to a scale of colours, which does not establish the real value of the amplitude, but only the interval of values wherein it is comprised. Third, the model employs elastic parameters of the timbers that, though plausible, are not necessarily equivalent to those of our real back. These restrictions can be obviously overcome but—we believe—only on a specialised level.

On the other hand, the advantage in a model is the chance to easily modify the geometric and mechanical parameters of the back, and understand how they affect the response, though within the mentioned approximation limits.

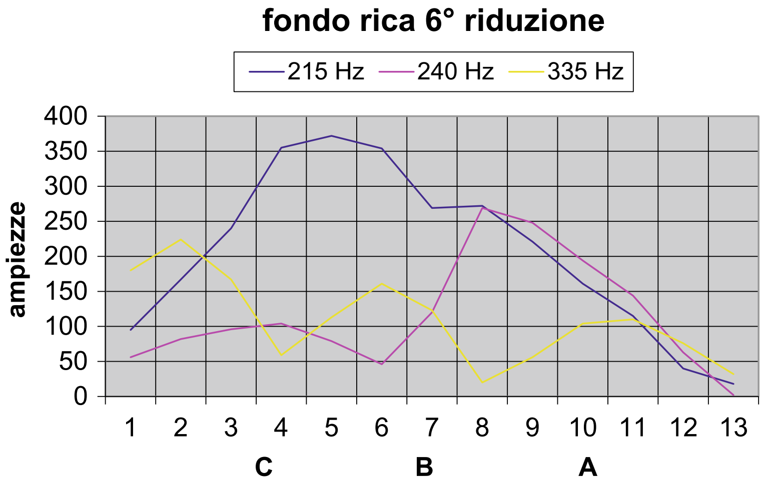
The two methods (Chladni and FEM) are not alternative but complementary. To identify the real characteristics of a specific back tightly fastened along its periphery, in specialised laboratories a *modal analysis* is executed that allows evaluation of the development of nodal lines at various modal frequencies, as well as the distribution of vibration amplitudes within the oscillating surfaces. The outcomes of these measurements and the employed machinery (for example laser interferometers) are mentioned in literature but, normally, are not available in a luthier's workshop, at least because of their high costs.

We suggest instead a method to execute a *simplified modal analysis* on a back fastened to the mould, requiring no other tools than the ones used for the formerly described analyses: a percussion pendulum and an analysis software able to carry out the Fast Fourier Transform. The method is conceptually very simple: on the diagrams drawn by the Chladni method we notice that the two nodal lines develop along some transverse axes (except the nodal line in mode $\langle 0\ 0 \rangle$ that runs along the perimeter of the back), while through the FEM models we see that in each of the vibrating areas the oscillating amplitudes gradually diminish toward the periphery, the *maximum values lying along the longitudinal axis*. Accordingly, if we excite the back in different points along the longitudinal axis and analyse their single output signals as recorded through the microphone, we get a number of sound emission spectrums, each associated to a specific measurement point. In fact the method is just the same as the one used to analyse the resonator properties. The difference is 'just' in repeating the measurement for each of the measurement points that divide the longitudinal axis of the back.

Now, from each of the emission spectrums, we select the values of the modal frequency we want to consider (those of mode $\langle 0\ 0 \rangle$, $\langle 0\ 1 \rangle$, $\langle 0\ 2 \rangle$), with the help of the Chladni patterns that—as we have seen—identify them accurately. For each of these *three* frequencies we report upon a graph the values of the vibration amplitude (vertical axis) referred to the measurement point (horizontal axis). The result is a diagram of the amplitude distribution along the longitudinal axis which, together

with the Chladni patterns, allows a complete analysis of the first (and most important) vibration modes of the back. Furthermore, the images obtained through the FEM simulation enable us to observe how the vibration amplitudes diminish, from the centre of each vibrating area along the axis of the back—where they are highest—toward the periphery.

The following image is the outcome of this analysis performed on the free back of our reference guitar.



On the horizontal axis are reported the 13 measurement points, and on the vertical one the related amplitudes issued from each of the three modal frequencies. On the horizontal axis the position of the three braces with respect to the measurement points are also reported, where C is the brace just below the middle of the lower bout.

- In mode ⟨0 0⟩, *the frequency is 215 Hz*. Amplitude has no absolute minimums (or, in other words, no transverse nodal lines are present—as we can see on the Chladni pattern regarding this mode); it remains high in the lower bout, while diminishing toward the upper bout. The maximum amplitude falls within the waist and the middle of the lower bout, as confirmed by the FEM simulation.
- In mode ⟨0 1⟩, *the frequency is 240 Hz*. Amplitude presents a single absolute minimum positioned between brace C and brace B—hence under the waist. The minimum corresponds to the nodal line visible in the Chladni pattern for this mode, as well as to the nodal area shown by the FEM simulation. Please notice that this minimum does not coincide with the position of brace B. The oscillation amplitude mainly develops in the upper bout, starting from the waist, as we can see by the FEM simulation.
- In mode ⟨0 2⟩, *the frequency is 335 Hz*. Amplitude presents two absolute minimums in the measurement points 4 and 8. The first one lies approximately in the position of brace C, the second about halfway between brace B and brace A. The two amplitude minimums correspond to the two nodal lines shown in the

Chladni pattern for this mode, and to the two areas where—as from the FEM simulation—the vibration amplitude is low. As already observed, at this frequency the nodal lines divide the back into three vibrating areas, clearly visible both on the Chladni pattern and the FEM simulation. The representation of the modal amplitude provided by the graph shows the position of maximum and minimum vibration amplitudes with respect to the position of the braces.

We point out that the transverse nodal lines do not coincide with the position of the braces, as we would empirically expect. In fact, at typical frequencies of the free back the braces are not at rest but vibrate, being the reason why, by adjusting the height of each brace, we can control the modal frequencies as well as the distribution of amplitudes along the longitudinal axis of the back.

In conclusion, the modal analysis is useful to establish the main frequencies as well as amplitudes, primarily along the longitudinal axis but also in the transverse directions, taking into account the outcomes of the Chladni pattern analysis and—in case—the FEM simulations.

To better understand the practical meaning of this analysis, we must keep in mind that the back is not only set into vibration over a small area—as the soundboard is by the string (at the saddle of the bridge). The entire surface of the back can go into oscillation by means of the air motion in the body. Taking as an example mode $\langle 0\ 0 \rangle$, if the response is flat along the entire longitudinal axis, this means that the motion of the air can set the back into vibration in *each* point along the axis, hence the whole surface can be excited. If, paradoxically, the maximum response were *restricted to a limited area*, the motion of the air would only be able to excite the back in *that* area; consequently, only part of the surface would be able to vibrate effectively and partake in the coupling with the soundboard.

Furthermore, where amplitude is larger, the impedance encountered by the source of excitation is smaller, and the back has a higher propensity to effectively respond to the stimulation received by the air in the body. So it is necessary, at least in the main vibration modes, to have a large vibration amplitude.

In summary, the modal analysis we have suggested has the purpose to optimise the response of a free back through progressive adjustments of the braces, keeping in mind two targets:

- Reaching the deemed optimal values of the modal frequencies.
- Getting a balanced development of amplitudes and—at least in the three main vibration modes ($\langle 0\ 0 \rangle$, $\langle 0\ 1 \rangle$, $\langle 0\ 2 \rangle$)—high amplitude values.

We will see in the second part what actions allow to reach these results through progressive adjustments of the braces, or at least to strike a balance that the luthier, according to one's own experience, will deem suitable.

5.6 Contribution of the Back to Resonances

We have seen how to measure the response of a free back and get the modal analysis of the first resonances, considering that this response can be controlled by modifying the height of the braces. Now the question is whether, and how, this back optimised through a series of regulations is able to contribute to the global sound quality of the instrument.

To answer the question, we refer to the resonator schemes reported in Appendix 5.1, where we show that, *structurally*, the back is equivalent to the soundboard: both interact with the air in the body and with the soundhole, that is to say with the Helmholtz resonator, and both—at least at the main resonance—can be outlined like simple oscillators (mass, spring, loss coefficient—including radiation resistance toward the surrounding environment).

But the analogy between back and soundboard is only outward, and this for two reasons.

First, the back has very different elastic and mechanical properties than the soundboard, because of different timber nature and different bracing. Normally, the natural frequency of a free back is above 200 Hz (we have seen that, according to the modal analysis, in our reference guitar it is set at 215 Hz) while the soundboard natural frequency is usually comprised between 130 and 170 Hz (155 Hz in our reference guitar).

The second reason is that the soundboard is excited by an *external* force applied by the string on a limited area (the bridge) while the back is excited by an *internal* force originated by the energy exchanges between soundboard—air—back and soundboard—sides—back. The whole surface of the back is influenced by this internal force, which legitimizes our persistence in stressing the need to have vibrating areas covering the whole surface, and with great oscillation amplitude, in order to get a good interaction between back and soundboard through the air in the body.

The outward analogy between back and soundboard helps to understand that the back, by its vibration, besides moving the air in the body, radiates energy toward the external environment, so contributing to the global sound field generated by the instrument. This is an often forgotten but not negligible effect.

We will consider now what comes about when the back is coupled to a frame composed of sides and soundboard. It is appropriate for this analysis to apply an external force to the back. In our tests the external force is generated by the pendulum or an electromagnetic device, acting at the level of the bridge in order to maintain the analogy with the measurements executed on the soundboard.

5.6.1 Resonances of the Back. Mode ⟨0 0⟩

We have formerly suggested some measurements on the soundboard, and how to use the results to evaluate and optimize the resonator parameters.

We recall them here for the reader's convenience.

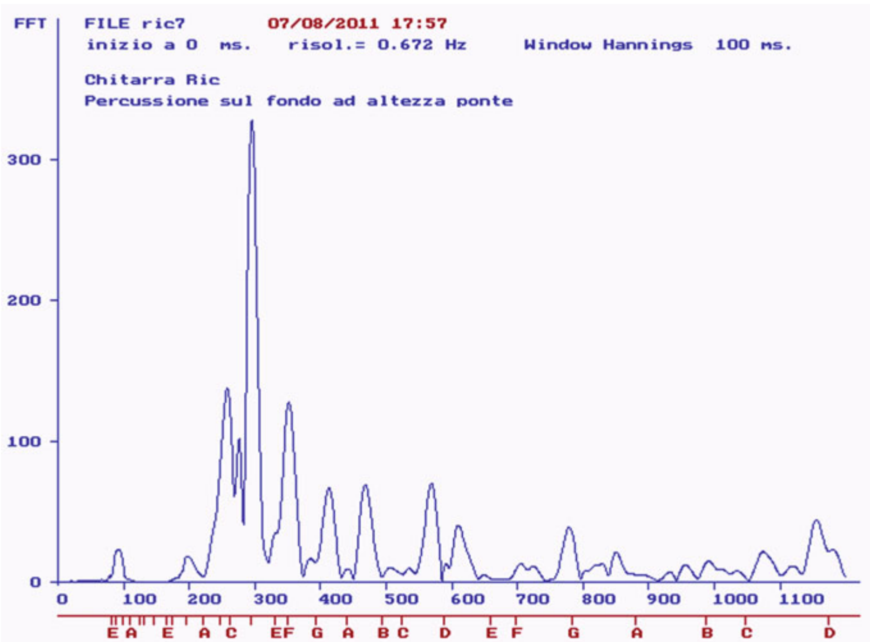
- *Resonator response obtained by exciting the soundboard at the middle of the saddle.*
- *'Covered soundhole' resonator response, obtained through soundboard excitation.*
- *Resonator response like the former, with additional mass.*

We suggest now a fourth measurement, adding up to the previous three.

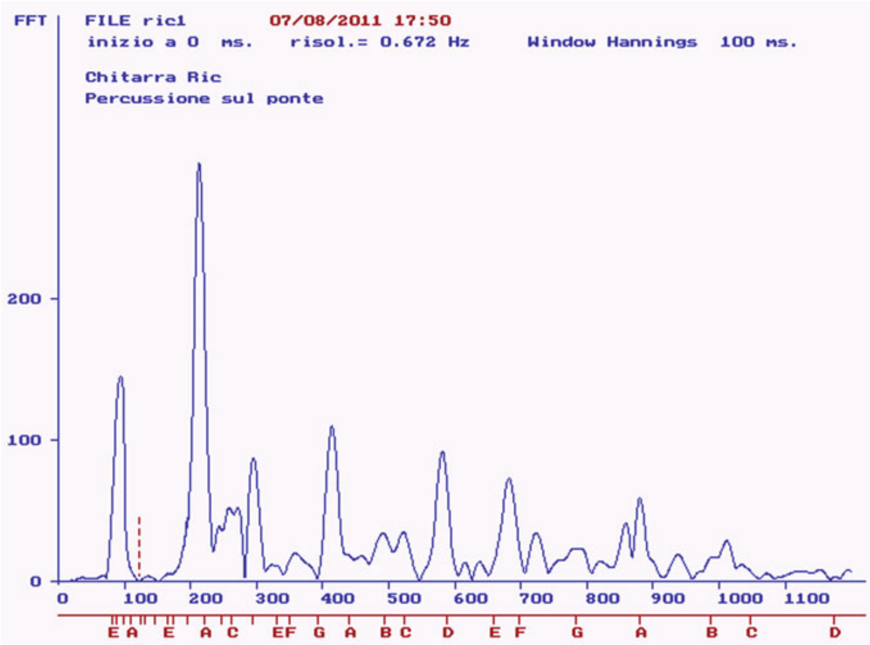
- *Resonator response obtained by exciting the back at the level of the bridge.*

The purpose of this measurement is to highlight the resonances of the back when it is attached to the body, and understand how they take on the role of *global resonances of the instrument*.

The following image represents the result of this analysis made on the reference guitar.



To interpret correctly the response obtained through excitation of the back at the level of the bridge, we must compare it with the already presented graph resulting from excitation of the soundboard at the saddle, reported again below for convenience. This diagram effectively sums up the resonator behaviour when set into vibration by the string, and therefore it depicts the *global response* of the instrument.



The first important resonance to be observed in the response of the back arises at about 258 Hz. This resonance—at the frequency we will call $\mathbf{F_{B0}}$—derives from the natural frequency of the free back in mode ⟨0 0⟩ which, as seen, in this instrument arises at 215 Hz. We will call this frequency $\mathbf{F_{b0}}$. On the formerly presented diagram of the modal analysis, both the frequency and the amplitude distribution of the free back in mode ⟨0 0⟩ are clearly visible.

The back, oscillating under the action of the external force, when attached to the frame opposes the stiffness of the air in the body, which in fact adds up to its natural stiffness. As a result, the resonant frequency rises: in our reference guitar the resonance of the back goes from $\mathbf{F_{b0}}$ (215 Hz for the free back) to $\mathbf{F_{B0}}$ (258 Hz, when the back is attached to the frame). This is *exactly* the same phenomenon we pointed out in the soundboard, when we observed that the soundboard motion, too, interacts with the air in the body; this increases the resonant frequency from 155 Hz (when the soundboard moves in free air) to 213 Hz (when the soundboard becomes part of the resonator and, in its motion, opposes the stiffness of the air in the body). Once again we are dealing with the analogy between back and soundboard, this time not merely structural (concerning mechanical components) but dynamic as well (concerning acoustic behaviours due to the same phenomenon—the interaction between air and soundboard and between air and back).

The phenomenon we have described explains the reason why a free back, whose natural resonance $\mathbf{F_{b0}}$ in mode ⟨0 0⟩ manifests at 215 Hz, when it is attached to the body resonates at the frequency $\mathbf{F_{B0}}$ (258 Hz). If this phenomenon only pertained the behaviour of the back, it would be just an inconsequential curiosity. In fact, at the

frequency $\mathbf{F_{B0}}$, the impedance in the point where the force applies to the back is very low, and its oscillation velocity becomes very high. This 'propensity' of the back to vigorously vibrate at the frequency $\mathbf{F_{B0}}$ turns into a similar propensity—on the part of the soundboard—to *vigorously vibrate at about the same resonant frequency* $\boldsymbol{F_{B0}}$ *of the back*. In other words the soundboard, which is part of the resonator, at the same frequency $\boldsymbol{F_{B0}}$ presents a resonance we could call *resonance due to the back* or *resonance induced by the back* which, in every aspect, is one of the guitar own resonances, just as the two basic resonances we have previously considered in detail.

On normal functioning the soundboard undergoes an *external force* due to the string excitation, and generates in turn an *internal force* that puts the back into resonance by means of the air in the body. The back in turn enforces its own resonance into the soundboard. If the natural frequency of the back is higher than the soundboard frequency ($\mathbf{F_{b0}} \rangle \mathbf{F_p}$), as is normally the case in the design of the guitar resonator, the resonance *induced by the back* arises at a higher frequency than the soundboard basic resonance.

This is nothing new: in Sect. 5.3 we have seen that when two uncoupled oscillators (whose original frequencies were f_a and f_b) are connected through an elastic medium, they bring about a new system where the original frequencies tend to *diverge*. The two new frequencies f_1 e f_2 that distinguish the coupled oscillator respectively fall left and right of the original frequencies f_a e f_b.

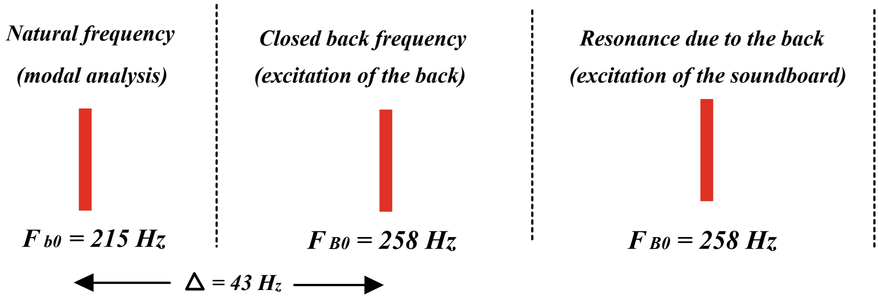
In our reference guitar the two primary oscillators are the soundboard (whose resonant frequency with the soundhole covered is $\mathbf{F_p} = 196$ Hz) and the back (whose resonant frequency is $\mathbf{F_{b0}} = 215$ Hz) coupled to each other by an elastic medium that is the air in the body. The new frequency $\mathbf{F_{B0}}$ at 258 Hz (the *resonance due to the back*) stems from the interaction between these two oscillators, and is placed right of the natural frequency of the back $\mathbf{F_{b0}}$, so $\mathbf{F_{B0}} \rangle \mathbf{F_{b0}}$. At the same time $\mathbf{F_p}$ is slightly pushed downward (by some Hz).

The specular case, when $\mathbf{F_{b0}} \langle \mathbf{F_p}$, has no practical meaning and we will not analyse it, except from observing that, in this case, $\mathbf{F_{B0}}$ would take place at *lower* frequencies than $\mathbf{F_p}$.

The divergence between the frequency of the primary oscillator ($\mathbf{F_{b0}} = 215$ Hz) and that of the coupled oscillator ($\mathbf{F_{B0}} = 258$ Hz) depends on the *coupling coefficient between back and air in the body*, which is conceptually akin to the formerly considered *coupling coefficient between soundboard and air in the body*.

The last dynamic analogy between back and soundboard is that, in the resonator response obtained through excitation of the *soundboard*, the natural frequency of the back $\mathbf{F_{b0}}$ presents itself as an *antiresonance*. In many occurrences—as in our reference guitar—this resonance can be hardly seen on the graph, being very close to the basic resonance of the soundboard.

The following scheme is a graphic description of the relative positions of the natural frequency $\mathbf{F_{b0}}$ (in mode $\langle 0\,0 \rangle$), of the frequency $\mathbf{F_{B0}}$ obtained through excitation of the back, and of the frequency obtained through soundboard excitation, coinciding with $\mathbf{F_{B0}}$.



Summing up the former reasoning:

– We found out important analogies (both structural and dynamic) between the behaviour of the back and that of soundboard. The back presents a natural resonant frequency $\mathbf{F_{b0}}$ in mode ⟨0 0⟩ (215 Hz in the reference guitar) that can be evaluated through the modal analysis.
– When the back is glued to the frame, namely when we close the resonator body, this frequency shifts to the value $\mathbf{F_{B0}}$ (258 Hz in the reference guitar). If, as usual in standard designs, the natural frequency of the back falls at a frequency higher than that of the soundboard (i.e. $\mathbf{F_{b0}} > \mathbf{F_{P}}$), the 'closed back' resonance arises at a higher frequency than the original natural one, so $\mathbf{F_{B0}} > \mathbf{F_{b0}}$. The gap between $\mathbf{F_{B0}}$ and $\mathbf{F_{b0}}$ ($\Delta = 258 - 215 = 43$ Hz) depends on a *coupling coefficient between back and soundboard*, conceptually akin to the *coupling coefficient between soundboard and Helmholtz resonator* (formerly observed when dealing with the formation of basic resonances).
– In the soundboard, too, we find a resonance *induced by the back*, or *due to the back*, at the frequency $\mathbf{F_{B0}}$. At this frequency the instrument generates a sound pressure due to the oscillation of its vibrating surfaces, just like at basic resonances.
– Finally, we observed that, in the global response of the resonator, an antiresonance *at the natural frequency of the back* $\mathbf{F_{b0}}$ takes place, yet normally difficult to be measured, being close to the basic resonance of the soundboard.

Lastly, we focus on issues that are perhaps the most important from the point of view of construction: what value shall we establish for the natural frequency of the free back $\mathbf{F_{b0}}$? Or else, how can we do so that the frequency of the fastened back $\mathbf{F_{B0}}$ takes place in a *proper position* with regard to the basic frequency of the soundboard? And, by the way, what is this 'proper' position?

These questions stem from the fact that we have given so far a *qualitative* explanation for the formation of the resonance $\mathbf{F_{B0}}$ due to the back, referring to the mechanism of coupled oscillators. But this same mechanism also enables a *quantitative* evaluation, based on the models presented in the appendix.

The next formula links the closed back frequency $\mathbf{F_{B0}}$ to the free back frequency $\mathbf{F_{b0}}$, but it also accounts for the basic resonances and the Helmholtz resonance. Therefore, it includes all of the parameters involved in the interaction between back, air, and soundboard:

$$F_{B0}^2 = F_1^2 + F_2^2 + F_{b0}^2 - 2F_h^2$$

If we put into the formula the already known values concerning our reference guitar

- $\mathbf{F_1}$ = 93 Hz (basic resonance of the air)
- $\mathbf{F_2}$ = 213 Hz (basic resonance of the soundboard)
- $\mathbf{F_h}$ = 129 Hz (Helmholtz resonance)
- $\mathbf{F_{b0}}$ = 215 Hz (natural resonance of the back in mode ⟨0 0⟩).

We get an *estimated* value for $\mathbf{F_{B0}}$ of 258.8 Hz, equal to the *measured* value ($\mathbf{F_{B0}}$ = 258 Hz).

The foregoing formula (of which we do not provide analytical proof) is congruent with the fundamental property of the coupled oscillators we have already run across: *the sum of the squares of the resonant frequencies in the coupled system is equal to the sum of the squares of the resonant frequencies in the uncoupled system.* In this particular case the soundboard (with its basic resonances), the Helmholtz resonator (with its resonance), and the back (with its natural frequency) come all into play.

The formula can be reversed to ascertain what the natural frequency $\mathbf{F_{b0}}$ of the free back should be in order to get the target frequency $\mathbf{F_{B0}}$ for the closed back:

$$F_{b0}^2 = F_{B0}^2 - F_1^2 - F_2^2 + 2F_h^2$$

This is the most useful version of the formula for the optimization of the back. It implies a practical procedure we outline now, deferring to the second part a thorough investigation of the subject:

- Suppose having optimized the resonator according to the procedure described in the previous paragraphs (position of the resonances and optimal quality parameters). As already mentioned, this optimization procedure can be developed with the aid of a rigid or semi-rigid provisional back, temporarily fastened to the frame, and working on an open back setting as well.
- Once a satisfactory result has been reached, the luthier must decide where to place the first resonance due to the back ($\mathbf{F_{B0}}$). At this stage the luthier's sensibility and knowledge of other instruments (not just guitars) will be most helpful. At least, this must be kept in mind: this resonance is meant to cover a range of the instrument response located right of the second resonance of the soundboard, and fairly close to it, but not too close in order to prevent potentially detrimental interferences or superimpositions. In the reference guitar, the divergence between the soundboard resonance ($\mathbf{F_2}$ at 213 Hz) and the first resonance of the back ($\mathbf{F_{B0}}$ at 258 Hz) is equal to 45 Hz, a value that ensures a proper gap (though it could be even smaller).
- Once the $\mathbf{F_{B0}}$ has been settled, the formula enables us to calculate the frequency $\mathbf{F_{b0}}$ that the free back shall assume in mode ⟨0 0⟩. This is the goal to aim to, acting on the braces and evaluating results, step by step, through the modal analysis.

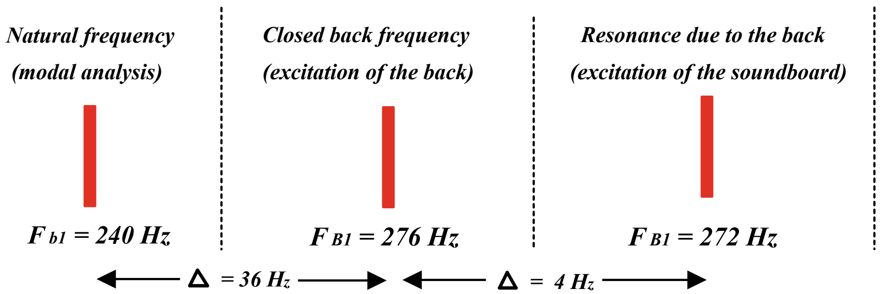
5.6.2 *Resonances of the Back. Modes ⟨0 1⟩ and ⟨0 2⟩*

Analogous reasoning apply to the other two resonances of the free back (not fastened to the frame) in modes ⟨0 1⟩ and ⟨0 2⟩ .

Many of the statements made for mode ⟨0 0⟩ are also true for these higher frequency modes.

– Looking at the modal analysis diagram we can see that, once the optimization accomplishments have been carried out, mode ⟨0 1⟩ of the reference guitar back arises at 240 Hz (which, by analogy, we will call $\mathbf{F_{b1}}$). When the back is glued to the frame, a corresponding closed back resonance arises at about 276 Hz (we will call this $\mathbf{F_{B1}}$), clearly visible on the response graph.
– The gap between the free back frequency $\mathbf{F_{b1}}$ (240 Hz) and the closed back resonance $\mathbf{F_{B1}}$ (276 Hz) is now equal to $\Delta = 276 - 240 = 36$ Hz, smaller than in mode ⟨0 0⟩ (where it was 43 Hz). The gap is smaller because now the divergence between the frequency of the soundboard $\mathbf{F_p}$ and the modal frequency of the back is greater, and so the degree of coupling between back and soundboard (through the air in the body) is lower.
– Here again mode ⟨0 1⟩ of the back at the frequency $\mathbf{F_{B1}}$ is able to *induce* a resonance that can be observed on the response curve obtained through soundboard excitation at the bridge (resonance *due to the back*). The frequency we find in the soundboard is slightly lower than that of the back (272 Hz) because—in this situation—the link between back, air and soundboard is less tight, owing to the reason explained in the previous point.

The next scheme is a graphic representation of the relative positions of the natural frequency of the back in mode ⟨0 1⟩, of the frequency obtained by exciting the back and the frequency obtained by exciting the soundboard.

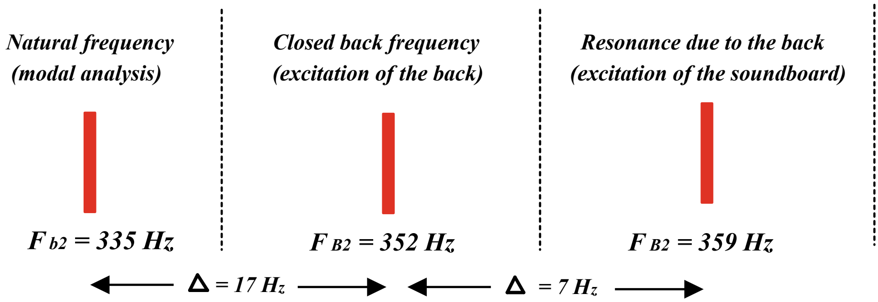


The modal analysis has shown a third significant vibration mode of the free back—mode ⟨0 2⟩—taking place at 335 Hz (by analogy we will call this $\mathbf{F_{b2}}$). Once again, observations here are similar to what previously stated:

– When the back is glued to the frame, a corresponding closed back resonance, evident on the response graph, manifests at about 352 Hz. We will call this $\mathbf{F_{B2}}$.

– In this case the coupling between back and soundboard is even lower and, accordingly, the gap between the frequency $\mathbf{F_{B2}}$ and the frequency $\mathbf{F_{b2}}$ is now just $\Delta = 335 - 352 = 17$ Hz. We remind that the gap was $\Delta = 258 - 215 = 43$ Hz in mode $\langle 0\ 0 \rangle$ and $\Delta = 276 - 240 = 36$ Hz in mode $\langle 0\ 1 \rangle$.
– Here, again, mode $\langle 0\ 2 \rangle$ of the back at the frequency $\mathbf{F_{B2}}$ is able to *induce* a resonance *due to the back*, that we also find on the response curve obtained by exciting the soundboard at the bridge, at a slightly higher frequency than that measured on the back (359 Hz). Once more, at this frequency, the instrument generates a sound radiation from back and soundboard (and soundhole, though much inferior).

We recall here the scheme of the relative positions of frequencies in mode $\langle 0\ 2 \rangle$.



As a conclusion, the gap between the free back frequency (that of the modal analysis) and the closed back frequency (measured exciting the back at the level of the bridge) depends on the degree of coupling between back and soundboard. This coefficient, in turn, depends on the gap in Hz between the frequency of the back and that of the soundboard: the greater the gap, the lower the degree of coupling and the gap *between closed back frequency and free back frequency*.

Each of the closed back frequencies induces a resonance visible on the soundboard (we called it 'resonance induced by the back' or 'resonance due to the back'). We observed that, when the coupling is tight, the resonance induced in the soundboard practically matches that of the closed back, while it slightly diverges if the coupling is loose. This is intuitive: if the coupling between back and soundboard is loose, the soundboard is less restrained by the back, and therefore can move more freely.

5.6.3 Other Resonances from the Closed Back Response

The global instrument response (obtained through soundboard excitation at the bridge) highlights the two basic resonances (the resonance of the air $\mathbf{F_1}$ at 93 Hz and that of the soundboard $\mathbf{F_2}$ at 213 Hz), as well as the antiresonance at 129 Hz (corresponding to the Helmholtz resonance $\mathbf{F_h}$). We will not resume the topic of the basic resonances (already investigated in detail), except from observing that they

show a very small amplitude in the response of the back. In fact, the coupling of the back with the Helmholtz resonator is loose, and the back behaves as if it were quasi rigid; in this situation it is natural that, on excitation of the back, the basic resonances—though present and visible—do not stand out significantly.

The important conclusion we draw is that, *in the low frequency field* where the basic resonances prevail (up to about 200 Hz), *sound radiation is almost completely brought about by the soundboard and the air in the body through the soundhole, while the back plays a restricted role*. *In the mid register* (up to about 400 Hz) *the back contributes to sound radiation both directly and through the resonances it induces in the soundboard. On the other hand, the contribution of the soundhole tends to fade away beyond 300 Hz.*

To conclude this review of the guitar resonances in the mid-low register we wish to point out, in the global response of the reference instrument, a powerful resonance taking place at 295 Hz. This is the resonance in mode ⟨1 0⟩, distinguished by a single nodal line running in the longitudinal sense. The two vibrating surfaces of the dipole vibrate in antiphase, and their sound emission, adding up, tends to nullify. In some instruments this resonance is not effective, so it does not appear in the overall response. In this instrument it comes about instead with notably large amplitude, because of the dipole asymmetries (different vibrating surface and/or different vibrating mass in the two vibrating sectors) leading to a *non-zero* net sound pressure. This essentially depends on the asymmetric bracing of the soundboard (a notion that will be deeply discussed in the second part).

We find the same resonance in the response curve of the back, still with a considerable amplitude. In this instance the resonance in mode ⟨1 0⟩ is *induced by the soundboard in the back* through a mechanism similar to the one we previously described: while formerly the resonances of the back in modes ⟨0 0⟩, ⟨0 1⟩, and ⟨0 2⟩ enforced the soundboard resonances, now is the turn of the soundboard (in mode ⟨0 1⟩) to enforce the resonance of the back.

5.7 Conclusions

In this chapter we have considered the behaviour of the guitar in a range of frequency spanning from around 80 Hz (the E on the sixth string) up to around 400 Hz (the G# on the first string), that is to say in a register we classified as 'mid-low'.

This register embraces many of the most significant resonances for the quality of sound in a guitar:

- *Basic resonances* (*air* and *soundboard resonance*). These two resonances *always* appear in every guitar, since their presence is related to the resonator own structure, which is in fact composed of two flexible tables (back and soundboard) enclosing a volume of air that communicates with the exterior through the soundhole.
  We have examined the influence of the basic resonances on the tones produced by the guitar, recalling Chap. 1 where we found out what parameters determine

sound quality (like sustain or attack and decay transients), in turn being related to the resonator physical parameters.

We have pointed out that the basic resonance of the air is properly excited by even much higher pitched tones. So this resonance gives the sound texture a background colour that is well known by both luthiers and guitar players.

But guitars are not equivalent as for sound quality just because of two basic resonances appearing in the mid-low register. Therefore we have suggested some measurements on the instrument and developed a criterion that—based on the results of these measurements—allows evaluation of the *objective* parameters that are relevant to the quality of sound. These parameters are referred to the physical model of the guitar resonator reported in the appendix. The suggested measurements are: the one with the soundhole covered, the one with the soundhole covered and an additional mass on the soundboard, and the normal open soundhole measurement. Based on a fair number of analysis executed on top, average, and poor quality instruments, we have established a range of values that each of these parameters, in an excellent instrument, should conform to. We have also provided some optimization criteria that the luthier can directly apply or adapt to one's own sensibility, once the operation mechanism of the guitar resonator and the meaning of the typical parameters has been understood. This analysis can be applied during construction of one's own instrument, as well as to finished instruments (by other luthiers if need be) for an objective evaluation of their quality and an inquiry into the structural elements that brought about that result.

We have finally presented a diagram that allows to ascertain the expected values when the natural resonance of the soundboard is known. We consider this a very useful approach for the optimization of the soundboard under construction.

- *Resonances 'due to the back'*. We especially insisted on the issue of the back which, intentionally or not, is generally underestimated. We believe that the back, when correctly sized, can give a significant contribution to the quality of sound.

  We introduced a method, the modal analysis—that will be fully developed in the second part—which allows a graphic reading of the amplitude and extension of the vibrating surfaces along the longitudinal axis, at least for the three most important resonances (those of modes $\langle 0\ 0\rangle$, $\langle 0\ 1\rangle$, and $\langle 0\ 2\rangle$). This representation offers the chance to work on the bracing of the back, optimizing it according to the criteria we have referred to (and that we will summon up later on).

  When the back is glued to the frame, each of the free back vibration modes brings about a new, 'closed back' vibration mode, that can be read on the response curve resulting from excitation of the back. In each vibration mode, the divergence between the free back frequency and the closed back frequency depends on the coupling coefficient between back and soundboard. This coefficient depends in turn on the divergence in Hz between back and soundboard frequency. However, knowing the free back frequency, it is necessary to estimate the closed back frequency: the formula we introduced allows to calculate the frequency of the back glued to the frame when the frequencies of the free back and of the basic resonances are known. Then we have observed that, when the back is fastened, each of the vibration modes of the back induces an equal or very close soundboard resonance. These resonances due to the back set soundhole and soundboard into

vibration, contributing to the overall sound radiation of the instrument and adding to the radiation produced by the back itself.

- The *resonance in mode* ⟨1 0⟩. This resonance, too, is sometimes underestimated, since it only arises in the sound response if, by means of special structural expedients, an asymmetry of the dipole, hence a significant acoustic output is achieved. This vibration mode normally appears around 300 Hz (as in our reference guitar) and sets the back into vibration as well: in this case we have a resonance arising from the soundboard that is induced onto the back, where it results in a large amplitude oscillation.

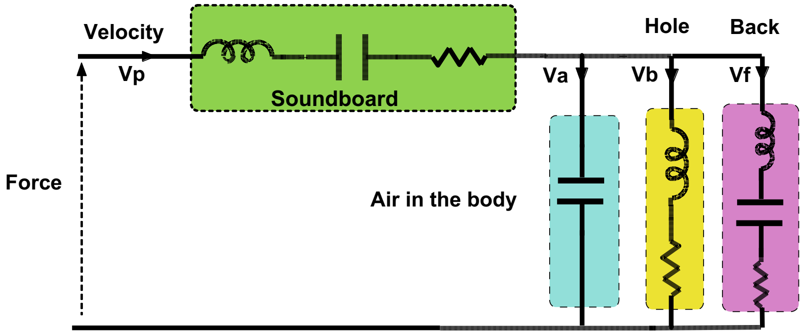
We referred this analysis to the results obtained from a high quality Garrone guitar. The luthier who, based on one's own production instrument, wishes to retrace the path and measurements we have suggested, will probably get different outcomes. This is natural, since every instrument holds a particular character and unique features that distinguish it from all other instruments. However, the notions expounded here have a general legitimacy that goes beyond the analysis of some specific instrument. These concepts can be really helpful for the analysis and optimization of an instrument under construction or, in case, for the examination of a finished instrument. This, at any rate, is our aim!

The mid-low register we have examined (between 80 and 400 Hz) is certainly crucial for the sound quality of a guitar. This register involves the fundamental and the first harmonics of low tones (at least up to the A at 220 Hz) and the fundamental of tones up to G# on the first string. Anyway, the sound response of a guitar is also deeply dependent on the response in the upper-mid register and in the highest register, where the resonator behaviour relies on different mechanisms than those we have so far described. But this is a different story, that will be related in the next chapter.

Appendices

Appendix 5.1: The Resonator Model

In the next scheme the fundamental resonator components (soundboard, back, Helmholtz resonator) are represented in such a manner as to highlight both their physical structure and their interaction.



Back and soundboard are represented in this model as simple oscillators featuring mass, stiffness, and loss coefficient. The soundhole is shaped like an element defined by a mass of air and a loss coefficient that, along with the elasticity of the air in the body, define the Helmholtz resonator.

In developing the model equations we adopted the *electro-acoustic analogy*, that allows to relate acoustic parameters (mass, stiffness, loss factors, vibrating surfaces) to electrical parameters (inductance, capacitance, resistance). According to this analogy, the force applied on the acoustic system corresponds to the voltage applied on the equivalent electrical circuit, while the velocity in the acoustic system corresponds to the current in the electrical circuit. Furthermore, capacitance corresponds to stiffness, inductance to mass, electric resistance to loss coefficient. The electrical parameters involved in the model also depend on the *vibrating surfaces* of the resonator components (soundboard, soundhole, back) that are relevant to sound radiation.

This is not the only possible description of the phenomena involved in the guitar resonator. We opted for this one because it allows to apply techniques that are normally employed in the study of electrical circuits, and so to extend them to acoustic systems.

According to this electro-acoustic analogy, the relation between *force* applied and velocity V_p in the application point (usually the bridge) is the *acoustic impedance* Z_r of the resonator at the bridge:

$$Z_r = \frac{Force\ applied\ on\ the\ bridge}{Velocity\ at\ the\ bridge}$$

This impedance is a function of the frequency: it is very low at resonance conditions (where velocity is maximum) and very high at antiresonances (where velocity is minimum).

Generally, the resonator response property is not expressed as *impedance* Z_r but rather as *mobility*, which is the reciprocal of impedance, so

$$Mobility = \frac{1}{Z_r} = \frac{Velocity\ at\ the\ bridge}{Force\ applied\ on\ the\ bridge}$$

The advantage of the representation in terms of mobility instead than in terms of impedance is that, on the mobility diagram, the resonances assume very large amplitudes, while the antiresonances show very low amplitudes. This is in fact a more intuitive representation, the one we used in reporting the resonator measurements and in tracing a graphical description of them.

We recall the expression of the *reflection coefficient* ρ introduced in Chap. 2:

$$\rho = \frac{R_c - Z_r}{R_c + Z_r}$$

where R_c is the *characteristic resistance* of the string, defined as:

$$R_c = \frac{Longitudinal\ Tension}{Waves\ travelling\ speed} = \sqrt{\mu T}$$

At resonant frequencies (where the resonator impedance is low) the wave that propagates along the string, from the nut towards the bridge, is mostly absorbed and exploited by the resonator; the available energy is returned as sound pressure through the oscillation of the resonator surfaces (soundboard, back, soundhole), while the reflection is too scarce to support the oscillation of the string that, as a consequence, quickly damps down.

At antiresonances (where the resonator impedance is high) the incoming wave at the bridge reflects almost completely without energy absorption on the resonator part, so the oscillation amplitude of the vibrating surfaces is small.

The force in the previous scheme is the *force that the string actually applies on the resonator, net of reflection*.

By the previous scheme we understand that the force received by the soundboard in the application point (usually the bridge) is used in part to excite the soundboard and in part to excite the other resonator components (air, soundhole, back). The fraction exploited by the soundboard determines its displacement velocity V_p in the application point, while the residual fraction determines the displacement velocity V_b (of the air in the soundhole), V_f (of the back) and V_a (of the air in the body).

So the three effective radiant surfaces of the resonator (soundboard, soundhole, back) move respectively at velocity V_p, V_b, and V_f. Each of them oscillates an *air volume* whose oscillating energy depends on both the extent of the working surface and the oscillating velocity of the air volume itself.

The motion of each of these air volumes brings about in the environment a sound pressure that is individually generated by each of the resonator radiant surfaces. At a certain distance from the body, the *sound field* generated by the instrument is the result of a combination between amplitudes and phases of the sound pressures due to the various vibrating surfaces.

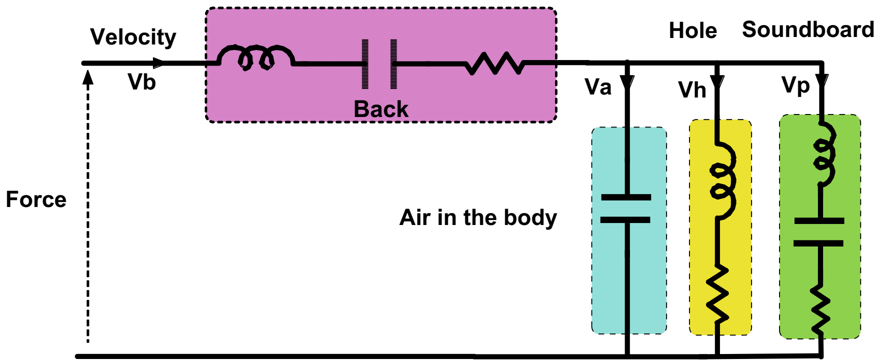
When the surfaces that define the resonator oscillate in the surrounding air, they meet a *radiation resistance*, that is the resistance of the air to be set into vibration. This resistance causes a loss of oscillating energy, hence a damping of the oscillation

in the vibrating surfaces. In the model it is merged into a single resistance parameter with the losses due to viscous friction in the material.

On the previous scheme we notice that the resonator soundhole is defined by the mass of air it contains and by the loss coefficient that—as already observed—also takes into account its radiation resistance. The impedance of the element that represents the soundhole grows with the frequency, since the mass of air (the inductance in the equivalent electrical circuit) tends to oppose the quick variations of the force applied (the voltage at the ends of the equivalent electro-acoustic circuit). Accordingly, as the frequency increases, the velocity of the air in the soundhole V_b falls progressively down to zero. When this velocity is very slow or—in case—nil, the soundhole, as far as acoustics is concerned, is virtually closed. In the frequency range that involves the basic resonances and the first resonances of the back, the soundhole is working and offers an effectual contribution to sound radiation while, beyond this limit, its role gradually comes to an end.

If we impose a very high stiffness value on the back in the preceding scheme, we get a situation where the back is perfectly rigid. It can be helpful, during construction of the instrument, to couple provisionally the soundboard—possibly glued to the frame—with a rigid back. This procedure can help in optimizing the soundboard when still accessible and modifiable, i.e. not yet secured with an elastic back. Yet in some instruments the back is kept intentionally rigid. The 'rigid back' scheme enables studying these kind of instruments as well.

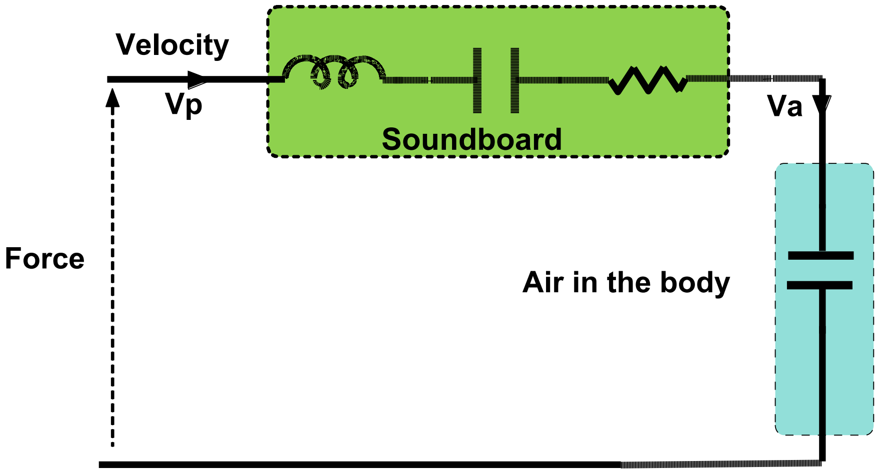
On the previous scheme we can see that back and soundboard are *structurally* similar, being both shaped like simple oscillators characterized by mass, stiffness, loss coefficient, and vibrating surface. This analogy calls for the examination of a case where back and soundboard mutually *exchange their roles*, according to the next scheme.



Here the force is applied on the back (usually at the level of the bridge) and no more to the soundboard as before. The values of the distinctive parameters of the model do not vary, in comparison with the previous case, since they are related to the physical nature of the resonator components. Conversely, the model response changes: in this situation, the resonances of the back stand out above all. This setting

allows in fact a study of the contribution of the back to the global resonator response, and an optimization of the placement of the resonances of the back with respect to those of the soundboard.

The last scheme concerns the case when the soundhole is closed and the back is rigid (or deemed so). In this situation the soundboard operates on the air in the body, counteracting its elasticity. The force applied on the soundboard is exploited partly in exciting the soundboard, partly in oscillating the air. Air and soundboard oscillate at the same velocity. Clearly, there is no airflow from the body interior towards the outside, so the sound emission is entirely due to the soundboard.



This simplified system features a single resonance at a frequency we will call $\mathbf{F_p}$. As already observed, we need to know the $\mathbf{F_p}$ to assess the other resonator parameters: chiefly the vibrating mass and, secondly, the vibrating surface and the coupling coefficient.

Appendix 5.2: Verification of the Model Outcomes

The exactness of a model must always be verified by comparison between experimental results and evaluations that the model itself provides. In other words, we must make sure that the model outcomes are sufficiently reliable.

For this purpose we refer to formerly mentioned data, obtained by measurements executed on a reference guitar, that we recall here for the reader's convenience:

- $\mathbf{F_1}$ (air resonance) = 93 Hz,
- $\mathbf{F_2}$ (soundboard resonance) = 213 Hz,
- $\mathbf{F_h}$ (Helmholtz resonance) = 129 Hz.
- $\mathbf{F_p}$ (covered soundhole resonance) = 196 Hz.

In addition, the value $\mathbf{F_{p0}}$ of the soundboard natural frequency:

- $\mathbf{F_{p0}}$ (soundboard natural frequency) = 155 Hz,

that was available during the instrument construction. In many cases (e.g. finished instruments) a direct measurement of this parameter is obviously not possible; as an alternative, the $\mathbf{F_{p0}}$ can be obtained by estimation via the model.

The first fundamental relation we employ after development of the model equations is:

$$F_p^2\,(estimated) = F_1^2 + F_2^2 - F_h^2 \quad so \quad F_p(estimated) = \sqrt{\left(F_1^2 + F_2^2 - F_h^2\right)}$$

This equation allows to *estimate* the covered soundhole resonant frequency of the resonator when we know the two basic resonances (F_1 and F_2) and the Helmholtz resonance F_h, and also allows to assess the inaccuracy of the model by means of the known (*measured*) F_p.

Setting these data into the equation above we get F_p (*estimated*) = 193.3 Hz, in front of a *measured* $F_p = 196$ Hz (covered soundhole measurement on the reference guitar).

The error between the estimation and the experimental value is equal to 2.7 Hz: a tolerable discrepancy, considering that we are comparing experimental data with calculated ones.

The preceding formula is congruent with the theory of coupled resonators we have formerly cited, whereby *the sum of the squares of the resonant frequencies in the coupled system is equal to the sum of the squares of the resonant frequencies in the uncoupled system*. As for the resonator, the frequencies of the coupled system are the two basic resonances $\mathbf{F_1}$ and $\mathbf{F_2}$, while the frequencies of the uncoupled system are $\mathbf{F_p}$ and $\mathbf{F_h}$.

The second fundamental equation taken from the model is

$$F_{p0}(estimated) = \frac{F_1 F_2}{F_h}$$

This equation allows to estimate the natural frequency of the fastened soundboard, when we know the two basic resonances $\mathbf{F_1}$ and $\mathbf{F_2}$ and the Helmholtz resonance $\mathbf{F_h}$.

Setting the given values of $\mathbf{F_1}$, $\mathbf{F_2}$, and $\mathbf{F_h}$ into the equation we get the model estimation of the soundboard natural frequency: $\mathbf{F_{p0}}$ (*estimated*) = 153.6 Hz. The experimental value measured on the reference guitar is 155 Hz. Once again, the error between the model estimation and the experimental value is very small (about 1.4 Hz).

The values estimated through the model, either regarding the covered soundhole resonance $\mathbf{F_p}$ or the soundboard natural frequency $\mathbf{F_{p0}}$, are very close to the experimental findings. This means that the model is correct, apart from expectable, little differences between calculated and experimental results. These differences are due to approximations introduced in order to describe the real functioning of the guitar resonator in a simplified and computable manner.

However, the model is sufficiently reliable for our purpose of evaluating the quality parameters in an instrument finished or under construction.

Chapter 6
Upper Resonances

Abstract This chapter focuses on the upper modes of the guitar in the range of 400–800 Hz, where the formation of the resonances outcomes from the coupling of the higher modes of the air with the higher modes of the top and back plates. The resonances which typically one can observe in this range are due to the modes of the plates, and to their phase and amplitude in relation to the air modes. Hence the coupling can be analysed looking to standard FEM models of the plates in comparison to the air mode shapes. The chapter ends with a graph encompassing all the various components contributing to sound production and their interaction in various frequency bands.

In the classification we will use, the low-mid register includes the fundamental and the first harmonics of low tones (at least up to A at 220 Hz) and the fundamental of the tones up to G# on the first string—therefore the frequencies between 80 and 400 Hz. The upper-mid register spans from 400 to 800 Hz, so involving the octave from G# to the high G# on the first string.

In concluding the previous chapter we stated that the resonator behaviour, in the upper-mid register, is related to different operating mechanisms than those accountable for the low-mid register response. Let us consider this differences:

- In the low-mid register, the two basic resonances take place (one of the air and one of the soundboard), owing to the coupling between soundboard and Helmholtz resonator. We also pointed out the presence of a series of resonances that depend on the interaction between the vibration modes of back, soundboard and air in the body, and a resonance due to the soundboard in mode $\langle 1\ 0\rangle$. The formation of these resonances basically depends on the Helmholtz resonance, on the first resonances of the soundboard in modes $\langle 0\ 0\rangle$ and $\langle 1\ 0\rangle$, and on the first resonances of the back in modes $\langle 0\ 0\rangle$, $\langle 0\ 1\rangle$, and $\langle 0\ 2\rangle$.
  The upper-mid register resonances are instead due to
  – Higher vibration modes of the *air in the body* (already examined in Sect. 4.1).
  – Vibration modes of the *soundboard*, at frequencies higher than those involved in the low-mid register; these frequencies were already examined in Sect. 4.2. As

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the frequency grows, the typical nodal lines of these modes assume an increasingly composite character, and tend to gather in characteristic areas of the soundboard—namely the area under the lower bout of the body—but also around the soundhole.
  - Vibration modes of the *back*, wherein the nodal lines are not only transverse (like in the simple modes $\langle 0\ 0 \rangle$, $\langle 0\ 1 \rangle$ and $\langle 0\ 2 \rangle$ that we have previously examined) but are also affected by the bending of the braces along their axis (see Sect. 4.3).

- In the low-mid register, the sound emission from the soundhole is especially high at basic resonances. As the frequency grows, the air meets a rising obstacle (an *impedance*) to its rapid flow through the soundhole. This also hampers the sound radiation from the soundhole: calculations (by the mathematical model) reveal that, at 400 Hz, the sound radiation from the soundhole is at least 20 times lower than at basic resonances. We can reasonably presume that, in the upper-mid register, sound radiation is only brought about by back and soundboard or, when the coupling with the back is poor, chiefly by the soundboard (possibly coupled with the air in the body, when conditions for a good interaction exist).
- The resonances manifested in the low-mid register, individually dependent on the vibration modes of the resonator components (soundboard, air, back). In the upper-mid register, two vibration modes close in frequency, which are *excited at the same time*, may be present: on the response diagram, these two resonances look like *merged together*. This phenomenon becomes especially evident as the frequency rises: in the high and in the highest register the instrument behaves as a continuous resonant system where the resonances, though manifesting a low amplitude, contribute to sound radiation and to timbre characteristics.

First of all, we must point out that the 400 Hz frequency, which separates the low-mid register from the upper-mid register, is not a barrier: there is a large grey area, wherein the two mechanisms coexist. For instance, we have seen in Chap. 5 that, in the low-mid register, a resonance due to the back takes place at 359 Hz, which we have ascribed to the interaction between the soundboard at its natural frequency, the air in mode $\langle 0\ 0 \rangle$, and the back in mode $\langle 0\ 2 \rangle$. In fact, both mode $\langle 0\ 0 \rangle$ and mode $\langle 0\ 1 \rangle$ *of the air* probably contribute to the formation of this resonance. In modern guitars (like Garrone's) the latter mode typically arises at 395 Hz (Sect. 4.1).

Reality is always more complex than our descriptions can convey, and so we can just represent it in simplified and partial ways. To examine the development of the resonances in the upper-mid register, we cannot rely on the model we used for the basic resonances and for the resonances of the back. Each resonance of the instrument must be examined taking into account the behaviour of the three fundamental resonator components (air, back, and soundboard), in order to understand how their characteristics interact and determine the formation of the resonances. The information we have is:

- Vibration modes of the air in the body, their own frequencies, and the progression of pressure waves inside the resonator body. We studied these modes in Chap. 4, where we observed that modal frequencies are tightly connected with the geometrical

dimensions of the body. We presented a measurement method that luthiers can use to evaluate air modes in their own model, by means of a sinusoidal signal generator, a loudspeaker placed in the instrument body, and a microphone.
- Upper vibration modes of the soundboard, which can be obtained by applying the Chladni method to the soundboard on the mould or, possibly, by a FEM simulation.
- Upper vibration modes of the back which, like the soundboard ones, can be obtained by the Chladni method or by the FEM simulation.

A good interaction between a vibration mode of the soundboard and a vibration mode of the air depends on the simultaneous presence of the following two conditions:

- The modal frequencies of air and soundboard must be close, i.e. they must *concord in frequency*. We do not specify how 'close': we will see hereinafter some examples of strong coupling, where the modal frequencies of air and soundboard are in close proximity, and cases where the coupling is weak, because of an excessive gap between the modal frequencies of air and soundboard.
- The soundboard must vibrate *in phase* with the air in the body. Generally, as we will see, a rising frequency brings about an increasingly complex pattern of the soundboard vibrating areas. Furthermore, the phase distribution takes on a matrix-like pattern, where the contiguous vibrating areas, separate by longitudinal and transverse nodal lines, vibrate in antiphase. The net effect of the contiguous areas on the air is nil, unless the surface/mass ratio of the zones that are in phase prevails on the surface/mass ratio of the zones in antiphase. We can compare this situation to a boat, where a group of people tries to push one way while another group rows against: the boat moves towards the goal if we have more rowers in the first group (increase of the 'in phase' vibrating areas) or these are stronger than the others (reduction of the mass).

In conclusion, the presence of two vibration modes of air and soundboard, that simultaneously *concord in both frequency and phase,* produces a resonance in the instrument response that effectively contributes to the sound radiation from the soundboard.

The modes of the back, just like the soundboard ones, couple conveniently with the modes of the air if *phase and frequency concordance* are simultaneously present. Back and soundboard are symmetrical components with regard to the air in the body, and they share a similar behaviour, despite obviously different mechanical parameters (elastic modules, density, thicknesses, bracing, etc.) and accordingly different vibration modes.

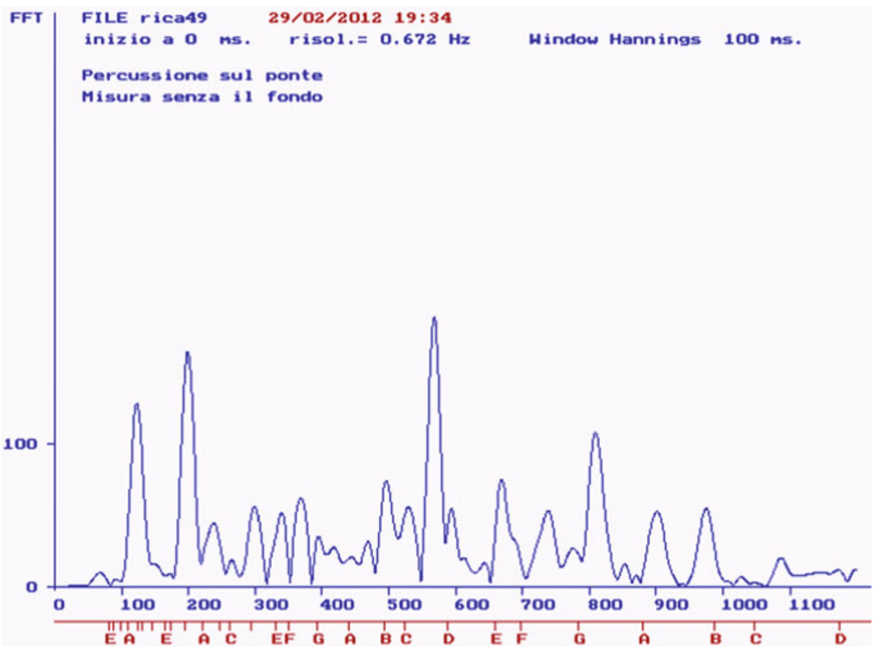
However, a fundamental difference exists: while the soundboard is excited by an *external* force (the string) the back can be excited by the air in the body and—to a minor extent—by the sides, that is to say by *internal* forces. This implies that, at frequencies where the back is *potentially* able to interact with the air, the coupling *actually* takes place only if the soundboard is able to excite the back; therefore the back needs a third condition, besides phase and frequency concordance with the air: a *synergy with the soundboard*. Only under these conditions we have a significant sound radiation from the back as well. In practice, as we will see, these conditions only occur for the back in the lower part of the upper-mid register (i.e. up to ca. 650 Hz) and not beyond.

While the interaction phenomenon between soundboard, air, and back determines the formation of the instrument resonances in the lower part of the register, beyond 650 Hz the resonances are due to the coupling between just the soundboard and the air or, possibly, just to the soundboard. The contribution of the back is limited (at least with the traditional three transverse braces).

To prevent confusion, we recall once again the very important role that the back plays in the formation of resonances in the low-mid register (see Sect. 5.6), whereas in the upper-mid register a frequency band exists—between about 650 and 750 Hz—where no synergy is present between back and soundboard and, consequently, the contribution of the back is poorer.

A significant aid to this investigation comes from the *response of the soundboard glued to the frame with the open back*. We have already discussed this setting in Sect. 5.6, highlighting the two resonances ‘of the bell’ and ‘of the soundboard’, which take place at low frequency. We also stated that, as the frequency grows, the sides (and linings) show an increasing tendency to behave like rigid structures. This means that, beyond a certain frequency (typically beyond 550 Hz) the soundboard glued to the frame with the open back responds as if it were perfectly rigid along its perimeter, and the modes we measure correspond to the fastened soundboard own modes. So the *response of the soundboard glued to the frame with the open back* is a useful element, of which we will largely make use hereinafter.

For the reader’s convenience, we show once again the response diagram of the soundboard glued to the frame with the open back.



Now we have reviewed all the means we will use to interpret the instrument response in the upper-mid register, and to explain the formation of the resonances here involved. First of all, we must cross-check the results obtained by different evaluation procedures (back and soundboard response, measurements of the air resonances in the body, FEM simulations and Chladni patterns of back and soundboard, open back response of the soundboard). Therefore, we must observe the results in light of the rules that ensure a good interaction between air and soundboard (frequency and phase concordance), not neglecting the rule that ensures a good soundboard-air-back interaction: the *synergy between back and soundboard*.

As we will see in the second part of the book, this investigation method also allows to understand how we can act on the design of back and soundboard, in order to optimize the resonator response in the upper-mid register.

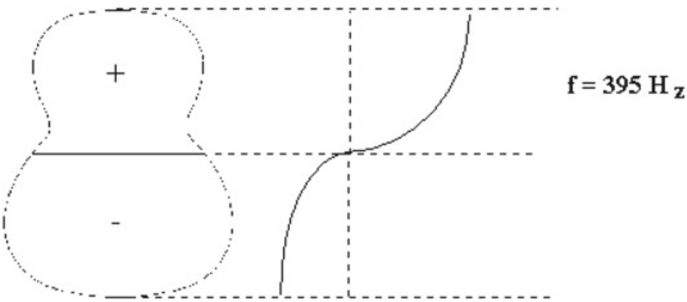
The characteristics of the previously examined reference guitar will guide us through.

6.1 Resonances in the Mid-High Register

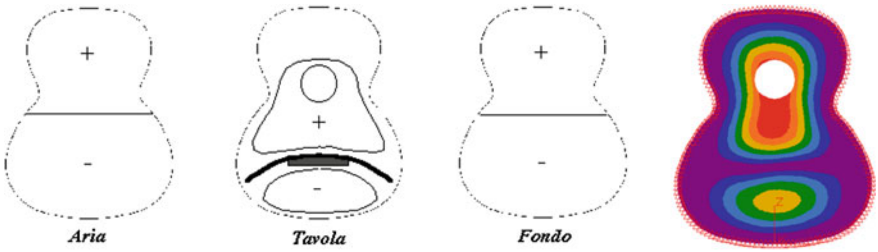
6.1.1 The Guitar Resonance in Mode ⟨0 1⟩

Looking at the response of the reference instrument—presented and discussed in Sect. 5.2—we notice that the first important resonance, with considerable amplitude beyond 400 Hz, arises at about 414 Hz. This is the resonance in mode ⟨0 1⟩, featuring a single nodal line that generally runs transversely through the bridge.

The formation of this resonance depends on the air resonance in mode ⟨0 1⟩ that we examined in Sect. 4.1. For the reader's convenience, we report again the illustration of the *quasi sinusoidal* progress of the pressure wave that, running along the longitudinal axis of the cavity, divides the resonator body into two volumes (above and under the waist). In the upper volume, the air vibrates in *antiphase* with the lower volume. Just under the waist, the pressure wave is nil and brings about a nodal line where the vibration amplitude is nil. In modern guitars, this vibration mode takes place between 380 and 400 Hz. Specifically, in the reference guitar, the air resonance in mode ⟨0 1⟩ takes place at 395 Hz, well matched with the value calculated from the instrument dimensions by the method described in Sect. 4.1.



Mode ⟨0 1⟩ of the air can be excited by mode ⟨0 1⟩ of the soundboard, provided that conditions favourable to a good coupling between air and soundboard exist. In the following figure, the first sketch on the right shows the oscillation amplitude of the soundboard in mode ⟨0 1⟩, obtained from the FEM model. We notice that, while the nodal line in the air runs at the waist, the nodal line of the soundboard runs approximately at the level of the bridge (second sketch on the left). This means that a phase concordance is present under the bridge and above the waist.



The surface of the soundboard that oscillates in phase with the air tends to amplify its oscillations, so supporting the coupling between air and soundboard. Vice versa, the surface that oscillates in antiphase hampers the motion of the air in the cavity. Consider pushing a swing: if the push is synchronized, the oscillation amplitude of the swing increases, while non synchronized pushes slow it down. But, at the moment we go for a push, we also receive a contrary push and, if we do not stand firmly, we will find ourselves swinging, too. In this simple test we are part of an interaction between two systems (us and the swing) that *mutually exchange energy*.

The two areas of the soundboard that vibrate in antiphase, respectively above and under the nodal line, produce a contrariwise displacement of the air in the surrounding environment and in the resonator body: while one part brings about a compression of the air, the other brings about a rarefaction of the air. The two vibrating surfaces constitute an acoustic dipole, analogous to the one we already met when dealing with the soundboard oscillation in mode ⟨1 0⟩ (Chap. 5).

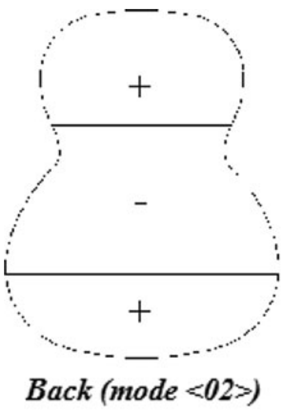
At a certain distance from the soundboard, compression and rarefaction of the air in the surrounding environment tend to mutually nullify, and so the sound pressure that reaches the hearer is quite scarce, *unless one of the two surfaces prevails on the*

other. If the two surfaces of the dipole have different dimension and vibrating mass, one of them (the one with the higher surface/mass ratio) is able to displace a greater air volume; therefore, the sound pressure that reaches the hearer is greater, net of the difference between the two volumes that vibrate in antiphase.

In practice, working on the bracing, the soundboard must be designed and made in such a way as to render the two vibrating areas asymmetric, in order to improve both the *radiation from the soundboard towards the environment* and the *coupling between soundboard and cavity*. This topic will be investigated extensively in the second part.

Mode ⟨0 1⟩ of the air also interacts with mode ⟨0 1⟩ of the back. By the normal 'three-bar' bracing, the nodal line of the back in mode ⟨0 1⟩ falls just under the waist (see Chap. 4), therefore very close to the nodal line of the air. The previous image shows a scheme of the nodal lines of soundboard, air, and back, and the related oscillation phases; we can see that the surfaces of the back, above and under the nodal line, vibrate in phase with the air.

In Sect. 5.7 we observed that, in this instrument, the frequency $\mathbf{F_{B1}}$ of the back attached to the resonator body occurs at 276 Hz. This is a quite too distant frequency from that of the air at 395 Hz, in order for good frequency concordance to take place. However, it is interesting to notice that the back can influence this guitar resonance in the vibration mode ⟨0 2⟩ as well (occurring in the reference guitar, as seen in the previous chapter, at 352 Hz). The following figure shows the real (Chladni pattern) and the schematic development of the nodal lines of the back in this vibration mode.



We can see that, approximately under the waist, the distribution of the oscillation phases of the back is very similar to the soundboard one. In other words the back—in its vibration mode ⟨0 2⟩—under the waist oscillates in synergy with the soundboard, and the modal frequency is close to the one of the air at 395 Hz. Therefore, conditions are favourable to a good interaction between vibration modes of back, air, and soundboard, determining the development of the guitar resonance in mode ⟨0 1⟩.

On the percussion response of the back (Sect. 5.7) we notice an outstanding resonance peak at about 412 Hz, which practically coincides with the one measured on the soundboard at 414 Hz. This confirms that, at this frequency, the coupling soundboard—air—back is conspicuous: back and soundboard vibrate at the same frequency, with great amplitude, and both contribute to sound radiation, while the soundhole plays a minor role.

We carefully investigated the case of the instrument resonance in mode $\langle 0\ 1 \rangle$, generally occurring just above 400 Hz. Many authors pointed out the importance of this resonance for the sound quality of the guitar in the upper-mid register; this is natural, if we consider that this resonance 'fills up' the instrument response for the tones played on the first string, at least from the open string E (at about 330 Hz) up to A on the fifth fret (at 440 Hz).

This resonance depends on the *longitudinal half-wave vibration mode* of the cavity, where the pressure wave develops a half sinusoid through the length **l** of the guitar body; but it also depends on the soundboard oscillation in mode $\langle 0\ 1 \rangle$ and, especially, on the position of the nodal line (typically running at the bridge). The formation of this resonance is enhanced if the areas that vibrate in antiphase, above and under the nodal line, are asymmetric. In this situation, both the *radiation from the soundboard towards the environment* and the *coupling between the soundboard and the air in the body* are improved.

We also observed that the back can favourably contribute to this resonance of the instrument in both mode $\langle 0\ 1 \rangle$ (where the nodal line runs at the waist and the back oscillates in phase with the air) and mode $\langle 0\ 2 \rangle$ (where, under the waist, a good synergy between back and soundboard occurs). This is confirmed by the presence, in the response of the back as well, of a remarkable resonance peak at 412 Hz, practically corresponding to the instrument resonance in mode $\langle 0\ 1 \rangle$ (at 414 Hz). This means that, *at resonant frequency in mode $\langle 0\ 1 \rangle$, a good sound radiation comes from both back and soundboard.*

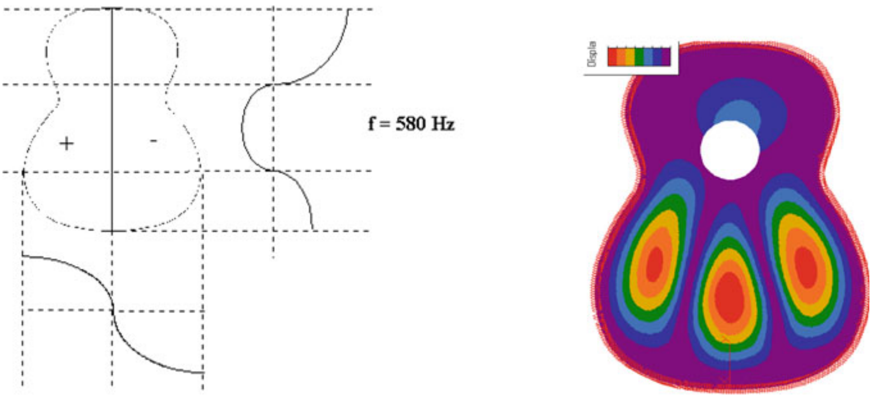
In concluding, the formation of the instrument resonance in mode $\langle 0\ 1 \rangle$ depends on the simultaneous excitation of many vibration modes that are able to positively interact with each other: these are the air mode ($\langle 0\ 1 \rangle$), the soundboard mode ($\langle 0\ 1 \rangle$), and modes $\langle 0\ 1 \rangle$ and $\langle 0\ 2 \rangle$ of the back.

6.1.2 The Guitar Resonance in Mode $\langle 2\ 0 \rangle$

In good quality instruments, the frequency response clearly shows a resonance peak between 550 and 600 Hz which, when present, supports the output of the tones played on the first string, at least between C (at 523 Hz) and D# (at 622 Hz). In our reference guitar this resonance appears at 581 Hz, as we can see on the response graph of the percussion on the bridge (Sect. 5.2).

This resonance peak is due to the coupling between the air resonance in mode ⟨1 0⟩ and the soundboard resonance in mode ⟨2 0⟩.

Let us consider again the distribution of air pressure waves inside the body in mode ⟨1 0⟩. Along the transverse axis of the body a half-wave pressure is present, determining a *transverse half-wave resonance*, and maximum values (positive and negative) are positioned on the sides, close to the lower bout of the body. In the longitudinal sense, the pressure distribution presents two nodes, respectively at the level of soundhole and the lower bout, while the maximum pressure value occurs at the extreme upper and lower limits of the cavity. We have seen that the resonant frequency of the air (580 Hz) can be calculated on the base of the dimensions of the instrument plane, and it coincides with the value measured experimentally.



The previous left image represents the pressure waves in the body at this frequency, while the right image reports the outcome by the FEM model of a soundboard that vibrates in mode ⟨2 0⟩. Three vibrating areas are visible, separate by two sectors where the vibration amplitude is practically nil. These sectors identify the two typical longitudinal nodal lines of the soundboard in mode ⟨2 0⟩.

In this simulation the vibrating areas are located under the soundhole, and so the soundboard only operates in the area under the waist, while above the waist it is practically idle. In many traditional bracing styles, the brace under the soundhole restricts the soundboard vibrating surface, and confines it to the area from the waist down. In other bracing styles (like in Garrone guitars) some braces extend beyond the brace under the soundhole, and also vibrate parts of the soundboard above the soundhole. In other words, this kind of geometry allows the two lateral antinodal areas of the soundboard to extend beyond the brace under the soundhole, which is a beneficial situation for the coupling with the air in the body. We will provide examples of this in the second part.

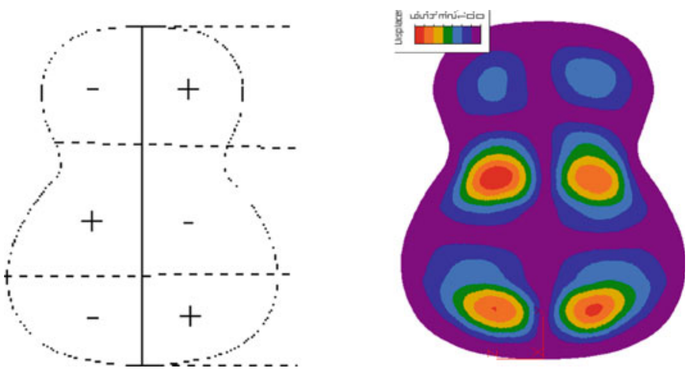
But, in order for the resonance peak between 550 and 600 Hz to be present in the instrument response (i.e. to have a good coupling between soundboard and air in the cavity), the two formerly mentioned conditions are required:

- The resonant frequency of the soundboard in mode $\langle 2\ 0\rangle$ must be close to that of the air (*frequency concordance*). The soundboard of the reference guitar manifests this resonance at 568 Hz, therefore very close to the air resonance at 580 Hz.
- The peripheral antinodal areas of the soundboard vibrate *in phase*, so they tend to simultaneously compress (or decompress) the air volumes right and left of the longitudinal axis, which in turn vibrate in antiphase. In order to have a non-zero effective *net* influence of the soundboard on the air, the two *peripheral* antinodal surfaces (included their possible extension over the soundhole) must have different areas, which means that *the bracing design must render them asymmetric*. If the two peripheral areas were identical it would be highly improbable, for the soundboard oscillation in this vibration mode, to succeed in driving the air into a specific direction; there would be no interaction between air and soundboard, but only a feeble radiation from the soundboard. Hereinafter we will provide examples of asymmetric bracing with respect to the instrument longitudinal axis.

The response of the reference guitar back shows a resonance with notable amplitude at 570 Hz.

We must keep in mind that we have so far considered the back like a structure featuring only transverse nodal lines (like $\langle 0\ 1\rangle$ and $\langle 0\ 2\rangle$), due to the transverse nodal lines that hinder its bending along the longitudinal axis. This model can properly explain the coupling between back, soundboard and air in the low-mid register, as well as the instrument resonance in mode $\langle 0\ 1\rangle$, which we have considered above. But, as the frequency grows, the braces ‘consent’ to be bent in the transverse sense, too (since their *impedance in the length direction* diminishes), allowing the formation of vibration modes that also feature longitudinal nodal lines. This explains the presence of a significant resonance at 570 Hz, and others at higher frequency.

The left side of the next illustration represents the FEM simulation of the back.



If we do not consider the upper area (where the vibration amplitude is scarce), the oscillation of the back in the lower part displays two nodal lines, one transverse and one longitudinal, that define a vibration of the back in mode $\langle 1\ 1\rangle$. A longitudinal nodal line is visible, meaning that the braces can bend along their length, following the natural vibration mode of the back.

The left image shows the typical chequered pattern of the phase distribution that we always find, due to the intersection of the horizontal and longitudinal nodal lines.

In the sectors that separate the back under the waist there is a remarkable phase concordance in the oscillation of back (in mode $\langle 1\ 1 \rangle$) and air (in mode $\langle 2\ 0 \rangle$). Therefore, we have the prerequisites for a good coupling between soundboard—air—back.

In conclusion, various vibration modes that positively interact with each other contribute to the formation of the resonance in mode $\langle 2\ 0 \rangle$: mode $\langle 1\ 0 \rangle$ of the air at 580 Hz, mode $\langle 2\ 0 \rangle$ of the soundboard at 568 Hz, and mode $\langle 1\ 1 \rangle$ of the back at 570 Hz. As an overall result, a significant resonance of the instrument appears at 581 Hz, leading to a good sound radiation from both back and soundboard.

6.1.3 Resonance of the Back at 610 Hz

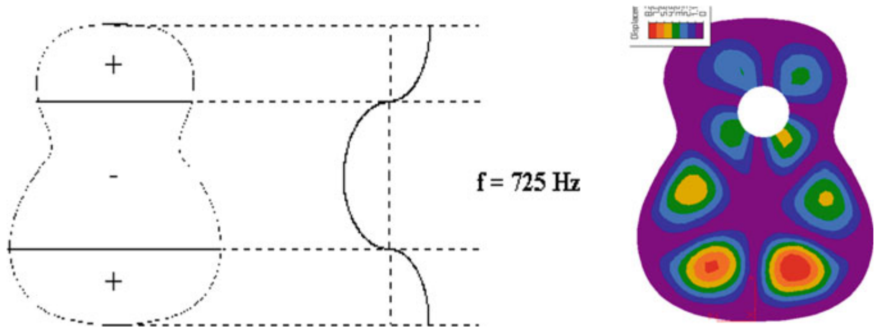
Looking at the response of the back reported in Chap. 5, we notice a resonance at 610 Hz, where the value of the sound radiation from the soundboard is very small. Quite obviously, this resonance is due to a coupling between back and air that—in this case—does not involve the soundboard. As a confirmation, we notice from the 'open back' measurement that, at this frequency, the soundboard manifests no resonance (Sect. 5.6).

Therefore, we could neglect this resonance of the back—which does not affect the instrument response—except as an additional instance of the complexity involved in the guitar functioning within the upper-mid register. We have seen in the previous paragraph that a significant resonance of the instrument is present at 581 Hz, owing to the coupling between soundboard—air—back. But, if we slightly increase the frequency, the soundboard does not partake in the phenomenon any more: the synergy between soundboard and back fails, and a resonance takes place at 610 Hz, only due to the (feeble) interaction of the back with the air in the body. This resonance is evident only by exciting the back and—at least in this instrument—it does not effectively contribute to the instrument sound radiation at this frequency.

6.1.4 Resonance of the Guitar at 680 Hz

Many instruments feature a remarkable resonance between 670 and 690 Hz. In our reference guitar, we find it at about 680 Hz. This resonance can be associated to the soundboard vibration mode which, under the soundhole, presents a longitudinal nodal line and a quasi transverse nodal line. This is a $\langle 1\ 1 \rangle$ mode restricted to the area under the waist. Visible on the FEM simulation of a guitar soundboard in this vibration mode, the antinodal areas gather in the lower part of the soundboard (under the soundhole) in a 'matrix-like' pattern we are now familiar with, but also involve the area around the soundhole. Please notice the characteristic 'daisy' shape of nodal

lines and vibrating surfaces around the soundhole. In the soundboard *open back* response this resonance takes place at about 670 Hz, very close to the instrument resonance at 680 Hz.



Readers, who have patiently followed our reasoning so far, will easily understand that the soundboard frequency in the mode ⟨1 1⟩ under the waist (at 670 Hz) is too distant from that of the air in mode ⟨1 0⟩ (at 580 Hz, as seen above) in order to have a frequency concordance. On the other hand, an interaction between the soundboard and mode ⟨0 2⟩ of the air at 725 Hz (whose characteristics are illustrated in the previous left image), is possible.

Mode ⟨0 2⟩ of the air features two transverse nodal lines that define three air volumes with alternate vibration phases. In the longitudinal sense, the air pressure inside the cavity is comparable to a *full sinusoid* that, while it takes maximum values at the cavity extremes along the longer axis, it is nil at about the level of the soundhole and of the lower bout. As we have seen (Sect. 4.1) there is an optimal congruence between the resonance calculated from the geometrical parameters of the body and the directly measured one.

The same arguments we have previously expounded lead to the conclusion that, in this case as well, phase concordance between the mode of the soundboard and the mode of the air is present, provided that the soundboard vibrating areas are made asymmetric with respect to the surface/mass ratio of the four vibrating areas.

Nearby this frequency, no resonance takes place in the response of the back, and so no synergy between back and soundboard. Only the soundboard and the air in the body partake in the formation of the instrument resonance at 680 Hz. In this frequency range, involving the tones between E and F# beyond the 12th fret, the sound radiation in this instrument is only due to the soundboard.

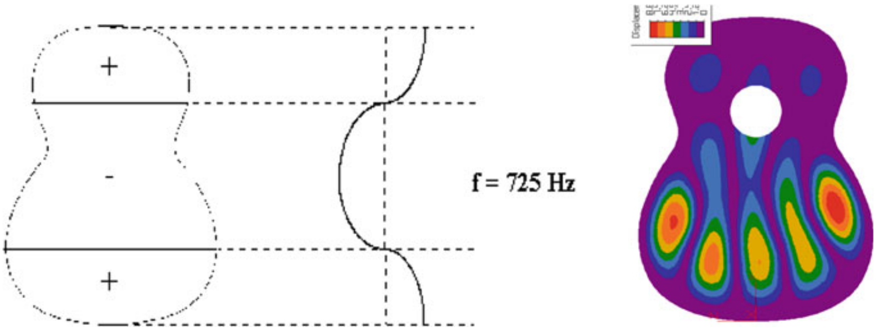
6.1.5 Resonance of the Guitar at 723 Hz

In the reference guitar, and in other high quality instruments, a considerable resonance takes place between 700 and 750 Hz. This is clearly an important resonance, since it contributes to a brilliant timber of the instrument in the upper-mid register.

In our guitar, this resonance appears at about 723 Hz, as we can see on the response graph (Sect. 5.2) obtained by exciting the instrument on the bridge.

The formation of this resonance is due to the interaction between the soundboard resonance in mode ⟨4 0⟩ and the air in the body in mode ⟨0 2⟩, the same mode we have already encountered in the foregoing chapter.

For the reader's convenience, we report in the next figure (left image) the distribution of the pressure waves inside the body in mode ⟨0 2⟩ of the air.



On the left image we see the FEM simulation of the guitar soundboard in mode ⟨4 0⟩. We notice five vibrating areas extending up to the soundhole, separate by four quasi longitudinal non-vibration areas. The oscillation phases of the vibrating areas alternate according to the matrix pattern we have already observed in other cases.

The concordance between the frequency of the air (725 Hz) and that of the soundboard (738 Hz measured on the frame without the back) is very good. Furthermore, in this vibration mode of the soundboard, the two peripheral areas and the central one oscillate in phase, while the other two vibrate in antiphase; this is advantageous to a good concordance between the phases of air and soundboard.

The response of the back shows a resonance at 724 Hz, yet with very limited amplitude. Clearly, there are no favourable conditions for a good coupling between back and air.

Summing up, the interaction between the soundboard frequency at 738 Hz and the air frequency at 725 Hz is favourable to the formation of a resonance of the instrument at 723 Hz. In the development of this resonance, only air and soundboard are involved, and sound radiation is almost exclusively due to the soundboard.

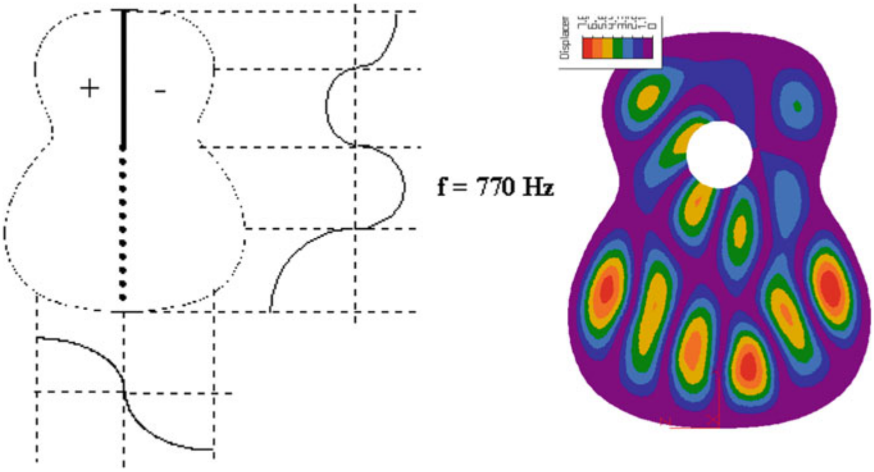
6.1.6 Resonance of the Guitar at 780 Hz

The last resonance of the guitar, in the upper-mid register we are dealing with, appears at about 780 Hz, involving the fundamental of G on the first string beyond the 12th fret (see the response diagram in Sect. 5.2).

Although—at least in the reference guitar—the amplitude is not very high, this resonance is relevant to the quality of sound.

In modern guitars, at about 770 Hz, we find a considerable oscillation in the air volume above the waist, where the oscillation energy concentrates. In the volume below the waist, the air is *forced* to vibrate at the same frequency and in the same mode.

This is *mode* ⟨*1 0*⟩ in the volume above the waist, already described in Sect. 4.1. The following figure (left image) sums up its characteristics, and shows the progress of the standing waves in the longitudinal and transverse direction.



The soundboard response shows a resonance at 776 Hz. The interaction between the resonance of the soundboard (at 776 Hz) and that of the air (at 770 Hz) produces a resonance of the instrument at 780 Hz. We are not able to exactly define the characteristics of the soundboard vibration mode at 776 Hz, but we suppose that the soundboard oscillates at this frequency in mode ⟨5 0⟩, if for no other reason, because this mode follows the previous ⟨4 0⟩ in a natural sequence of the guitar soundboard modes of vibration. This mode mainly develops in the lower part of the soundboard, though involving the region around the soundhole as well, and it features the already observed, typical alternate phases of the vibrating areas.

The image on the right represents the simulation of a soundboard in this vibration mode. Once again, the interaction between air and soundboard at this frequency is supported by an asymmetry of the vibrating areas right and left of the soundboard longitudinal axis, if present.

In the reference guitar, the response of the back as well shows an important resonance peak at about 780 Hz (see Sect. 5.7).

In summary, the interaction between the frequency of the soundboard (776 Hz) and the frequencies of air (770 Hz) and back (780 Hz) is advantageous to the formation of an instrument resonance at 780 Hz. At this frequency, both back and soundboard contribute to sound radiation.

6.2 Conclusions

In this chapter we have examined the formation of the guitar resonances in the upper-mid register which, in our classification, spans the frequencies from 400 to 800 Hz, therefore the fundamental of G# on the fourth fret of the first string up to G# beyond the 12th fret.

To interpret these resonances, we must cross-check the outcomes resulting from different evaluation means. These are not specific measurements; they are just the same we used for studying the guitar resonances in the low-mid register:

- *Resonator response obtained through excitation of the soundboard at the centre of the bridge saddle.*
- *Resonator response obtained through excitation of the back at the level of the bridge.*
- *Response obtained through soundboard excitation at the level of the bridge, with the open back.*

As for the evaluation of the resonances of the air in the body, in the low-mid register we just need to know the value of the Helmholtz resonance (129 Hz in Garrone guitars), while in the upper-mid register we also need to know the vibration modes of the air at higher frequencies. We previously provided indications on how to ascertain these modes: basic measurements, which can be executed once and for all on a finished instrument or on a mould reproducing its geometry, remaining valid as long as the luthier keeps on using that model. Anyway, we have seen again in this chapter the structure of the vibration modes of the air in the Garrone model, while in Sect. 4.1 we demonstrated that the associated resonances can be accurately calculated from the geometric dimensions of the body.

As for vibration modes of back and soundboard, we largely employed *FEM simulations* to show what they look like at different modal frequencies. Though FEM modelling software is available on the internet (possibly limited versions) it is probably easier for the luthier to use the Chladni method, which provides basically equivalent images. We will talk about that in detail in the second part.

The results obtained by means of these evaluations must be observed in the light of the 'rules' that ensure a good interaction between air and soundboard (*frequency and phase concordance*) without neglecting the one that ensures a good interaction between soundboard—air—back (*synergy between back and soundboard*). If the conditions implied by these rules are present, we will get a particular resonance and a good sound radiation from back and soundboard or—at least—from the soundboard alone.

Understanding the mechanism for the formation of a particular resonance is a first important step to optimizing it. We have mentioned some construction expedients, but this topic will be fully developed in the second part.

Through this chapter we have analyzed the resonances in the upper-mid register of the same reference guitar considered for the low-mid register. Obviously, the purpose was not to show the response of a particular instrument, but rather to work out an investigation method that luthiers can apply to their own production instruments. Carrying out this investigation, luthiers will probably find some differences (and many analogies, too) with respect to the data we have provided: this is natural, since there are not two instruments with the same characteristics, no matter how similar they can be.

We feel confident that the method we developed and, above all, the 'guidelines' expounded in the second part, will allow luthiers to correctly *interpret* and *optimize* their instrument response.

Both the low-mid register (between 80 and 400 Hz) and the upper-mid register (between 400 and 800 Hz) are definitely strategic for the quality of a guitar. Nonetheless, quality also depends to a large extent on the response in the *highest register*, covering tones beyond G# on the first string.

Responses from high quality instruments feature great amplitude resonances up to 1800 Hz and beyond. Such high frequencies do not correspond to the fundamental of tones that can be played on the guitar, but enhance the upper harmonics of the tones produced in the lower registers as well as in the highest register, assuming an important role for the brightness and balance of the sound.

Unfortunately, at the moment, we are unable to establish the mechanisms that determine the formation of the instrument resonances in the highest register. We are working upon it, so, as regards this issue, to be continued!

6.3 Overview of the Resonator Functioning

We are now at the end of a long journey through the operating mechanisms of the guitar resonator.

We have examined the elementary oscillators, the simplest structures into which the three fundamental components of the resonator (soundboard, air in the body, back) can be split up. We have studied the acoustic properties of these components—considered as separate objects—and the global characteristics of that structure—the resonator—brought about by their interaction.

Now, the patient reader who has followed us through this path, will probably appreciate a map displaying all the visited places, and providing an overview of the land we have explored.

It all begins from the string. When the player plucks the string, after blocking it on the fret at a distance **l** from the bridge, the string goes into vibration in its fundamental mode at the frequency

$$F = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$$

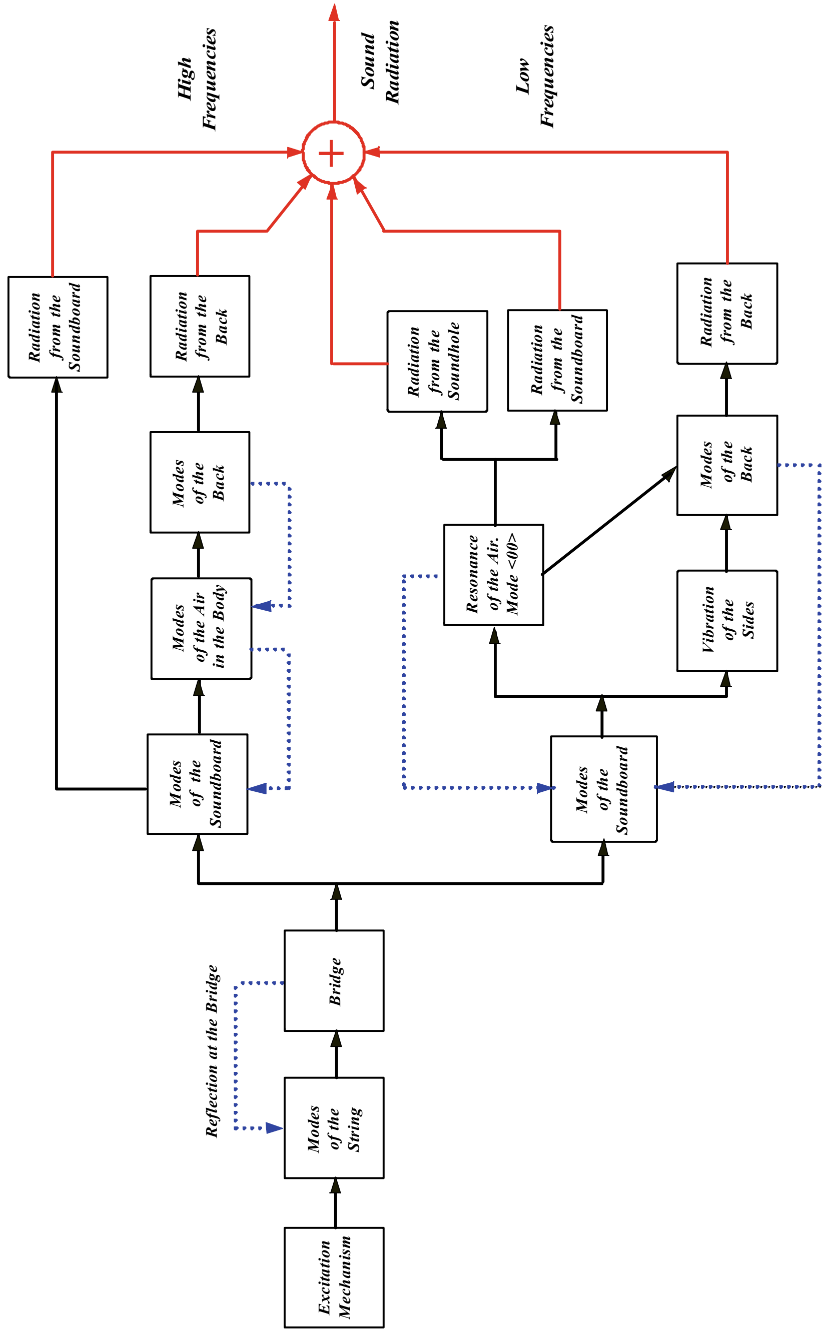
which corresponds to the pitch of the played tone ($\boldsymbol{T}$ is the tension of the string and $\boldsymbol{\mu}$ is its linear mass, as seen in Chap. 2). The string oscillation is the result of the sinusoidal vibration modes, whose frequency is an integer multiple of the mode associated to the fundamental frequency. Yet not all of the possible modes are present in the string motion 'recipe', depending on the excitation mechanism, i.e. how the string is plucked by the performer. We will not cover this topic, since it pertains to the playing technique rather than to acoustics or design and construction of the guitar. We just remind, as an example, what we know from ordinary experience: the sound changes depending on the distance from the plucking point and the bridge. We explained that in Chap. 2.

The action of the string on the bridge turns into a series of *force impulses*, replicating at the frequency of the fundamental (that of the played tone). The shape of the impulses sums up the overall harmonic composition of the string oscillating motion (namely vibration modes and their amplitude).

But not all of the force available at the bridge is exploited by the resonator: part of it is given back (or reflected) to the string, entering a mechanism of reflections (at the nut and at the bridge) and absorptions (at the bridge). This process only ends when all of the string oscillating energy has been used up by the resonator, or dissipated in string or nut losses. In the next diagram, this reflection is sketched by a blue arrow from the bridge to the string (all *indirect* actions between the components are represented in the diagram by a blue arrow, while *direct* actions are represented by black arrows).

The parameter that describes this phenomenon is the *coupling coefficient*, which represents the reflected part of the force with respect to the incoming force on the resonator:

$$\rho = \frac{R_c - Z_r}{R_c + Z_r}$$


Excitation Mechanism
Modes of the String
Bridge
Reflection at the Bridge
Modes of the Soundboard
Modes of the Air in the Body
Modes of the Back
Radiation from the Back
Radiation from the Soundboard
High Frequencies
Sound Radiation
Low Frequencies
Modes of the Soundboard
Resonance of the Air. Mode <00>
Vibration of the Sides
Modes of the Back
Radiation from the Soundhole
Radiation from the Soundboard
Radiation from the Back

The coupling coefficient involves the characteristic resistance of the string R_c that, as seen in Chap. 2, is a constant ($R_c = \sqrt{\mu T}$) only dependent on tension and linear mass of the string.

But now, in the expression of the coupling coefficient, the resonator impedance Z_r is also present, taking us into the real functioning of the guitar resonator.

The resonator impedance is defined as the relation between velocity and force applied on the bridge:

$$Z_r = \frac{Force\ applied\ on\ the\ bridge}{Velocity\ at\ the\ bridge}$$

This is not a constant factor: it is very complexly connected to the frequency of the signal provided by the string. At certain frequencies, the resonator impedance is very low, the velocity at the bridge is very high, and the force that the string applies on the bridge is almost completely exploited by the resonator. The coupling coefficient is equal to 1. These are the *resonant frequencies* of the resonator. At certain frequencies, impedance is very high, the velocity at the bridge is almost nil, and the force that the string applies on the bridge reflects almost completely without absorption by the resonator. These are the *antiresonances*, where the coupling coefficient is equal to -1.

So the impedance present at the bridge fully defines the resonator behaviour. *Mobility* is normally used in preference to impedance, which is the reciprocal parameter:

$$Mobility = \frac{Velocity\ at\ the\ bridge}{Force\ applied\ on\ the\ bridge}$$

This parameter allows an easier reading of the outcomes: in mobility diagrams, the resonances look like pronounced peaks, while the antiresonances look like deep valleys. We, too, referred to mobility rather than impedance.

At this point of our journey, we have seen that the harmonic components of the string motion (the *motion recipe*) depend on how the string is plucked (the *excitation mechanism*). These components are somehow influenced by the resonator, according to the impedance present at the bridge. Some components correspond to antiresonances of the resonator and are hindered, because just a small part of the incoming wave is absorbed and exploited by the resonator, while the residual fraction reflects into the string. Other components are enhanced, being close to some resonance of the instrument. At these frequencies, the incoming wave is almost completely absorbed by the resonator, and just a little part of it goes back to the nut.

The previous diagram is divided up into two parts, which describe the resonator functioning respectively at low and high frequencies—therefore, according to the classification we have suggested, in the low-mid register (between 80 and 400 Hz) and in the upper-mid register (between 400 and 800 Hz).

In Chap. 5 we have thoroughly described the resonator functioning in the low-mid register. We have seen that the available force at the bridge (net of reflection) is partly used for the excitation of the soundboard vibration modes, while the remaining part excites the air in the body in its natural mode ⟨0 0⟩, whose resonant frequency depends both on the volume of the air in the body and on the geometry of the soundhole.

The interaction (or coupling) between the soundboard natural frequency in mode ⟨0 0⟩ and the natural frequency of the air in the body (blue arrow in the diagram) determines the formation of the two basic resonances of the instrument, which typically appear between 90 and 110 Hz (*air resonance*) and between 180 and 220 Hz (*soundboard resonance*). These two basic resonances *always* appear in all guitars, independent on the quality and characteristics of the instrument, since their presence is connected to the resonator structure, which is basically composed of two flexible tables (back and soundboard) enclosing an air volume that communicates with the exterior through an opening—the soundhole—in one of them.

As for sound quality, clearly not all guitars are equally valuable just because they manifest two basic resonances in the low-mid register. After studying in detail the mechanism of formation of the basic resonances—by also using a mathematical model of the resonator (Chap. 5)—we identified a number of *objective* parameters to define the quality of an instrument:

– Vibrating mass
– Vibrating surface
– Soundboard mean stiffness
– Coupling coefficient
– Vibrating surface/vibrating mass ratio.

To evaluate these parameters we suggested three measurements on the instrument:

- *Resonator response obtained through excitation of the soundboard at the centre of the bridge saddle.*
- *Closed back resonator response obtained through soundboard excitation.*
- *Closed back response, as the previous, but with additional weight on the soundboard.*

We provided some simple formulae, whereby these parameters can be calculated from measured data, and established the limits wherein each of them should fall in a good instrument. We also evaluated these parameters in a number of high, average, and poor quality instruments. This is therefore an a posteriori evaluation, to be executed on a finished instrument or one with a provisionally closed back (which still allows optimization actions during construction). But we also suggested an a priori method, allowing to evaluate the basic resonances from the known *Helmholtz resonance* (that can be measured once a mould has been established, and will not change as long as that mould is used), the *soundboard natural frequency* (measurable on the mould) and the *coupling coefficient*.

As for the back, some players and guitar makers consider its contribution to the quality of sound as totally negligible, preferring a completely rigid back or—at most—a standard one. We recognize the pros and cons in this approach, yet we

believe that the back can effectively contribute to the instrument quality, provided that it is properly calibrated, in order to suitably match the soundboard characteristics.

By the previous diagram we understand that the modes of the back can be excited by mode $\langle 0\ 0\rangle$ of the soundboard, whether directly through the sides (which are elastic components, too) or indirectly through the air in the body in its mode $\langle 0\ 0\rangle$. *At low frequency*, only the first natural modes of the back are involved (those measured when the back is fastened to the mould) featuring transverse nodal lines (modes $\langle 0\ 1\rangle$ and $\langle 0\ 2\rangle$) in addition to the ring mode—where the nodal line runs along the edge (mode $\langle 0\ 0\rangle$). Once the back is glued to the sides, so enclosing the resonator body, important interactions come into play between back and soundboard (blue arrow in the diagram):

- First of all, the fastened back influences the basic resonances (of *air* and *soundboard*): with a flexible back, the basic resonances are lower by some Hz with respect to a completely rigid back setting, since the air compliance in the body grows because of the elasticity of the back.
- Secondly, when we glue the back onto the frame and close the resonator body, the vibration modes measured on the mould (see Chap. 5) bring about new and different 'closed back' vibration modes; the discrepancy between the modal frequencies of the back on the mould and those of the 'closed back' depends on the *coupling coefficient* between back and soundboard, which we have numerically estimated on the base of the coupled oscillators model.
- Finally, we have seen that each of the vibration modes of the closed back induces into the soundboard a resonance at the same frequency (or at a very close frequency). The discrepancy is related to the coupling coefficient between back and soundboard. These are the resonances due to the back, covering the frequency band between the basic air resonance and the boundary of the low-mid register.

To highlight the resonances of the back when fastened to the resonator body, and see how they assume the role of *instrument global resonances*, we suggested an additional fourth measurement:

- *Resonator response obtained by exciting the back at the level of the bridge*

In high quality instruments, within the low-mid register the resonator response also features a significant resonance at about 300 Hz, which is not due to the back but, basically, to the soundboard in its mode $\langle 1\ 0\rangle$. This resonance is particularly evident if the two parts of the acoustic dipole, which determine the mode $\langle 1\ 0\rangle$ of the soundboard, are rendered asymmetric by an appropriate design of the soundboard bracing.

The previous diagram shows that, in this register, the *sound radiation* is due to the motion of the three vibrating surfaces of the guitar resonator (soundboard, back, and soundhole) facing the surrounding environment. The individual contributions of these components in the diagram are represented by red arrows. However, we wish to remind that, as the frequency grows, the impedance of the soundhole also grows and, at the same time, the velocity of the airflow through it diminishes. As a consequence,

the sound radiation from the soundhole tends do decrease as the frequency increases, until it becomes very low at the upper end of the register.

The upper part of the diagram describes the resonator operation at high frequencies, i.e. in what we have defined as the upper-mid register spanning, in our classification, between 400 and 800 Hz. In this frequency range, the operation mechanisms of the resonator are rather different from those that rule its workings at low frequencies:

– While, at low frequency, only the air resonance in mode $\langle 0\,0 \rangle$ is involved in the resonator operation, at high frequencies even the upper modes of the air intervene. In Chap. 4 we have examined the formation of these modes inside the resonator body, both by means of direct measurements and by estimate from the real dimensions of the body.
– At low frequency, the soundboard only operates in the ring mode ($\langle 0\ 0 \rangle$) and in mode $\langle 1\ 0 \rangle$ (but only under 300 Hz). At high frequencies the upper modes come into play, from mode $\langle 0\ 1 \rangle$ up. As the frequency grows, the soundboard oscillation develops into a growing number of antinodal vibrating areas, which mainly concentrate in the lower part of the soundboard. The natural frequencies can be evaluated by measuring the response of the soundboard fixed on the frame with the open back.
– At low frequency, each global resonance of the instrument is due to the interaction of three specific modes: one of the air, one of the soundboard, and one of the back. At high frequencies, the natural resonances of air, back and soundboard tend to gather. Therefore, some resonances of the instrument may depend on the simultaneous excitation of more than one mode.
– At low frequency, the back operates in the transverse modes $\langle 0\ 1 \rangle$ and $\langle 0\ 2 \rangle$, in addition to mode $\langle 0\ 0 \rangle$. At high frequencies, composite modes come into being, presenting nodal lines 'across' the braces as well. Besides, in the upper-mid register the back is excited by the soundboard only through the air in the body, while the sides become too rigid to partake in this action.
– Definitely, at high frequencies, no sound radiation comes from the soundhole (see the previous diagram). The sound is conveyed by the soundboard (whether directly or coupled with the air) and by the back, if a favourable interaction subsists between soundboard, air, and back itself.
– Finally, at high frequencies, some resonances have an especially effective development, if conditions are present for a good interaction between soundboard and air (*phase* and *frequency concordance*) while a third condition (*synergy between back and soundboard*) ensures a good interaction of the back with the air and the soundboard, and a good sound radiation.

In concluding, what concerns the player and the luthier is how the instruments sounds.

In Chap. 1 we examined the characteristics of the guitar sound, both in the time domain (the waveforms) and in the frequency domain (spectra and spectrograms) and we identified the main properties of high and top quality instruments, as defined by people, namely players and listeners, who experienced the sensation of the guitar sound.

The next step was to relate subjective quality criteria to objective sound parameters and, consequently, to physical and acoustic attributes of the instrument.

Even though, probably, a universal best instrument for every player, every listener, every piece of music, and every location does not exist, we tried to expound in detail an investigation method: luthiers can apply it to their own instruments, evaluate results, and interpret it according to their own ‘concept of sound’.

Chapter 7
Analysis Systems

Abstract This chapter introduces the reader to the tools which are necessary for the implementation of the instrument. In detail, it describes an hardware measurement system including the acoustic hammer and other devices used to excite the finished instrument or part of it. The Chladni method is also outlined as complementary to the acoustic hammer method. Photographs and dimensions of the tools are given in order to allow duplicating them. A software system—custom developed—allows gathering a number of data from the measurements, like spectrograms, third of octave graphs, waterfall, decay plots. Resonances and antiresonances are displayed in two ranges (respectively up to 1000 Hz and 10,000 Hz) with a resolution of about 0.6 Hz.

When we beat the soundboard, or the back of a finished instrument with the hand, we set the soundboard and the inner air into vibration.

The sound we hear enfolds the whole configuration of the instrument resonant frequencies, although we can hear just a scanty part of it.

Both the soundboard and the back, in the classical guitar, are fastened to the sides by the gluing on the linings: these are the conditions we need to replicate in order to execute all of the desired quality controls.

In order to verify the various steps of construction, before the final assembly, we need to build a suitable testing structure that simulates the final bond to the sides. We will call this structure a “mould”, having the dimensions of the guitar frame and behaving as such, binding the soundboard or the back, or both, during check-up analyses.

The construction of the mould, as well as other check-up tools, will be described in detail in Chap. 10, to which the reader may refer all the time.

The system works perfectly, yielding results that do not diverge very much from those obtained from back and soundboard after the final fastening to the sides.

The mould can be provided with a fretted neck and machine heads, in order to test the back and the soundboard as if they were already glued to the sides, and to taste some of the sound characteristics that the instrument will feature.

This method implies a bridge already glued to the soundboard. In fact, since this element is definitely necessary and its exact position can be easily guessed

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beforehand, its gluing is recommended prior to any other procedure. After that, the main braces and the fan braces will be glued.

The Acoustic Hammer Now we can describe the method used for quality control during the construction of these important components.

The method is based on some considerations.

If we excite a soundboard with a signal at a certain frequency, we obtain a response at *that frequency*; if the excitation signal contains two frequencies, we obtain a response at *those two frequencies*. If we use instead an excitation signal containing all the frequencies comprised in a large band, we get the global response of the examined back, or soundboard, with respect to all the frequencies comprised in that band. This signal is called *impulse*: a very short signal, whose amplitude can set the soundboard into motion. Therefore, a device must be conceived to drive the soundboard with a proper *impulsive* excitation.

This impulsive excitation (or *acoustic hammer*) method is employed in many measurement domains; the signal issued from the hammered object is acquired by common PC software. After that, a special computer program calculates the Fast Fourier Transform (FFT), and marks all the resonance peaks of the object hit by the acoustic hammer.

This method recalls the "Tap Tones", traditionally used in lutherie, where the experimenter holds in a vibration node (between two fingers) a bond-free table, while lightly tapping it in order to check the sound.

Yet, the "Tap Tone" method presents various drawbacks:

- The soundboard is analyzed without perimeter bonds, therefore in a situation very different from the real one.
- Results cannot be shared with other people.
- The replication of the measurements is questionable.

Therefore, we preferred the acoustic hammer for our tests, averting all the disadvantages of the tap tones which, moreover, offer no practical indication.

In order to perform the analysis we need

- A mould
- An elastic support to hold the guitar
- A pendulum
- A microphone
- A microphone preamp
- A computer program for the acquisition of the signal
- FFT processing software.

This method allows to verify and modify at any time the physical conditions of back or soundboard, until the desired outcome is achieved.

The Chladni Method

Using the same mould, which fastens the soundboard along its perimeter, we will

also obtain very helpful indications by means of the method, devised by the German physicist Ernst Chladni, consisting in exciting the soundboard with a loudspeaker or an electromagnetic exciter. Chladni, who lived in the second part of the 18th century, had obviously no electronic tools available, so he used to rub a violin bow against the soundboard edge in order to set it into vibration.

This method highlights the fundamental vibration modes, and also allows inspection of their geometry and their resonant frequencies, in order to set them at the correct frequency and with the correct amplitude values.

This does not substitute the formerly described method, but can support it, giving some additional, useful, information.

7.1 Elements of the Measurement System

The above-mentioned inspections require the following components:

7.1.1 A Mould

Please refer to Chap. 10 for full information about building the mould.

7.1.2 The Elastic Holder

While the back or the soundboard of the guitar can be analyzed by means of the mould—which binds them along the perimeter—the finished guitar, or even the frame alone, with just the neck and the soundboard joined to it, can be inspected by hanging the whole on a stand with an elastic means, as shown in the next picture. The guitar is suspended from two properly sized elastic bands, this way being able to vibrate free of any damping due to contact points.

The stand was made from aluminium profiles purchased in a DIY (Do-It-Yourself) store. The vertical strut and the stabilizing cross bar at the level of the sides are 20 mm $\times$ 20 mm. square tubes. The base is composed of a 60 mm $\times$ 30 mm tube resting on two 20 mm$\times$ 20 mm tubes for the feet. At the top of the vertical strut is applied the suspension fork, composed of two M6 Thread bars screwed into a vertically sliding 30 $\times$ 15 cross strut. At the ends of the stabilizing cross bar and at the bottom end of the vertical strut are glued three foam rubber pads, in order to help stabilizing the instrument, once it has been placed on the stand. The pads must only flick the instrument, without pressing it.

7.1.3 The Pendulum or Acoustic Hammer

The pendulum, as made by the authors, is composed of:

– a 16 cm square base, weighing at least 5 kg.
– a 62 cm long vertical strut, made from a 20 mm× 20 mm anodized aluminium profile, and fixed to the base by an inner M8 Thread tie-beam.
– a 30 × 15 mm transverse bar, measuring 24 cm in length, which can slide along the vertical strut.
– a boxwood head, holding the pendulum, and fixed on the free end of the transverse bar.
– a duraluminium stem, measuring 5 mm across and 75 mm in length, with an M5 Thread end.
– a boxwood pendulum weighing 17 g in total. At the end of the pendulum, a hard plastic striker.

The pendulum oscillates on two ball bearings 5 mm Ø.

On the head that holds the pendulum a little arm is also attached, bearing a sexagesimal Vernier scale that establishes the starting point of the pendulum before percussion. This way the angle is constant, and the percussion on the soundboard yields replicated results. Using this percussion pendulum it is possible to set into vibration either the back and the soundboard of the guitar, as well as any other object whose FFT one wants to determine. As for the guitar back and soundboard, they must be fastened along their edge—just like when glued to the sides—by means of the mould (as described in Chap. 10). For the partially finished guitar—when the soundboard or, possibly, both soundboard and back are glued to the frame—it is instead convenient to use the elastic support, in order to correctly perform the analysis. The pendulum is set on the lock and then released, whether manually or, preferably, by an electromagnetic releasing device. The height of the arm that sustains the pendulum must be settled so as to get the percussion in a proper point: for example the bridge saddle, when dealing with the soundboard, where the string conveys vibrations onto the resonator; one or more points, as required for the modal analysis of the back.

7.1.4 The Microphone

A quality microphone should be used. The harmonics of the guitar sound span from less than 80 Hz (air resonance) up to over 10 kHz. So the microphone response must be flat up to at least 50 Hz, in order not to get erroneous measurements right from the beginning of the test.

The microphone must be placed at the level of the soundhole, 80 cm from the soundboard surface, in order to capture the sound emission from both soundboard and soundhole. These are not mandatory positions, but the experimenter must fix the relative positions of microphone and pendulum with respect to the guitar soundboard

once and for all, so that the measurements performed on the same instrument under different conditions, as well as the measurements performed on different instruments in different moments, can be replicated and compared with each other.

For the same reason, it is recommended to fix once and for all the overall gain, keeping it constant. The overall gain is achieved through various hardware and software amplification stages between the microphone signal and the computer file recording.

This also includes the gain of the possible microphone preamplifier, and other amplification factors that can be regulated via software.

Overall view

Guitar holder

Pendulum

Pendulum support

7.2 The Software

The microphone is connected to the computer sound card; any sound card in modern computers has suitable characteristics, and so the signal acquired from the microphone can be sampled at 44,100 Hz (CD quality) and recorded as an audio file. This is the file processed by the analysis software.

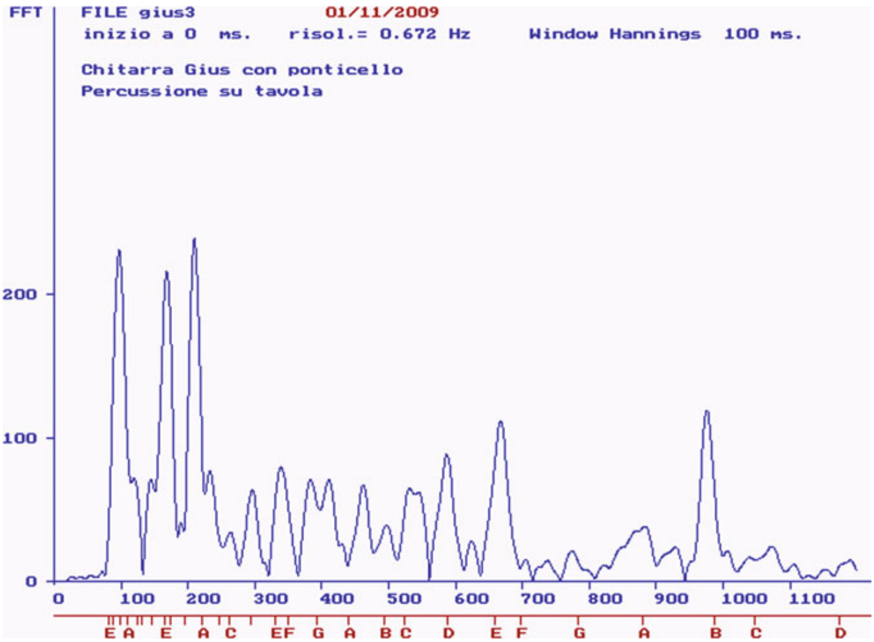
We use a computer program that we have specifically conceived and developed for the analysis of the guitar sound. Nonetheless, on the web and in shops, general purpose software is available that can perform, in whole or in part, the functions provided by our own program.

As a guideline to the choice, we summarize here the functions that our application provides:

- The program determines the spectral composition of the signal with a resolution of about 0.6 Hz in the frequency band up to 1200 Hz or, with lower resolution, up to 10,000 Hz.

Within these range of frequencies, both the resonances and antiresonances of the sound are displayed. The algorithm is known as the Fast Fourier Transform. The signal is examined within a limited (and variable) time window, for instance 100 ms. By scrolling the window along the signal timeline we can see the variation of the spectrum over time, hence how the amplitude of the resonances decays.

The following example shows the spectrum of the signal obtained from a finished guitar.



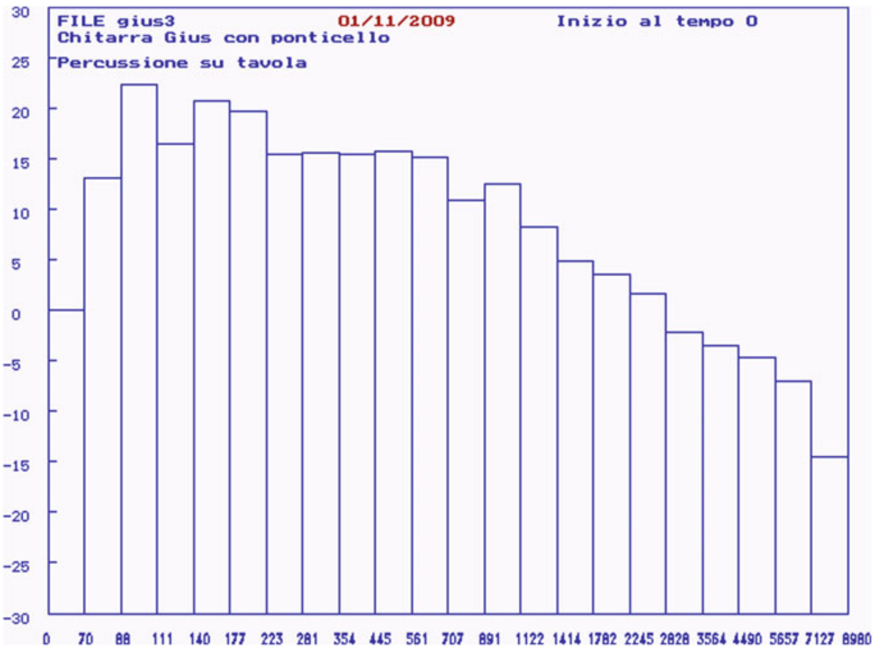
Please notice that, parallel to the frequency scale, another scale shows the position (in frequency) of the tones. This is to verify if any resonant frequency coincides with a played note. As we have pointed out in the first part, this situation must be averted, since a tone coinciding with a resonance is—most probably—a *wolf tone*!

The user can select the desired resonances, whose frequency, amplitude, and quality factor Q are calculated by the program.

Various representations of the spectrum are displayed, of which the most interesting is the *third of octave* one. An octave (the interval between, for instance, 100 and 200 Hz, or between 1000 and 2000 Hz) is divided up into three bands, each one extending for *one third of the octave*.

Obviously, should we represent the frequency on a linear scale, the octave between 1000 and 2000 Hz would cover a segment of the scale ten times longer than that of the octave between 100 and 200 Hz. On the contrary, in our representation we use a *logarithmic* scale, where the segments covered by the two different octave intervals have the same length, and the three bands in each octave have the same width. Each of the bands has a *middle frequency*; as an international convention the middle frequency of the reference band is 1000 Hz.

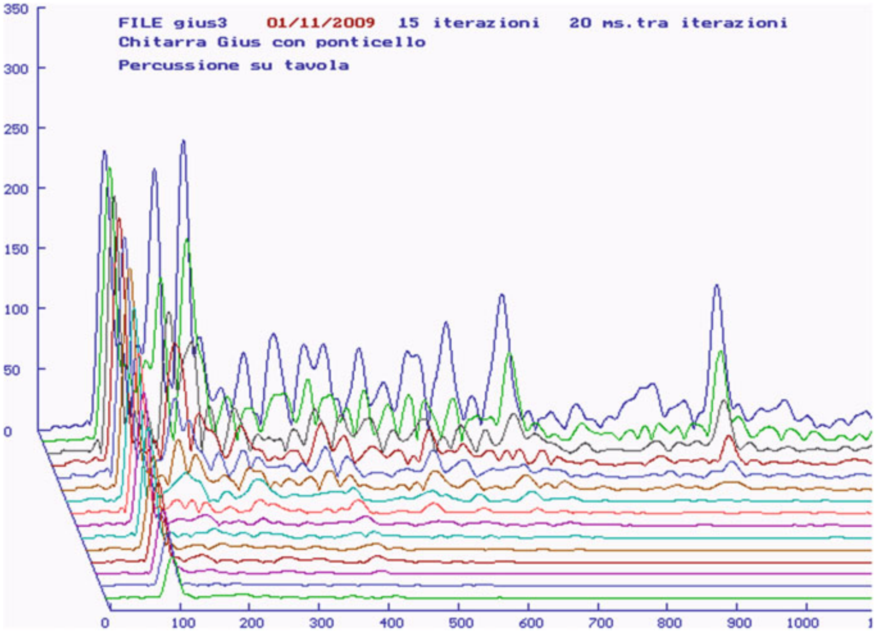
An example will clarify the former reasoning: the following graph is the third of octave representation of the same instrument whose spectrum was previously represented on a linear scale.



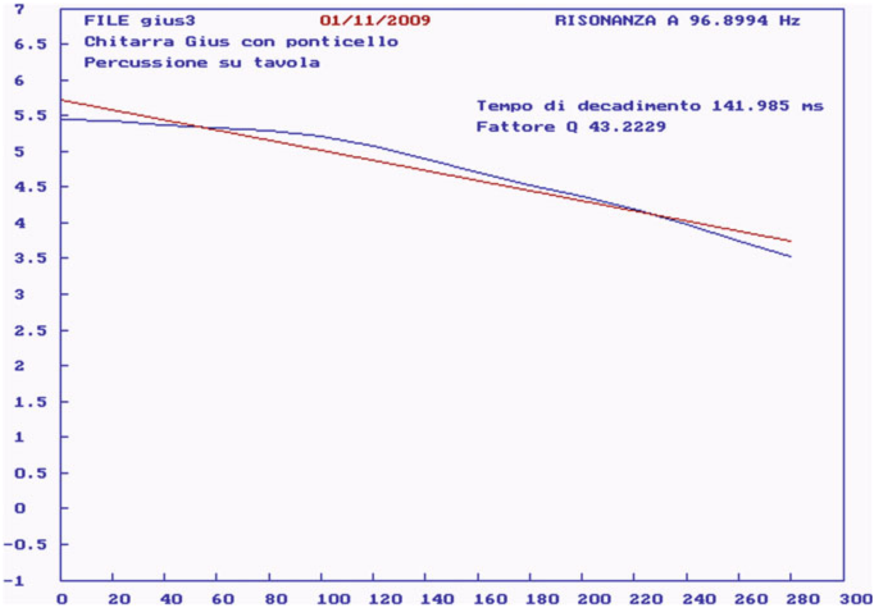
It is evident that, on the logarithmic frequency scale, the third of octave bands have the same width, and each of them is defined by an initial value and a final value.

The software associates each band to the average amplitude value of the response in that band, still on a logarithmic scale. The meaning of this representation is the following: in a howsoever extended frequency band can exist resonances, whose response is enhanced, or antiresonances, whose response is hampered, but the average response in that band assumes an especially important global significance, which summarizes the meaning of the specific resonances within the band.

- The diagram known as *Waterfall chart* is a 3D representation of the changes occurring in the signal spectrum over time: the spectrum representations are superimposed (and staggered for clear viewing) forming a sort of 'solid' surface, with frequencies displayed on the horizontal axis and values of the single spectral lines, calculated at different moments, displayed on the vertical axis. The following example shows how our software elaborates the Waterfall chart.



- It is important for the luthier to know the position and amplitude of the resonances, but also their decay rate: in fact, an excessively rapid damping of certain resonances often implies that some tones die out abruptly, and this is clearly a flaw in the instrument. The resonance decay time corresponds to the sound *sustain*. Our software allows to calculate the decay time in each of the selected resonances, therefore to determine whether the sustain is appropriate, or actions on the soundboard are required. We report the decay time diagram of the air resonance in the reference guitar.



This diagram clearly shows how the peak value of the air resonance (at about 97 Hz in this guitar) diminishes in the course of time. We stated elsewhere that, in theory, the amplitude decline of a resonance follows an exponential law, or

$$A(t) = A_0 e^{-t/\tau}$$

where A_0 is the initial amplitude of the resonance, $A(t)$ is the resonance amplitude at the time t, and τ is the *time constant*, which defines how quickly the resonance amplitude drops down.

The former diagram shows (on a logarithmic scale) the initial values of the resonance amplitudes and their values at subsequent times. It also allows to identify the time constant value (142 ms); in addition to that, the quality factor Q (equal to 43 at this frequency) is calculated. The analysis of decay times is obviously performed on all of the selected resonances.

Finally, the whole data are transferred into a report file, serving as a record of the measurements and a guideline to intervene, if necessary, on the design.

Chapter 8
Quality and Evaluation Methods

Abstract This chapter introduces the concept of progressive controls and adjustments to be done along the various phases of the guitar construction, in order to achieve a target quality. An example is given, demonstrating how—in four soundboards—the main resonance evolved from bare board through the progressive steps of adding rosette, bridge, braces, and gluing to sides.

The issue of quality in musical instruments—especially the classical guitar—involves different aspects that need to be separately examined, in order to eventually obtain a picture as compliant as possible with the expectations of the instrument owners, hence players.

Good structural qualities are obviously required to endure usage, passage of time, and adverse weather conditions. That is not our concern in this book, since it has been long investigated, and partly answered, in traditional lutherie. We just recommend to plan the aging of the timber intended for manufacturing even the most trivial details, and also to store it some days ahead in the workshop at constant temperature and humidity. Specifically, the relative humidity must be kept around 50%.

Secondly, but not secondary, the instrument should be very *user-friendly*. So the fingerboard, the neck, and the dimensions of the body should comfortably suit most of the players.

The *tuning machines*, serving to stretch and keep the strings in tune, should be free of frictions due to incorrect assembly or scarce structural quality: on the contrary, tuning the instrument should require the smallest energy, meaning that the involved elements work unhampered.

Finally, the most important parameter: sound emission quality, which is the leading attribute as far as playing is concerned. Dealing with sound quality, one has to consider a number of parameters, like

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- Balance—both in power and timbre richness—between the tones played along the fingerboard;
- absence of sharply decaying tones;
- proper *sustain*—being a compromise between emission power and duration of the sound;
- easy execution of the natural harmonics;
- quality figures presented in Chaps. 1.5 and 1.6.

In this book we proposed a construction method based on controlling, at any step of the manufacture, that the parameters relative to the frequencies involved, to the amplitudes of the main vibration modes, and to the Chladni nodal patterns, are objectively congruent with the indications drawn from the experience with this method. Therefore we can say that, generally, the compliance with these parameters surely brings about very good results, even though little differences in the sound may come from the kind of wood, and from the manufacturing method.

Now the quality control will explicitly concern the outcome of adjustments required on the guitar components according to regularly performed analyses.

First of all we must establish what parameters distinguish a quality instrument, and then verify if they have been achieved.

As we have thoroughly explained and demonstrated in Chap. 5 (about the resonator as a global system), the most important parameters in determining the quality of a classical guitar are frequencies and relative amplitudes of the fundamental resonances.

Though secondary, outward appearances and usage characteristics are also distinguishing features between high and low quality instruments. However, these aspects fall outside the focus of the authors in this book.

Since quality control begins by testing the suitability of the materials to be employed, we refer this issue to Chaps. 9.4–9.7, concerning the choice of timbers and related analyses.

Later, and after initial fabrication procedures—like calibrating the soundboard thickness, inlaying the rosette and its reinforcement, fixing the main braces to the soundboard—it is possible and recommended to begin the control of the soundboard resonant frequencies, in order to verify that they suit the current stage of construction. The following table reports some indicative values derived from our experience in building instruments credited with good quality.

Since the suggested evaluation method implies a continuous use of results obtained by Chladni method and acoustic hammer analyses, the instrument construction will proceed through a number of data verification, in order to verify the formation of ample and properly shaped vibration modes, whose resonant frequencies must fall within the limits repeatedly mentioned in this book.

The settings considered here—specifically soundboard on mould or frame, with or without the back—are relative to four instruments under construction. The measured frequencies refer to the vibration mode $\langle 0\ 0 \rangle$. This is also called 'ring mode', owing to the characteristic circular shape of the nodal line, which runs parallel to the outer

edge of the soundboard, and very close to the bond where the soundboard is glued to the sides through the linings.

Indicative frequencies, in Hertz, of a Soundboard on the mould

	Bare soundboard on the mould	With hole, rosette and reinforcement	With bridge glued	With main braces	With semi-finished fan bracing	With finished fan bracing	With rigid back	Glued to the frame without the back
P1	73.34	74.02	72.67	94.88	137.9	155.2	195.1	181.8
P2	70.65	64.59	59.88	94.88	141.3	178.9		181.0
P3	73.34	99.59	96.22	118.4	135.2	156.7		192.4
P4	72.5	74.02	95.22	117.7	168.2	159.4		191.7

In the event of significant discrepancy between any of the measured values and those reported on the table, we can rectify by adding some braces—if the frequency is low—or by reducing the height of the same braces—if the frequency is too high. During this procedure, we recommend to always seek a condition of asymmetric stiffness in the two halves of the soundboard; this is meant to get different oscillation amplitudes in the vibration modes featuring two parts that oscillate in antiphase, for instance modes ⟨1 0⟩ and ⟨0 1⟩.

Tests involving the back can only be executed when data are reliable and congruent with the indications listed in the previous table.

The reported data must be regarded as indicative, though excessively diverging values are sometimes inappropriate because, beyond those limits, the guitar will not work anymore according to the repeatedly arranged and verified design.

If the ring nodal line of mode ⟨0 0⟩ runs some cm apart from the edge, the soundboard is not elastic enough along the perimeter, and must be therefore rectified and improved. This can be done by thinning it down along the periphery, in a band covering 2–3 cm towards the centre of the soundboard, or by further lowering the ends of the braces near the sides. We remind that the ring mode is crucial for the guitar quality, so it must be broad and correspond to the right frequency.

While proceeding in the instrument construction, regular quality controls will be required, constantly monitoring the frequency of the other fundamental resonances, namely those of the soundboard, of the inner air, of the upper vibration modes, and of the back. This issue has been thoroughly discussed in Chap. 5.

We recommend the luthier once again to accomplish the gluing of the bridge before any other action. Actually, the bridge is an inseparable component of the soundboard; unlike a brace, it cannot be applied and then removed or relocated. Frequency controls would be nearly—if not entirely—useless without the bridge, since fixing it later would bring about a conspicuous variation of the recommended frequency characteristics.

When the soundboard, fully provided with all details and still fastened to the mould, reaches the right frequencies for the main vibration modes, a very important

test must be executed by applying a rigid back to the mould, in order to bring forth the body resonance. This rigid back is just a plywood panel, conveniently shaped as the mould profile and provided with a series of holes for the fixing bolts. The rigid back closes the rear of the mould, defining the same air volume that will be contained in the guitar, so that the oscillation of the air can take place. A rubber seal, or any soft material, is applied to ensure an adequate airtight closure.

The oscillation of the soundboard, vibrated by the acoustic hammer or the electromagnetic exciter, sets in oscillation the air contained in the body constituted by mould, soundboard and rigid back. The FFT analysis obtained from the percussion of the acoustic hammer highlights a resonance peak at about 100–105 Hz, which is in fact relative to the oscillation of the air.

This frequency will then stabilize at about 95–100 Hz, once the rigid back has been replaced by the final back of the guitar.

These results being achieved, the soundboard can be glued to the frame. In the rigid-back setting the resonant frequencies of the soundboard will noticeably change, growing by 10–20 Hz with respect to the open-back setting; even the struts that, glued to the sides, ensure the solidity of the main braces, can modify the soundboard resonant frequencies.

When all the targets planned for this stage of construction are attained, we can start verifying the frequencies that will arise by percussing the soundboard once the final back of the guitar is applied to the mould. This back will be both manufactured and tested with the modal analysis method, as illustrated in Sect. 13.1.

As thoroughly described in Sect. 5.7, frequencies and amplitudes of the back will be calibrated through the modal analysis, in order to get its frequency in mode

⟨0 0⟩, measured after gluing to the sides, 30–40 Hz higher than the soundboard fundamental frequency. This frequency of the back will contribute to filling some antiresonant areas that may come into being beyond the fundamental frequency of the soundboard (220–250 Hz) and up to 400 Hz.

The instrument finished with tuning machines, strings, nut and bridge saddle, will be checked up note for note in order to verify emission, sustain, presence or absence of harmonics in a feeble fundamental tone and, finally, timbre and power balance of each sound on each string.

We wish to advise a method for fixing some little flaws that can be detected during the finished instrument check up.

Frequently, even in quality instruments, a high tone like, for instance, F, F#, G, G#, A, respectively at the keys 13–14–15–16–17 on the first string, quickly decays after release. This can be at least in part amended by applying, first outside with double-sided tape, then inside with the glue, a little weight varying between 1 and 5 g. The convenient position of the weight, generally near the hole, must be patiently sought, possibly with the help of a guitar player.

While the faulty tone is repeatedly played, the luthier touches the soundboard with a finger until the point is identified where, by gently pressing, an improvement comes about.

Then the little weight is provisionally applied in that position and, if the improvement persists and the instrument actually works better, it can be glued to the inner side of the soundboard.

Chapter 9
The Modern Guitar

Abstract In this chapters a general information on the modern classical guitar is given. The shape is outlined by a set of arcs and radiuses (polycentric) providing an objective description of the profile. Then the general characteristics of top, sides, back, neck are given. Also the double tops with Nomex and their assembly are presented. The reader is introduced to a novel fingerboard profiling approach, which provides a graded relief. The chapter analyses the materials properties and the relevant physical characteristics (density, velocity of sound, elastic modulus) of various woods suitable for the boards, and presents a methodology for assessing the quality of woods through the acoustic hammer and the response of a sample. Results from not less than 24 samples of spruce coming from various sources are presented.

This book is meant to guide the reader through the big issues implied in classical guitar designing, to gradually understand and solve them. This requires some dedication and, above all, passion and independence from the many long-lived prejudices around the subject.

The modern classical guitar must be a versatile, balanced instrument, and loud enough to perform in small and medium-sized musical ensembles without amplification. These results, attainable at the end of the construction process, cannot be left to chance or to mere reproduction of the outlines and dimensions of a reference instrument, expecting to repeat the phenomenon of its acoustic features.

As a matter of fact, different attributes of the wooden parts (even when coming from the same tree), and seasonal ambience variations in the luthier's workshop, invalidate the efforts aimed at faithfully reproducing a former and worthy model.

Scientific approach, and analyses executed at any stage of manufacture, are at the current status of research the only way to get sure and very positive results.

Many variables depending from different parameters are involved in classical guitar construction. Therefore we suggest to focus on the most important ones during design as well as fabrication, neglecting those of an aesthetical rather than functional nature, in order not to complicate the management of problems that will come about in the course of construction.

For example, we suggest to choose a standard shape implemented by former luthiers, abstaining from recurrent change of the model, in order to turn aside

G. Cuzzucoli and M. Garrone, *Classical Guitar Design*,
https://doi.org/10.1007/978-3-030-32992-1_9

unknown elements which may engender nasty and unfamiliar troubles rather than improvements.

On the contrary, we will focus on using differently shaped braces, different thicknesses, different timbers, and so on, because all of these variations bring about new characteristics that will be verified by means of regular and fairly frequent controls during construction.

9.1 The Design—Getting Started

Many books available on the market display guitar shapes in use from Torres age to present, so making easier to choose the one we deem most satisfying, at least in dimensions and appearance of the body. The choice of woods is open to the luthier's aesthetical preference, as well as the design of a personal headstock that will represent a sort of author mark.

Obviously, the selection of timbers to be used for the sides and the back of the guitar must not be based only on appearance but, primarily, on physical parameters like stiffness/weight ratio (Brazilian rosewood—or Rio—is among the best in this regard), workability, availability, and price. Of course we advise not to use costly materials for the first attempts, in order to limit the loss in case of ruinous blunders due to inexperience.

The height of the sides affects the instrument response in the fundamental resonance of the body, i.e. the oscillating frequency of the air inside the guitar. The higher the sides, the lower this resonant frequency. However, this reduction of the fundamental resonance of the body does not necessarily impair the instrument performance at high tones, provided that proper actions are taken on the resonances of back and soundboard.

Anyway, we suggest to keep the height of the sides within standard levels, for playing comfort among other reasons.

Right from the beginning, we also must establish the scale length, represented in this instrument by the length of the vibrating string. The most common measure is 650 mm from nut to bridge, which is extended by a couple of mm to compensate the increase in string tension during usage; usually the compensation is 1 mm at the first string and 3 mm at the sixth string.

Once an agreeable and regular-sized model has been chosen, its outline must be transferred onto drawing paper and checked to correct possibly uneven curves.

The contour of the guitar, hence of its back and soundboard, can be revised by means of the circle sector method illustrated in the next paragraph.

9.2 Drawing the Guitar Shape—Circle Sectors

Begin by copying, as pointed out, the profile of the sides. Trace the guitar axis and, beginning from the top of the left shoulder, search for the centre of the first portion of the curve along the axis, by means of a compass. Then trace a straight line connecting this centre to the end of the first segment of the curve, on its left side.

Along this line, find the centre of the second curve sector and link it to the end of the same sector with another straight line. Then find the centre of the third sector on this line, and so on. This way the arches join up to each other smoothly and free of corners at the junctions.

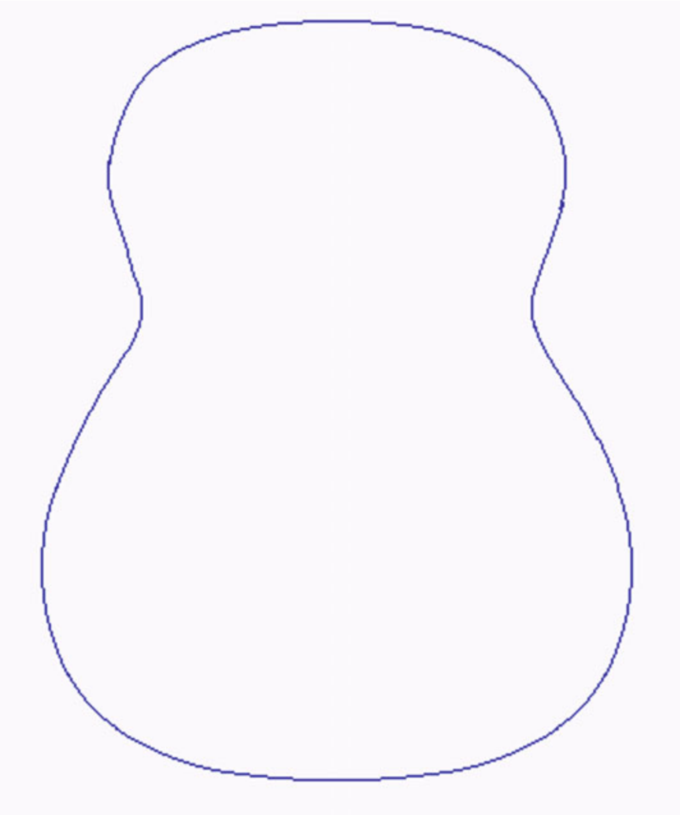
All the centres of the guitar profile will rest inside the shape, except the ones related to the curves of the waist, which will lie outside the profile.

Avert in any case straight segments in the profile of the sides, for instance near the tail block and near the neck block (the junction between sides and neck). They would be regarded as faults in the bending of the sides, conveying a sense of inaccurate construction.

At the conclusion of this stage of work, a set of data is available that provides an objective description of the guitar profile (radiuses, width of the arches, coordinates of their centres, etc.). These data can be collected into a table in many ways: the following one reports data relative to the profile of the Garrone guitar we have taken for reference, as resulted from computer processing aimed at rectifying little flaws that are inevitable in a fully handmade draw. The semi-profile is divided up into sixteen arches. The table reports the coordinates of the centre, and those of the initial and final points of every arch. Coordinates (expressed in cm) refer to the lower end of the body, where the sides meet at the tail block: the Y coordinate follows the longitudinal axis, and the X coordinate follows the transverse axis. Furthermore, the direction of the concavity is indicated, whether inwards (at the waist) or outwards.

The following figure reproduces the outline of the reference guitar, resulting from the reported data. These data allow faithful reproduction of the soundboard profile, and they can be employed in many ways. For example, as coordinates for a plotter to trace the soundboard profile, or else for a CNC (Computer Numerical Control) machine to cut out the exact shape. We used these data to create the FEM model of the soundboard, whose employ has been more than once exemplified in previous chapters.

Lastly, these data allow accurate calculation of both the soundboard area and the body volume: these elements—as we have seen—are involved in the calculation of the Helmholtz resonance.



Arch	Concavity: +1 = inner centre −1 = outer centre	Coordinates of the centre X (cm) Y (cm)	Initial point X (cm) Y (cm)	Final point X (cm) Y (cm)
1	+1	0 10.57	0 48.67	5.29 48.31
2	+1	3.13 32.92	5.29 48.31	10.36 46.67
3	+1	7.15 40.55	10.36 46.67	13.16 43.95
4	+1	2.82 38.11	13.16 43.95	14.57 39.85
6	+1	7.62 38.82	14.57 39.86	14.33 36.74
7	−1	−77.68 65.28	14.33 36.74	13.62 34.54
8	−1	961 −285	13.62 34.54	12.76 31.99

(continued)

(continued)

Arch	Concavity: +1 = inner centre −1 = outer centre	Coordinates of the centre X (cm) Y (cm)	Initial point X (cm) Y (cm)	Final point X (cm) Y (cm)
9	−1	17.64 30.36	12.77 31.99	12.69 28.98
10	−1	18.13 30.50	12.69 28.98	13.39 27.42
11	+1	−1.48 13.61	15.26 24.54	16.58 22.19
12	+1	−19.55 5.03	16.58 22.19	17.98 18.87
13	+1	3.91 13.68	17.98 18.87	17.27 6.87
14	+1	8.90 11.13	17.27 6.87	14.15 3.34
15	+1	4.09 18.26	14.15 3.34	6.91 0.49
16	+1	0.646 40	6.91 0.49	0 0

9.2.1 Outline of the Sides

The edges of the sides have different profiles: the one in contact with the soundboard, from the junction of the neck to the tail block, is perfectly straight. At the back, on the contrary, it must be cut following a curved line, obtained by fitting the shape of the back into the already bent side.

It is therefore necessary to use a back provided with glued braces, which give it the right curvature making easier to trace the rear outline of the sides.

The lower point of junction between sides and tail block should be the highest one, but we suggest to slightly reduce it, so as to confer the back, once glued, a somewhat arched appearance in both directions.

As an indication, the width of the sides can be 90–92 mm at the junction with the neck, 102 mm at the largest point of the soundboard, and about 100 mm at the tail block, where they meet.

With regard to the **tail block**, we wish to point out that this element has just the purpose to keep the sides together at their ends. The tail block is traditionally oversized, which is useless, or rather detrimental. In bowed instruments, where it was probably first employed, the tail block undergoes the tension of the strings through the tailpiece and the fixing button. This is not the case in the guitar, and therefore a simple joining plate, shaped as the soundboard profile in that point, works well enough. A pointless bulky tail block would be an obstacle for the air inside the guitar. In fact the air, as we have seen, goes into oscillation according to its own resonant frequencies, being deranged by anything in the way of its vibrations. For instance, this is why the

organ pipes, where sound is produced by resonances generated via the air driven by the bellows, are perfectly smooth and polished.

As for the thickness of the sides we remind that, when excessively thin, they can be easily bent but lack in stiffness, and vice versa.

A suitable thickness is between 1.8 and 2 mm, according to the stiffness of the wood.

Hot bending of the sides is basically unproblematic. The sides must be dipped in tepid water for some minutes, to facilitate the formation of steam when in contact with the hot iron. The vapour softens the wood and facilitates its bending. When we remove the hot iron from the side, we must keep the lumber bent until it has cooled down almost completely, in order to prevent it from straightening back. Maple wood must be bent very cautiously, and not excessively moistened. In fact its marbling, arranged in transverse patterns with respect to the grain, is a mark of beauty for this kind of lumber but also represents a discontinuity, which can be the cause of ruinous transverse breaches.

Finally, we suggest to perfectly smooth the sides before bending, this way making easier to finish them when joined to the frame.

In designing the headstock, which is as stated the distinctive mark of the luthier, remember to facilitate, by the inclination of the two slots that lodge the tuning pivots, the position of the string ends, so preventing an excessive angle between nut and tuning pivots. At its best, the string should depart from the nut at 90°, with no other angle except downwards. Lastly, the headstock to neck angle must be comprised between 11° and 12°. This is sufficient to let the strings keep contact with the nut; a greater angle could be aesthetically unpleasant.

The rosette was formerly meant to protect the border of the hole, which might break along the vein as a consequence of wood shrinkage due to aging or excessively dry conditions.

Custom turned it into a colourful but harmfully oversized ornament. We must keep in mind that the area of the soundboard where the rosette is inlaid involves especially high frequency vibrations, and so an outsized decoration can compromise the result. The same is true for the reinforcement on the inner side of the soundboard, which can be limited to a few mm large and lens-like circular band, maximum 1.5 mm thick.

9.3 Guitar Elements

9.3.1 The Soundboard

The soundboard is definitely the most important of the instrument components, whose design requires for this reason the highest care.

First, the choice of wood for the soundboard must be extremely meticulous. The traditional spruce (growing on the Italian Alps with excellent resonance quality), the American cedar, and many other tonewoods are suitable.

It is crucial to use lumber aged for some years, and to store it in the workshop for a time at reduced thickness (not more than 3–4 mm). This way the board is allowed

to release its inner humidity, and initiate the polymerization process of the resin contained in the resin canals.

Furthermore, before working it, the board must obviously be stored for some weeks at constant humidity (50–55%). But the recommended relative humidity ratio is highly dependent on the ambience where, presumably, the instrument will be used. The recommended humidity will be halfway between seasonal highest dampness and dryness values. However, the instrument becomes gradually unaffected by humidity fluctuations after some years.

For the first three or four years a special care must be taken in avoiding cracks in back and even soundboard, or detachment of braces from the back, due to high humidity conditions.

The soundboard thickness deeply influences the final result. Thicknesses just about recommended limits (2–2.5 mm) can be successfully employed, provided that the target of a proper stiffness/weight ratio is constantly considered. It is always preferable to limit the thickness and then increase the stiffness through properly sized braces. At any rate we recommend not less than 2 mm, to prevent some inconveniencies that will be later described.

As for the soundboard wood, boards with clearly visible and not too close veining should be preferably selected. The "vein" is in fact the winter growth ring, the most resistant but also the heaviest part, while the brighter portion, the sapwood, is the summer ring. Anyway, as described in the chapter about "rough boards", accurate indications about the selection of wood can be drawn through control of the board flexibility, and acquisition of all the relevant parameters.

Acoustics applied to musical instruments identifies the three fundamental oscillators of the guitar: soundboard, air inside the body, back.

The soundboard turns vibrations originated by the strings via the bridge into fairly ample oscillations, at least enough to engender acoustic pressures in the surrounding environment.

These pressures turn into audible sounds, with that typical complement of overtones that constitute the guitar timbre.

As thoroughly described in the chapter concerning the guitar resonator, the oscillations of the soundboard compress and vibrate the air contained in the body, which in turn sets the back into oscillation. The vibrations of the back in turn excite the soundboard, and so the coupling between back and soundboard enhances the instrument overall response.

This makes it clear that the sound-producing system of the guitar is very different from that of bowed instruments, where vibrations travel from soundboard to back through the sound-post, a small spruce dowel gently forced between back and soundboard.

Finally, we wish to point out that it is incorrect to define the soundboard as an element which *amplifies* the sound of the strings. The soundboard cannot amplify the sound because it has no source of energy available to *amplify* the sound. Simply, the energy that the musician applies on the strings is transferred to the soundboard through the bridge, which matches the string and resonator impedance; the resulting oscillations are vigorous enough to produce an acoustic pressure that we perceive as sound.

A traditional soundboard can be uniform in thickness or cone-shaped using a planer, and armed with a framework of wood strips arranged in various ways to form the bracing, as shown in illustrations at Chap. 11.2.

Or else it can be further stiffened, and at the same time lightened, by building it in two layers, with an interposed sheet of polymer honeycomb structure, called Nomex.

As far as we know, this construction method was conceived by two German luthiers engaged in well aimed experimentation of new construction techniques.

This structure demands a light bracing, being very stiff in itself. It was created for light and very resistant assemblies like airplane wings, rally racing cars, etc. It is produced in the form of very thick panels, with variously sized cells, and then cut into sheets according to customer's requests. Sheets sold for employ in lutherie are about 3 mm high, normally with 3 mm sided cells and about 0.1 mm thick walls. Nomex must be glued between two boards using a vacuum pump and epoxy or polyurethane glue.

To build such a soundboard we must make a gluing frame that will be then connected to a vacuum pump. Vacuum pumps are for sale at affordable prices, but a dismissed fridge pump can do well enough.

This frame, as shown in the picture, is composed by an aluminium or wooden structure, wherein an elastic rubber membrane is attached. Those used in hospitals are very good for the purpose. The rubber is glued with impact resistant adhesive and then framed by means of an aluminium tape. A closed-cell foam seal must be glued onto the lower edge of the frame, in order to prevent air infiltrations. On one of the smaller sides of the frame, a threaded hole will be drilled to insert the suction pipe.

The soundboard is then assembled using two high quality boards, thinned down to about 2 mm thickness. Strips of spruce, which can also be made out of old discarded boards, are glued onto the inner side of the lower board to leave room for the Nomex.

A support of the same thickness will be glued under the bridge, and a strip about 1 cm large will be glued as a joint-cover at the centre of the board. Now Nomex can be glued to the lower board, taking care that it does not exceed the retaining spruce strips. Since the finished soundboard must be no more than 3 mm thick, and the outer boards cannot obviously be thinned down to less than 0.6–0.7 mm, the inner part including Nomex must be reduced by means of a calibrating machine to about 1.5 mm.

Nomex, slightly spread with glue, is applied to a perfectly flat polystyrene panel, which will be covered with glue using a paint roller. Once the Nomex sheet has taken the glue, it will be inserted into the compartment prepared on the lower board, then the whole structure will be placed under the frame to be compressed through the vacuum generated by the pump.

The Nomex sheet must absorb the necessary amount of glue but, on the other hand, the glue should be absolutely kept from filling the hexagonal cells, to prevent a detrimental weight increase. For the same reason, the glue must not be spread on the board, but only on the honeycomb border.

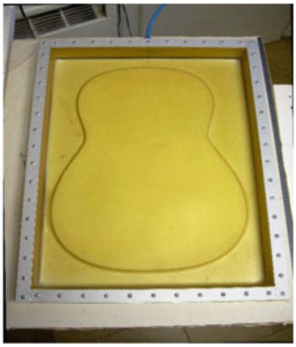
When the glue has dried up, the whole board will be calibrated to about 3.5 mm (2 mm for the board plus 1.5 mm Nomex). The same gluing procedure of Nomex

will be then repeated, spreading the glue directly on the honeycomb. Then the spruce layer intended for the outer side of the soundboard will be applied onto the lower side and vacuumed again for the final gluing.

The last stage is the calibration of the whole soundboard, executed by thinning down both the sides, step by step, until the desired thickness is achieved. The current thickness of both the boards can be always controlled observing the soundboard edges.

Soundboard

Lower board with Nomex



Gluing the soundboard

An instrument whose soundboard is designed according to this method delivers loud and very balanced tones over the whole relevant acoustic band. The attack is quick, and sustain fairly good, thanks to scarce inner friction losses in the structure of the soundboard itself.

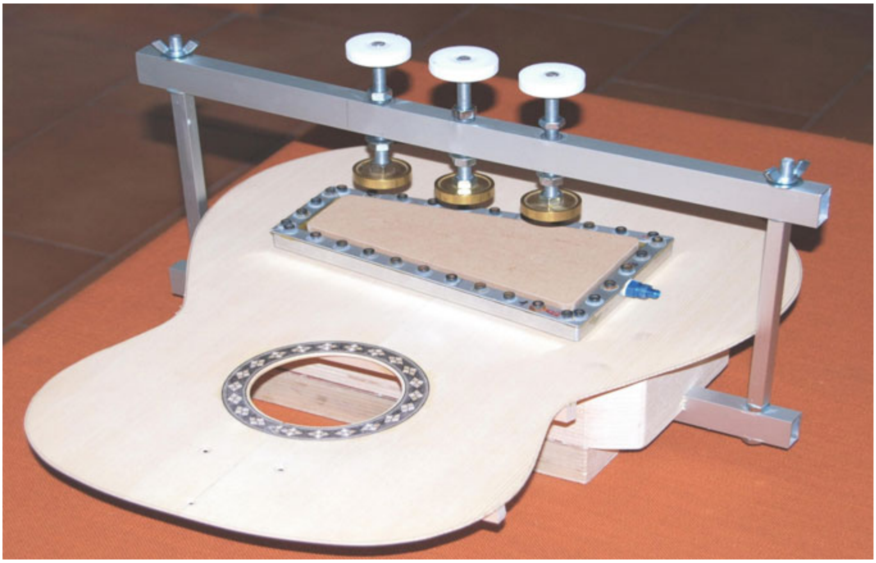
Once established what kind of soundboard we want to build, whether at constant thickness, cone-shaped or assembled with Nomex, we can go on to the design of the bracing pattern. Leaving this decision to the luthier, we recommend to proceed with the gluing of the bridge prior to any other action. This priority is based on practical reasons and on the quality control method we adopt.

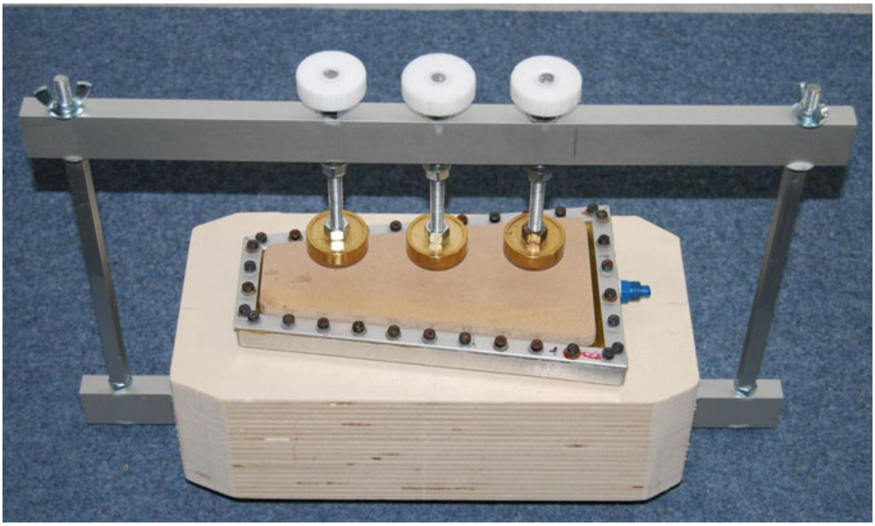
Gluing the bridge without any hampering fan braces on the inner surface of the board, is clearly advantageous. This way it is easier to find its correct position, and the absence of fan braces, whatever their arrangement will be, keeps the soundboard surface flat, favouring the adherence of the bridge.

In fact one of the possible reasons for the detachment of the bridge from the soundboard is an incomplete adherence of the surfaces. The glue employed for this purpose is an aliphatic resin that requires the surfaces to be very close, leaving room for just an imperceptible gluing layer. This is the reason why producers recommend very high gluing pressures, in the order of 10–15 kg/cm^2, based on the fact that the gluing surfaces are supposedly never perfectly even. It is therefore preferable gluing the bridge on a board not yet armed with fan braces.

The authors, in order to reach high pressures, have conceived the device illustrated in the next picture. This device is easy to build and use, and allows to apply very high gluing pressures.

The first picture shows the press ready for gluing a bridge on a soundboard. The bridge is hidden by the vacuum jig, which makes the pressure uniform, so avoiding even the slightest displacement upon clamping. After a few seconds it is possible to lower the screws, pressing directly on the jig through an interposed wooden block, as shown in the picture.





Here we can see the simple plywood basis. Reasons for gluing the bridge right from the start are:

- The bridge is a definitive and immovable element.
- Analyses executed on the soundboard without the bridge are not correct, since the bridge influences considerably the soundboard structure.

The position of the bridge on the soundboard must be accurately traced, since it determines the scale length.

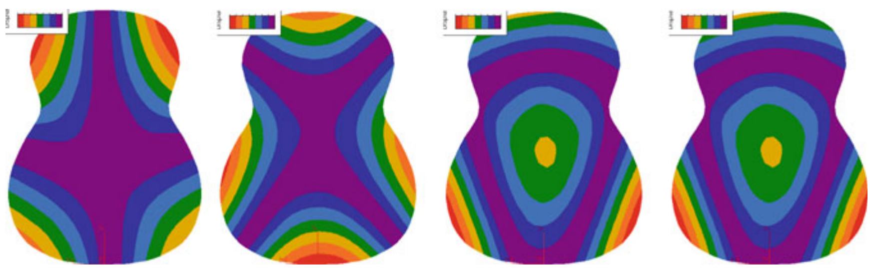
Then we proceed to set the position of fan braces, main braces, and hole reinforcement. As recommended, the height of the fan braces will be a little greater than what designed, to facilitate accurate stiffness regulation during later overall adjustments.

9.3.2 The Frame

This is the connecting element between back and soundboard. From an acoustic point of view, it represents the peripheral bond for these two components and, together with them, it forms the instrument body.

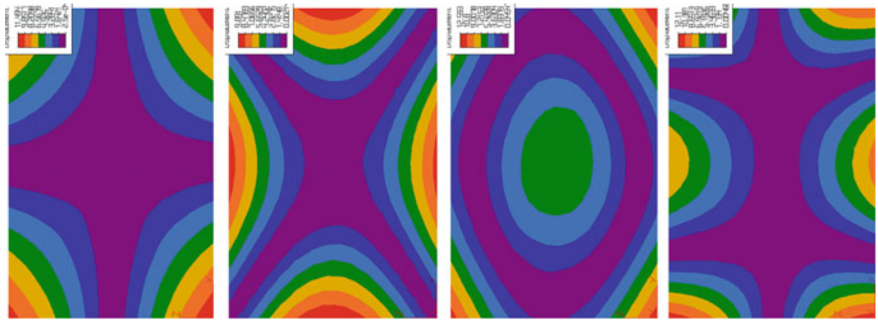
Let's look into implications. A board meant for sound production must be fastened along its periphery, in order for its typical vibration modes to be generated. Only the gong and some other folk sounding surface can vibrate with no peripheral bond.

An unfastened, or free soundboard, presents vibrating and non vibrating areas at the periphery. The following set of illustrations represents a soundboard that vibrates at different frequencies with modes typical of the free edge plate.



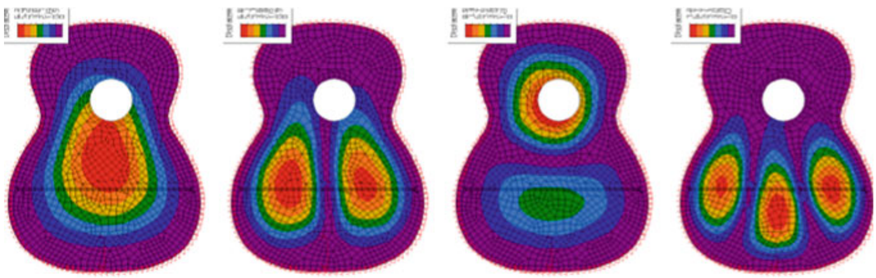
It can be useful to go back to the chapter about the resonator components, where the first modes of a soundboard are compared with those of a rectangular board made of Sitka spruce, measuring cm 30 × 20 × 0.25. Please notice, in the figures below, the analogies between the distribution of vibration modes in a guitar and in the rectangular board.

These images are all processed through FEM simulation of both the soundboard and the rectangular board.



The guitar soundboard, when joined to the sides through the linings, behaves as a *clamped edge plate*; this kind of bond is typical, for instance, in masonry, when a beam is fixed into a wall.

The third set of illustrations represents the same soundboard with this kind of "ideal clamping".



Between these two extremely opposed fastening conditions, free edge and clamped edge, a third one exists, called *simply supported edge*, where the board is allowed to rotate in any point along the border.

Actually, the guitar soundboard is subject to a kind of bond halfway between clamped and simply supported edge conditions, because the gluing made at the sides through the linings does not confer complete stiffness like the *clamped* bond does. The sides have marked elasticity, and the linings are more or less rigid according to their form (kerfed or unbroken). This can make the luthier conscious about the influence of such details on the behaviour of the soundboard.

Once realized that the frame, from the point of view of acoustics, represents the bond that favours correct vibrations of back and soundboard, let us also consider it as an element that confers volume, and shape, to the instrument body.

Usually the sides have a mean height of about 10 cm, and the volume of air they enclose is about 13,800 cm^3, or 13.8 l.

9.3.3 The Back

Generally, the back comes from the same lumber as the sides, usually with a mean thickness of about 2 mm. The typical bulging shape of guitar backs is obtained by gluing 3 or more curved braces to its inner side. These braces also serve to increase stiffness, this way driving the back to resonate at the desired frequencies.

The back as well is glued to the sides through the linings: these, increasing the contact surface, ensure a firmer gluing.

Like the soundboard, the back is normally built by joining two half boards but, depending on the size of the available lumber, it can also be built in three or four parts. It is advisable to reinforce the gluing with a spruce strip, usually made from remains of soundboards and cut out so that veins are transverse to length. This serves as *joint-cover* that strengthens the gluing tie.

The end of each brace of the back lies on a support glued to the sides, called strut, and made from spruce or any other wood, serving to increase the resistance of the gluing to the back.

9.3.4 The Neck

The guitar neck mainly accomplishes static purposes but, based on FEM dynamic models, it is believed it also marginally contributes to the overall timbre of the instrument. It is usually made of mahogany, cedrella or maple when the whole guitar is made of this same wood. In order to increase its torque resistance, a reinforcement made of hard wood (ebony or rosewood) is inserted inside. There are two ways for fitting the neck into the guitar body: the Spanish heel joint, where the sides are fixed into lateral slots directly cut in the neck, or a neck—body dovetail joint.

The Spanish heel method consists in cutting two slots, of the same thickness as the sides, where the neck ends. These slots lie on three different angles:

- Shoulder angle
- Top angle of the sides
- Neck assembly angle.

The first one only serves to confer the sides a little sloping angle with respect to the neck, so that they follow the subsequent curvature of the shoulders. Its value is about 3 degrees. The top angle of the sides depends from the outer profile of the neck, hence from the shape of the heel; it provides the maximum insertion of the sides in the slots, hence the maximum gluing surface. As an indication, the top end of the neck slots stop at 18 mm from the neck axis on the soundboard surface, and the bottom end at 5 mm from the base. The third angle is the one between the planes of neck and soundboard, allowing the construction of an even-thickness fingerboard. Without this little angle, which allows the neck to be, at the nut location, about 1.5–2 mm higher than the soundboard plane, the fingerboard should be built with a conic

profile: this implies thickness variations of 1–2 mm from end to end, resulting in additional problems in the construction process and unpleasant appearance.

Both the ways to fasten a neck to the guitar body work perfectly, and the choice only depends on the kind of tools available to the luthier. However, the Spanish heel is generally considered the most solid one.

At the upper end of the neck is the headstock, which slants by 11°–12° to the neck and bears the *tuning machines* as the string tuning devices are traditionally called. Until the early '900 a kind of the so-called *pegs* was in use, as it is still nowadays in bowed instruments and in flamenco guitars. The fingerboard, glued to the neck, is usually made from ebony (a non-deformable and long-lasting wood) and bears the frets; these divide the fingerboard into gradually shorter sections from the headstock to the hole, where it terminates. The fingerboard can be flat along its longitudinal axis, or slightly shaped to favour playability. It can also be a little convex in the transverse sense.

As for the construction of the neck, we only advise to use a non-deformable and properly aged kind of wood. Sometimes a leaden mass can be inserted into the upper end of the neck, under the nut and the headstock: this behaves as an acoustic impedance to keep the oscillating energy of the string from being partially transferred to the nut, at the expense of the available energy at the bridge. This stratagem improves the instrument loudness and sustain, but it obviously requires the inclusion of counterbalancing lead in the tail block, in order to prevent the instrument from weighing down on the player's left arm.

As for values, good results can be achieved by adding a 250 g leaden weight under the nut, and 400 g in the tail block.

9.3.5 The Fingerboard

As mentioned talking about the neck, the fingerboard is made from ebony in all quality instruments, because of its toughness and crushproof properties.

As stated, the fingerboard can be flat or shaped so as to favour playability (or *action*).

For many years the author has shaped its fingerboards according to the so called 'constant energy profile' devised by the Italian engineer Bruno Pizzigoni, University Professor at the Politecnico di Milano, to whom he wishes to manifest respect and gratitude.

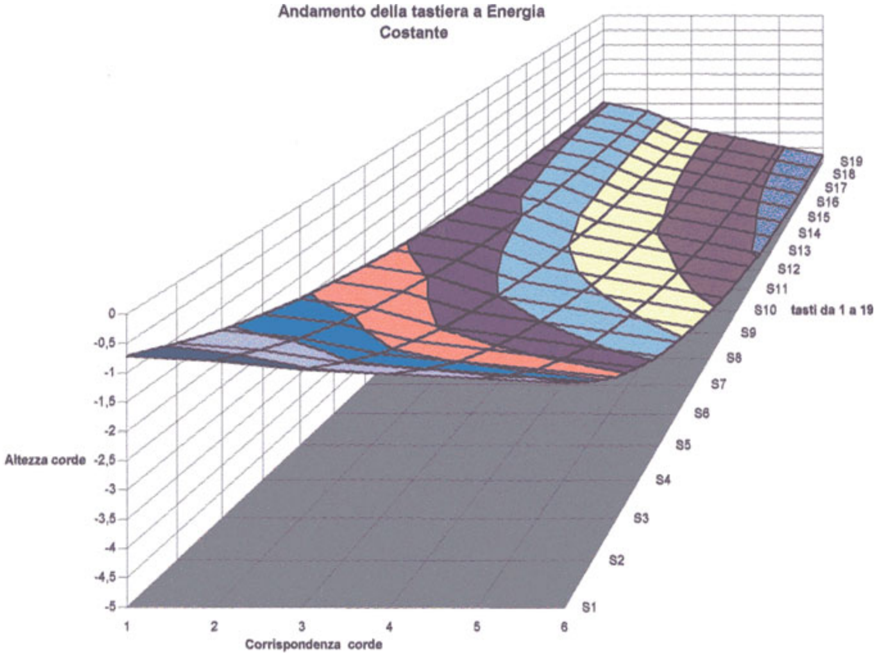
The parameters assigned to this fingerboard are: 12th fret clearance of 3 mm for the first string and 4.5 mm for the sixth string, the scale length being 650 mm.

Building this fingerboard is a very challenging task, considering that CNC machines are generally unavailable to luthiers, and shaping of the fingerboard has to be fully done by hand. The result is nevertheless noticeably superior in comparison with a flat fingerboard. In effect the latter, from the 10th fret on, leaves an excessive clearance between string and fret, making the performance harder. On the

contrary, the ‘constant energy profile’, thanks to its twisted shape, maintains the correct clearance up to the highest positions.

The diagram below presents:

– The string position (1st to 6th) on the horizontal axis
– The height of the strings on the vertical axis
– The position of the frets (1st to 19th) along the longitudinal axis.



9.3.6 The Bridge

This important piece of the guitar is usually considered a subsidiary component, as if with scarce acoustic significance. Besides its obvious string-fastening role, the influence of the bridge must be carefully pondered in the construction process.

Owing to its position in the centre of the soundboard lower bout, the bridge especially influences low frequency resonances. The vibration mode ⟨0 0⟩ has its antinodal area exactly at the centre of the soundboard, where this component is glued. The presence of the bridge tends to raise the natural vibration frequency of the soundboard, which is however partly compensated by the weight of the bridge itself. But this is unfortunately detrimental to the soundboard vibration system, so the mandatory target here is to thin the bridge down as far as possible.

Moreover, due to their horizontal extension, the bridge wings restrain the oscillation of the soundboard around its axis, so hampering the proper mobility of the soundboard in mode $\langle 1\ 0 \rangle$, which will be discussed in Chap. 11.4.

This mode usually plays a minor acoustic role, since it involves a dipole whose two vibrating areas operate in antiphase, this way producing scarce acoustic pressure (see pictures related to this mode in Sect. 11.4).

In order to avoid this undesired behaviour, a little asymmetry must be created in the dipole, by simply increasing the stiffness in one of the two oscillating poles.

Consequently, we must design the bridge wings so that they do not excessively hamper this vibration mode, which means they need adequate elasticity. For this purpose we advise using walnut, which combines proper hardness with high elasticity.

The authors have recently carried out successful experimentation of a wing-free bridge.

The bridge plate slot must be carved very carefully for a perfect fit with the saddle. We wish to remind that the saddle is the means of transmission of the energy stored in the strings to the soundboard, which will turn it into audible sound. Therefore, it is crucial to prevent losses due to incomplete adherence of the surfaces.

It also needs a slight backward slope to prevent the tension of the strings from pulling it forward.

9.3.7 The Saddle of the Bridge

Usually, the saddle is made of bone, since the rightful banning of ivory for wildlife preservation. Optimal sound transmission is achieved using *carbon fiber* material.

In the appendix we report the results of testing on different materials that are commonly used for guitar bridge saddles.

9.4 Selecting Woods

The right start to achieve outstanding results is, especially in luthiery, a pondered selection of wood for the soundboard.

Red spruce (Picea Excelsa) from the Italian and German Alps, as well as from Slovenia, is definitely ideal for the purpose. Paneveggio, in the Italian area of Val di Fiemme, is a location renowned for providing wood to great luthiers of the past. Western red cedar (Thuja Plicata) also yields good results, but has different acoustic properties that make it less suitable than spruce to produce the highest partials in guitar sound emission.

Summing up, the following tonewoods are generally recommended for the construction of a classical guitar **soundboard**:

- *Red spruce,* also called *resonance spruce*
- *American cedar*
- *Red wood* (Sequoia Semper Viridens)
- *Sitka spruce*
- *Engelmann spruce.*

and other, less valuable species.

All of the above mentioned are orthotropic woods. This means that, due to their growth ring vein running longitudinally, one can find significant differences in the elastic parameters of the wood, either in the transverse or the longitudinal sense.

The spacing and the thickness of the veins, hence the width of the annual growth rings, vary significantly from tree to tree, especially when they come from different locations: environmental conditions like altitude, exposure, and density of the forests influence the growth rate.

The next chapter reviews recommended parameters for the soundboard wood, and how to assess them.

The selection of wood for **back** and **sides** is highly conditioned by the quality/pricing ratio.

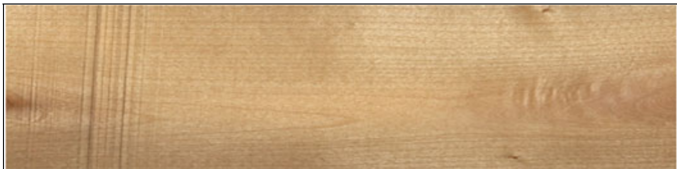
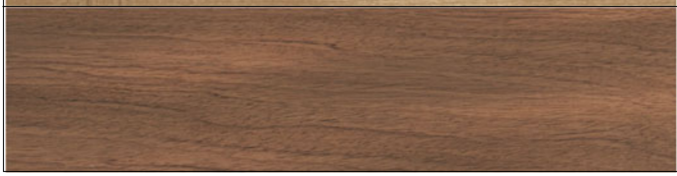
The absolute best wood is the one combining maximum stiffness to minimum weight. One of this kind is undoubtedly the Brazilian rosewood, in all its varieties.

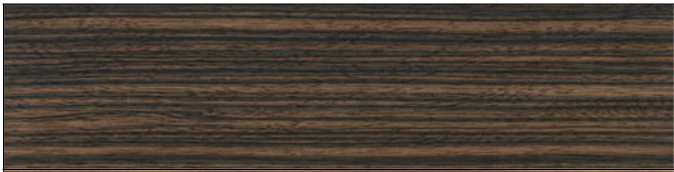
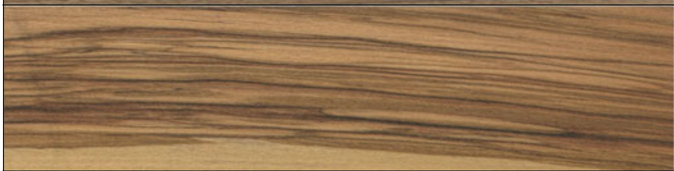
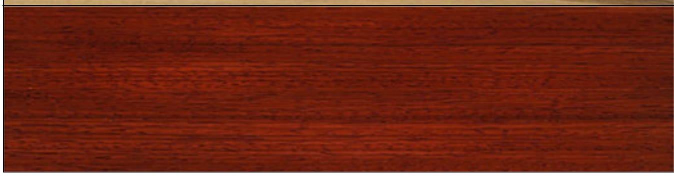
A good alternative is the European sycamore maple. Woods like Macassar ebony, Ziricote, Cocobolo, East Indian rosewood, and others, are mainly valuable for their appearance, though they do not match the quality of Brazilian rosewood.

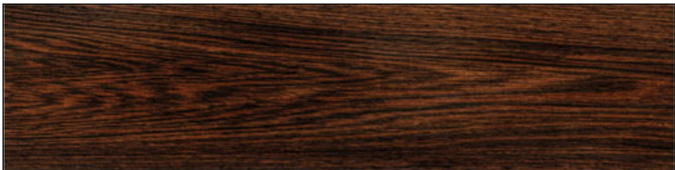
Unfortunately, the last one is too fragile. The sides often suffer longitudinal cracks; before bending, it is advisable to mend them by using an epoxidic glue, after which the lumber is ready for use.

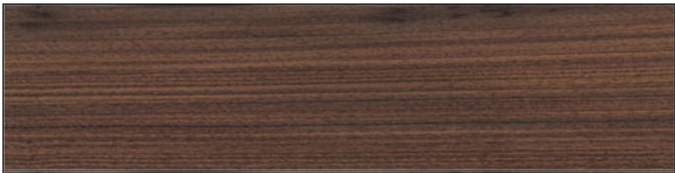
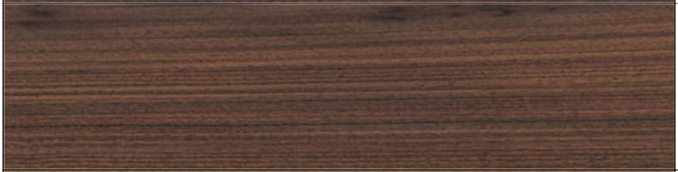
The wood for the **neck** should be chosen among the most immune to both size and torque deformation. This property is optimally featured in the African Mahogany, despite a seemingly unappealing look in its rough form. If we split it lengthwise and then interpose a strip of hard wood like ebony or rosewood we get a perfectly stable sandwich. Obviously, like all wooden manufacture require, they must be left drying at constant humidity for a long time, and possibly worked in two or three stages alternated by some extra drying days for perfect settlement.

Here are some pictures of woods commonly used in guitar building.

	Maple
	Bubinga

	Macassar Ebony
	Jacaranda
	Walnut
	Olive
	Paduk

	Brazilian Rosewood
	East Indian Rosewood
	Brazilwood

	Pau Ferro
	Pau Viola
	Western Red Cedar
	Zebrano

Inside the guitar are inserted various wood strips, called braces, whose purpose is to stiffen the structure, limiting weight at the same time. Especially for the soundboard bracing, spruce is very appropriate—provided that veins are perfectly straight and not too close; likewise the American cedar, which is a bit lighter, can work well.

For the tail block a virtually vein-free wood is advisable, owing to the required function: joining the sides at the bottom of the guitar. Excessively veined lumber like, for instance, spruce, could break along the vein endangering the integrity of the sides. The author normally employs the Brazilian Tulipwood, or similar varieties.

9.5 Characteristics and Parameters of the Woods

Among the different species of wood, those intended for the guitar soundboard need an especially careful evaluation of their physical parameters.

The wood for back and sides is mostly chosen on aesthetical basis. Nevertheless, it is convenient to know its density in kg/m^3, keeping it in mind for thickness calibration.

The main parameters of spruce, at least those relevant to readers and guitar makers, are:

- *Specific weight or unit weight "ρ"*
- *Mean spacing between veins*
- *Thickness of the veins*
- *Grain visibility*
- *Vein verticality.*

The specific weight (or unit weight) is easily determined dividing a sample weight by its volume. Since the weight of a wooden board varies significantly from the inner to the outer part of the log, we suggest testing the whole board.

Multiplying surface by thickness we get volume; dividing the weight of the board by the volume we get density. The soundboard surface adopted by the authors is 1473 cm^2.

The term *"**veining**"* indicates the alternation of *early* and *late wood* in the annual growth, consenting age estimation. Late wood has a noticeably darker colour than early wood. To know the exact width of the growth rings, measurements must be executed on the radial section, where growth stages show their actual thickness.

As for mass density, this is deeply influenced by the late wood thickness (the vein). For this reason, a mean value of about 400 kg/m^3 is eligible, since a lower value also implies low transverse stiffness. Transverse stiffness is in fact particularly dependent on the width of late wood and on the dimension of early wood.

The mean distance between veins is also important for a first sight evaluation of a spruce board quality. A well-known requisite for a good guitar board is vein regularity, save a small increase in spacing towards the periphery. Too thick veining patterns, with very narrow late wood bands, raise the board weight while not increasing stiffness. On the other hand, excessively scattered veining do not provide the transverse stiffness required for good acoustic results.

Large veins indicate a very long winter time and give the board a high stiffness, but also increase its weight.

So a middling width of the veins and of the early wood rings is the best condition for the board to offer good results in guitar building.

A mean width and spacing of the veins is therefore esteemed, respectively, in the range of about 0.5–0.6 mm and 1–1.2 mm.

The grain indicates the alignment of the wood cells, which normally run parallel to the axis of the trunk. As every experienced luthier has ever known, clearly evident grain at the surface of the board is definitely a sign of quality.

When wood is split instead than sawn, the trunk basically breaks following the natural arrangement of the cells which, as a consequence, move apart without rupture of the single units. This is obviously the most desirable condition in spruce meant for soundboard construction, as well as for braces and minor details.

The right grain direction is visible on the soundboard surface in the form of closely running waves (a few mm from each other), not always visible on the whole surface.

Finally, it is important that veins in the board run perfectly parallel, at least in its central part: this is the best condition in terms of transverse stiffness.

9.6 Parameter Control Methods and Criteria

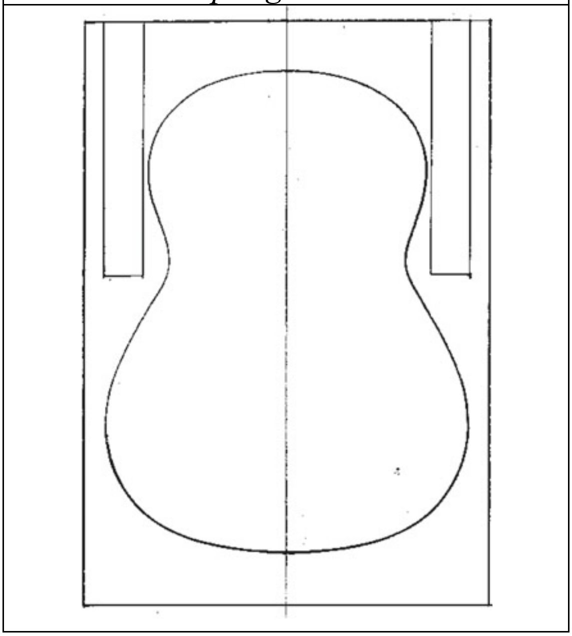
After choosing the soundboard on the base of the above-mentioned indications, two crucial parameters must be measured for board quality evaluation, namely:

The *elastic modulus* **E**, expressed in Newton/m^2
The *velocity of sound in wood* **C**, expressed in m/s.

Both can be measured either using appropriate electronic tools which exploit and measure the propagation velocity of ultrasounds in wood, or simply—and affordably to all guitar makers—by determining the deflection in a significant test sample of the board. The following sketch highlights the locations where the test samples must be extracted.

Test samples must be precisely calibrated in thickness, width, and length, then positioned on two sharp supports for the deflection tests. From the measured values we will then calculate the average. The load is a calibrated weight, laid down on the test sample in such a way that the interface between sample and load is minimal. At the bottom of the board a micrometer indicator will measure the deflection. In order to use the hereinafter reported formulae, we suggest to express all measurements in cm.

Sampling Location



As a reference, we report the following table of relevant parameters, and the values obtained from a sample bar.

Parameter	Symbol	Unit of measurement	Measured value
Width	**b**	cm	5.504
Thickness	**s**	cm	0.327
Length	**l**	cm	35.45
Deflection	**f**	cm	0.090

(continued)

(continued)

Parameter	Symbol	Unit of measurement	Measured value
Load	**p**	kg	0.200
Density	ρ	kg/m^3	454.52

Now we transfer these data into the following formulae (results in brackets).

$$\text{Moment of inertia (or rotational inertia)} \quad J = \frac{b\,s^3}{12} \qquad \left(0.0160376391\ \text{cm}^4\right)$$

$$\text{Elastic Modulus} \quad E = \frac{0.0208333\,p\,l^3 \times 9.8}{f\,J} \qquad \left(12\ 603\ 173\ 387\ \text{N}/\text{m}^2\right)$$

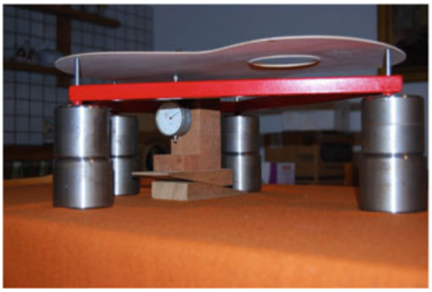
$$\text{Velocity of sound in wood} \quad C = \sqrt{\frac{E}{\rho}} \qquad (5265.79\ \text{m}/\text{s})$$

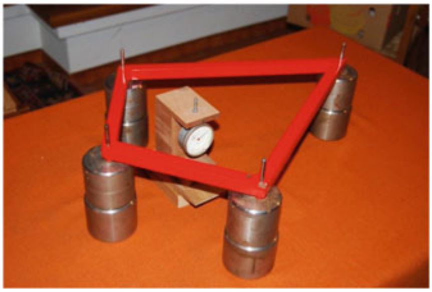
Once entered into an Excel spreadsheet, these simple formulae give automated response data, as the case requires, by simply changing the physical dimensions of the soundboard sample.

Obviously, boards with high values of **E,** and primarily of **C**, are recommended. This is because the board density value affects the resulting velocity C, so that the higher the value of C, the lower the board specific weight.

Good spruce boards can reach, in the longitudinal direction, values of E beyond 14.000.000.000 N/m^2 (or 14×10^9 N/m^2) and sound transmission velocity beyond 5600–5800 m/s.

Another important test we recommend is the measurement of the deflection in both longitudinal and transverse direction. Here again the board, resting on two supports, is tested along the grain and across. At the intersection of the two axes we place a weight—for instance 100 g—while a dial indicator placed underneath measures the deflection. Best boards obviously have smaller transverse deflection: this is generally to be found in boards where the grain runs at right angle to width when looking at the end of the board. Additionally, in best tops the fibers must run parallel to the surface in both of its halves, as a visual inspection can ascertain.





9.7 Evaluation of Rough Boards—Sample Testing Tools

In the previous paragraphs we explained the soundboard attributes required for best acoustic results. We will describe now the tests we carried out on a limited number of good soundboards, as they were all deemed by visual inspection.

The boards had been stored in the workshop for some months, and all were cut years before, so their aging was practically uniform. Analyses aimed at spotting the best boards by scientific methods, rather than just visually or through the "tap tones", were then executed.

The spruce boards, 12 in all, were firstly glued in the centre, as usual in the guitar board building process. They were then all calibrated to an exact 3 mm thickness, with the precaution to work upon both sides in order to ensure maximum uniformity. After that, the boards were trimmed to the same dimensions to favour comparison and avoid extra calculations.

We drafted a comparison grid, listing the following quality parameters:

- vein verticality
- longitudinal deflection
- transverse deflection
- run-out
- density
- number of veins per linear cm.

The longitudinal and transverse deflection was measured by means of a dial indicator placed under the board, a weight being positioned on the centre of the board upper side (at the intersection of the two diagonals).

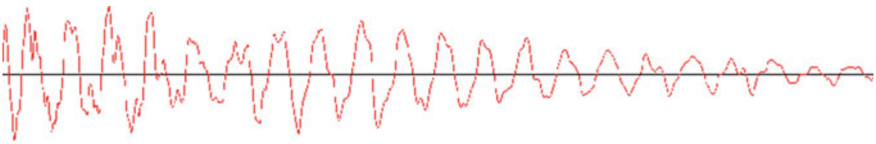
Run-out is the angle measured between the board surface and the wood grain. This is checked through observation of the grain direction in the board section. Density is expressed in kg/m^3. The number of veins per linear cm is the mean value calculated on half of the board.

This initial inquiry about the quality of the boards was followed by the analysis of the acoustic response, obtained by exciting each board in an antinodal area with the pendulum hammer described in Chap. 7, each board hanging in the same nodal point. In this percussion test, the FFT (Fast Fourier Transform) provided important information about the vibration propensity of these boards, about their harmonic content, about losses due to inner friction, and much more.

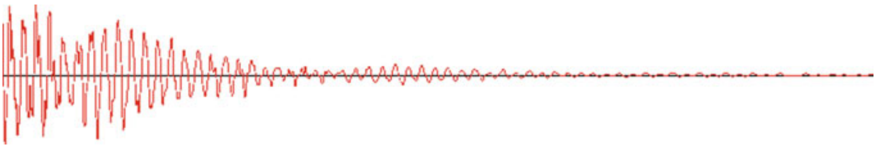
The first chart resulting from the percussion of the pendulum and the subsequent FFT analysis is the initial time response. What we notice in this response is a signal growth from 0 to maximum value in the span of about 0.5 ms, then the signal drops down to minimum value, reached after 1.5 ms. The width of the positive peak is related to the Q factor, hence to selectivity, and losses, of the fundamental oscillator. The width of the negative peak should equal the positive one if the system were perfectly elastic, or, in other words, if it could be able to shift continuously from the situation of potential energy storage to maximum kinetic energy, like an ideal pendulum. This does not occur in actual fact, both because of limited elasticity, and because of inner losses.

Considering the time response, we can say that the width of the positive peak is an indication of the board quality: the larger it is, the more effective the excitation of the board in contrast to its inner losses.

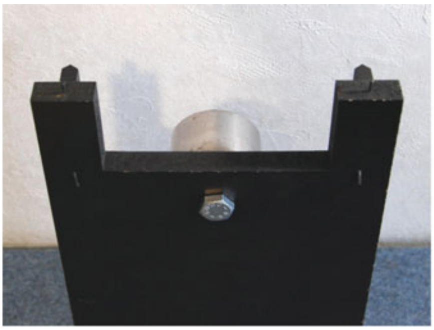
Another quality marker is the proportion between the amplitude of the two peaks, positive and negative: the more similar they are, the better the elasticity of the system.



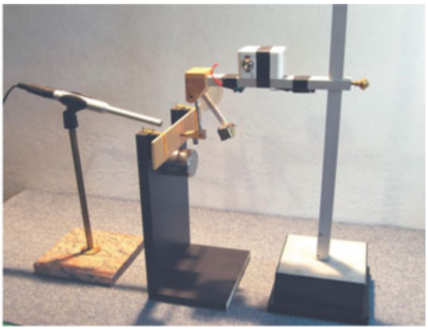
The time representation above illustrates the first 100 ms of the response. The following one covers instead the first 350 ms; here it is interesting to notice the amplitude modulation brought about by the beat of signals coming from the oscillators that compose the resonator.



After that we extracted two samples per board from the most relevant areas looking to the vein extension (see the previous paragraph). Subsequent to the usual calibration, the samples were attached to the testing structure shown in the photo, which holds them in a nodal area, with an antinodal area exposed to percussion.



Sample holder



Acoustic analysis of the sample

Record from rough board samples

Board	Sample no	Length (mm)	Width (mm)	Thick (mm)	Weight (g)	Density (kg/m^3)	Deflection (mm)	Load (g)	Distance between supports	'E' Modulus (N/mq)	Velocity 'C' (m/s)
1	1	270.1	41.75	3.04	15.3	444.6	0.55	200	260	13.363.445.309	5472.0
1	2	270.0	41.61	3.02	15.0	443.5	0.55	200	260	13.676.567.920	5553.2
2	1	270.2	42.25	3.02	13.9	403.0	0.67	200	260	11.056.967.063	5238.0
2	2	270.4	41.49	3.06	13.7	397.6	0.64	200	260	11.331.063.472	5338.4
3	1	270.3	41.95	3.07	14.7	422.0	0.53	200	260	13.400.944.222	5635.2
3	2	270.3	41.62	3.05	14.4	419.6	0.55	200	260	13.273.763.556	5624.4
Paneveggio	1	270.3	42.02	3.02	12.0	350.0	0.74	200	260	10.065.534.002	5362.9
Paneveggio	2	270.1	41.56	3.04	12.0	351.6	0.74	200	260	9.977.697.920	5327.1
35	1	270.3	42.10	3.04	10.7	309.3	1.06	200	260	6.876.218.189	4715.0
35	2	270.2	41.64	3.01	10.7	314.4	1.17	200	260	6.438.768.669	4543.0
40	1	270.1	42.28	3.00	13.3	389.6	0.68	200	260	11.105.821.918	5339.1
40	2	270.1	41.56	3.05	13.0	381.0	0.65	200	260	11.247.861.162	5433.4
41	1	270.2	42.30	3.03	11.0	317.6	1.14	200	260	6.426.658.992	4498.3
41	2	270.6	41.47	2.99	10.6	314.4	1.26	200	260	6.172.203.199	4430.8
X	1	270.3	42.33	3.03	15.3	441.3	0.54	200	260	13.557.775.772	5542.8
X	2	270.6	41.39	3.03	15.2	446.4	0.54	200	260	13.865.683.782	5573.3
XIV	1	270.2	41.22	3.03	12.1	358.5	0.84	200	260	8.950.415.602	4996.6
XIV	2	270.4	41.42	3.02	12.0	354.7	0.82	200	260	9.215.387.145	5097.1
XV	1	270.3	41.90	3.01	15.5	454.6	0.50	200	260	15.089.499.905	5761.3
XV	2	270.1	41.71	3.03	15.6	455.5	0.54	200	260	13.759.305.884	5496.1

(continued)

(continued)

Board	Sample no	Length (mm)	Width (mm)	Thick (mm)	Weight (g)	Density (kg/m^3)	Deflection (mm)	Load (g)	Distance between supports	‘E’ Modulus (N/mq)	Velocity ‘C’ (m/s)
XVI	1	270.2	42.00	3.03	12.6	366.0	0.73	200	260	10.107.839.209	5255.2
XVI	2	270.2	41.70	3.03	12.5	366.0	0.70	200	260	10.616.867.083	5385.9
XVII	1	270.8	41.75	3.04	13.3	387.0	0.82	200	260	8.963.286.488	4812.6
XVII	2	270.0	41.83	3.05	13.4	389.0	0.82	200	260	8.858.437.507	4772.0

The numeral repeated in the first column indicates that reported data refer to two different samples from the same board. The table highlights differences found in the 12 investigated boards.

The most relevant features are:

density "ρ", spanning from the minimum value of 311.85 in board 35 to 455.05 in board XV;

E *modulus*, spanning from 6.299.431.095 N/m^2 in board 35 to 14.424.402.890 N/m^2 in board XV;

velocity C, spanning from 4464,55 m/s in board 41 to 5629,83 m/s in board 3.

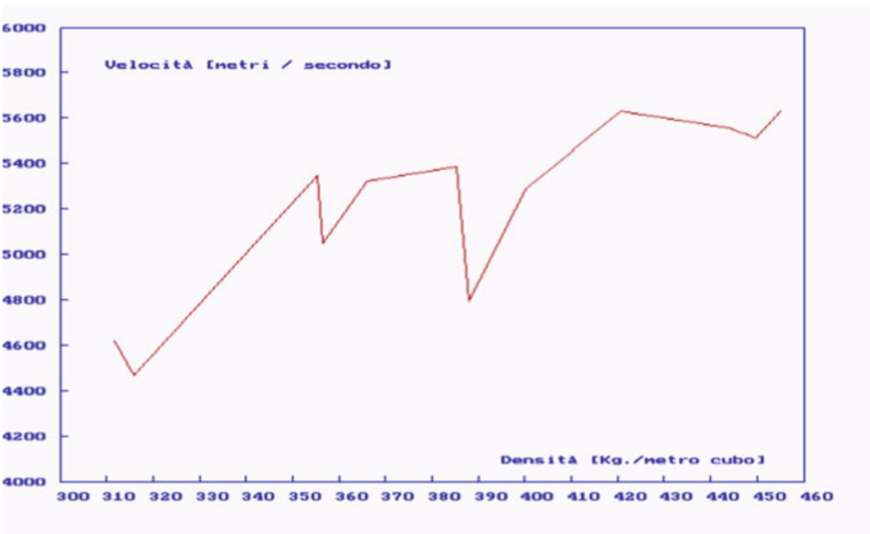
Discrepancies in data obtained from different measurements on the same board can be due to actual differences between the two sides, or to little errors in the manual calibration of the samples, executed by means of the micrometer calliper.

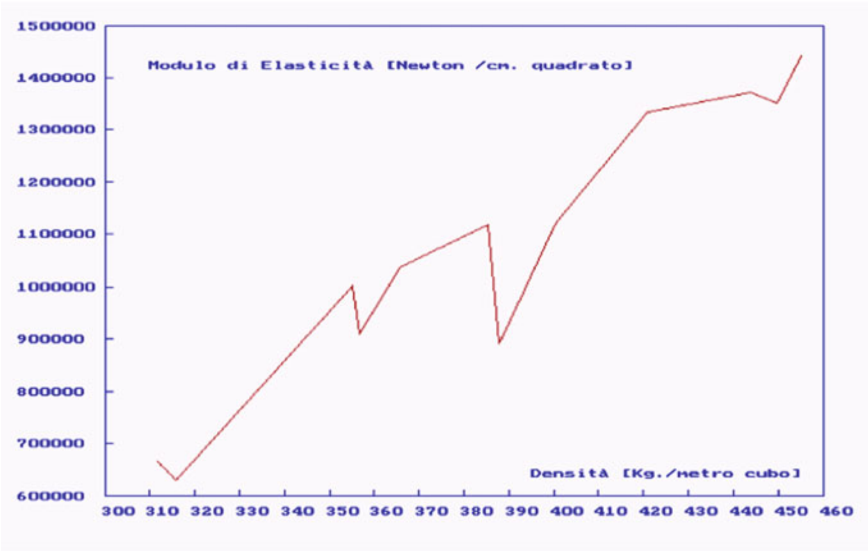
9.7.1 Practical Results of the Testing

The executed tests do not reveal any correlation between density of the fibres, namely the number of veins measured in a 100 mm large band beginning from the middle of the board, and the board specific weight.

On the contrary, the increase of the specific weight is related to a parallel though quite irregular growth of the E modulus, and to an increase in sound velocity along the vein.

The following diagrams highlight these features in the boards concerned.





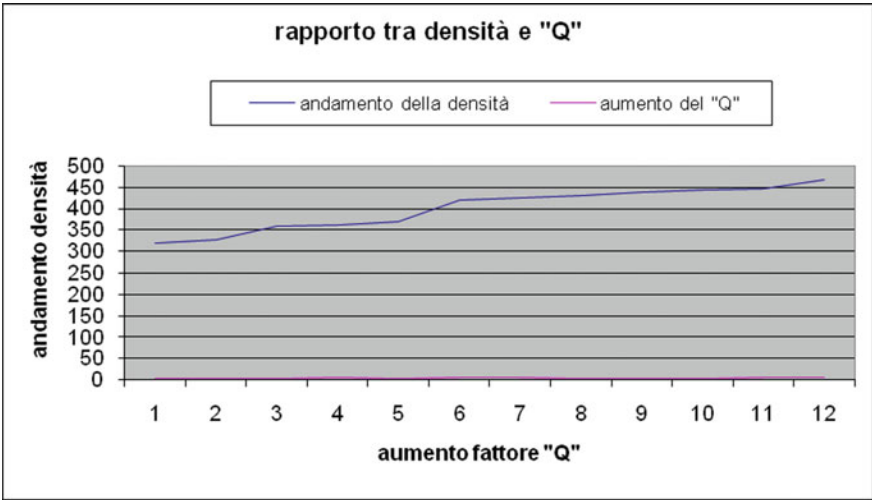
The two evident drops at 358 and 388 Hz must be ascribed to measurement errors.

Other significant information emerging from the analysis of rough boards is about the ratio between board density and quality factor "Q" (corresponding to the board resonance gain, hence to selectivity).

The quality factor "Q" grows along with density, though in a somewhat irregular manner.

This is evident in the following graph reporting:

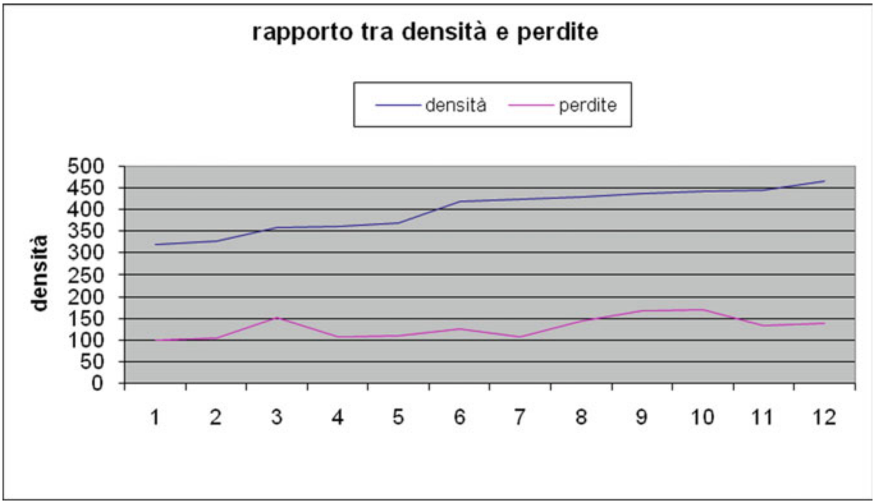
- The growth of 'Q' on the horizontal axle
- The density on the vertical axle
- The two lines in the graph highlight:
- Density development (blue)
- Increase of "Q" (magenta).



It is also interesting to notice that losses due, as mentioned, to radiation resistance and viscous friction between molecules, always grow along with density. In this case the growth is very limited.

See next graph highlighting:

– Density (blue line)
– Losses (magenta).



Chapter 10
Building and Using the Mould

Abstract The mould, as presented in this chapter, is a basic tool for progressive assessment of the quality in the guitar. On it, as a surrogate of the real sides, top and back can be mounted and the resonances can be evaluated through the acoustic hammer and FFT or, as an alternative, looking to the Chladni patterns. This chapter describes such a mould and provides several photos and construction details. Also given is an approach for building an electromagnetic exciter (to be used with the Chladni method) starting from a common speaker.

As often mentioned in previous chapters, most of the quality controls executed on the main components of the guitar (the soundboard and the back) are carried out by means of a ***mould***. This is a structure that provisionally substitutes the guitar frame, allowing to check these components with optimal accuracy, through quality controls relative to vibration modes and resonance peaks engendered during analyses. In effect the soundboard and the back are oscillators that, in order to work as designed, need to be fastened along their periphery, just as they will when glued to the frame—therefore to the sides—by means of the linings.

In order to reproduce this peripheral bond, we have built a plywood structure able to block the soundboard, the back or both, by means of two frames bearing each 11 bolts to provide the required pressure. The interior of the mould is the exact size as the guitar we are building, with ordinary linings glued inside. To protect the surface of the boards, the perimeter of the mould in contact with them was covered with artificial leather.

The use of the *mould* somewhat restricts the luthier's freedom to change too often the dimensions of guitars of its production, since making a mould for many guitar shapes may look like a laborious task. This is yet a low price to pay for the clearly conspicuous advantages that a regular use of the mould brings to the luthier.

The height of the mould must be equivalent to the mean height of the sides, in order to ensure the same air volume—between back and soundboard—as that contained in the guitar body. The mould can be provided with fretted neck and tuning machines, to consent an interesting preliminary test on the acoustic performances of the instrument at a very early stage of construction once the strings are mounted on the neck and tasted as in a real instrument. This is one of the plus points in the illustrated method,

G. Cuzzucoli and M. Garrone, *Classical Guitar Design*,
https://doi.org/10.1007/978-3-030-32992-1_10

because it allows us to verify the results attained at any time and, consequently, to carry out the adjustments required to achieve target parameters.

The clasp of the frames on back and soundboard should just be tight enough to block them; so a moderate fastening of the bolts is sufficient to prevent the boards from moving when tapped with the knuckles for a trial.

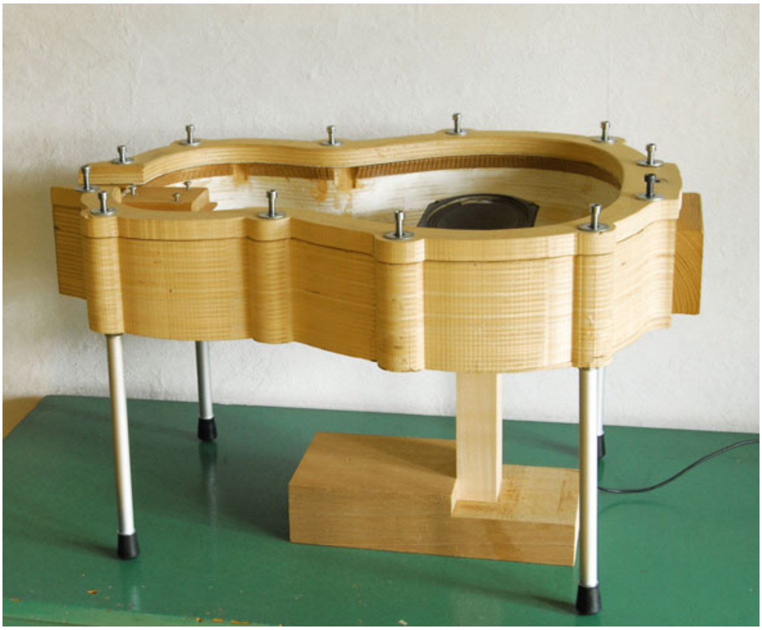
The interior of the mould can be plastered and varnished, in order to level the typical irregularities of plywood surfaces.

We definitely advise inserting threaded bushes for the bolts, otherwise normal wood screws would soon spoil the holes.

Two kinds of control are essentially possible with the mould:

- Analysis of the vibration modes, through the *Chladni method.*
- Analysis of the resonances, through the *FFT.*

In order to check the vibration modes, we must excite the board by means of a loudspeaker or, better, a specific electromagnetic exciter. In either instance, the mould must be laid horizontally on a flat surface, resting on supports at least 20 cm high. This is to prevent the formation of an air cushion that may interfere with the vibrations of the board.



The mould

When using a loudspeaker, this must be placed under the board and facing it. A little amount of not too fine-grained sand, or other particles deemed suitable by the luthier, is then spread over the board. Modulating a low frequency generator connected to the loudspeaker or the electromagnetic exciter, we will try to excite all

of the fundamental resonance modes of the soundboard. Preferably, the excitation should be operated under an antinodal area.

This is the simplest method, with the inconvenience that the whole board is simultaneously excited and, consequently, not all of the vibration modes will be thoroughly highlighted.

The same result can be achieved by means of an electromagnetic exciter, which can be easily built using a little speaker (about 10 cm in diameter) carried over from an old radio or TV set. The speaker must be cleared of the paper cone and the supporting metal framework, which can be easily removed using a suitable tool. We must take care that no metal particles, possibly generated by the cut, fall into the mobile coil compartment. However, if the speaker is intact, it is provided with an appliance that keeps the coil in place and sealed at the same time, just to prevent dust or other foreign objects from disturbing the normal oscillations of the coil itself.

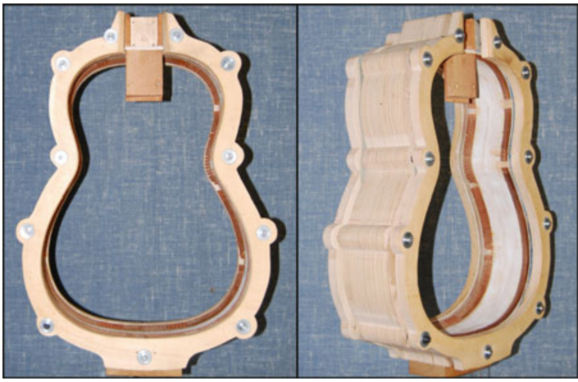
Lastly, a little truncated cone made of soft wood (e.g. spruce) and measuring about 10 mm at the base and 6 mm at the top is attached to the centre of the mobile coil. On this top we stick a piece of double-sided tape, serving to convey vibrations from the exciter to the soundboard or the back; in fact the exciter needs to be just in contact with the board under examination, exerting no pressure on it, in order to allow full oscillation of the mobile coil. The exciter assembled in this way will be fixed with bolts to a supporting bar (for instance a 30 mm$\times$ 15 mm aluminium profile), to be laid on the mould by means of two interposed wooden blocks of suitable height.

With respect to the loudspeaker placed under the whole board, the advantage in this method is that the electromagnetic exciter can stimulate carefully selected points, namely the antinodal areas of the modes we wish to highlight, by simply moving the point of contact.

To be executed with this method, called Chladni method after the name of the German physicist who first employed it, the modal analysis requires the use of a sinusoidal signal frequency generator, and a power amplifier providing at least 20 W. The frequency generators are available on the web in the form of software but, in preference, it is better to use an hardware generator, self made from some kit or purchased.

The amplifier can be taken from an old car stereo set or purchased in a specialized shop.

The second kind of analysis that can be executed with the mould shown above is the FFT of the response, obtained through acoustic hammer percussion of the soundboard (or the back) and captured by a microphone. This extremely important method allows a deeper inquiry into the vibratory features of the guitar components, as described in Sect. 7.1 (which illustrates parts, construction and usage of the acoustic hammer).



The mould

The electromagnetic exciter

10.1 Mould/Frame Conformity

We have used the above described mould for years and to great advantage, though there is no denying that the mould reproduces the guitar frame functions, but with some restrictions. In effect, the walls of the mould are much stiffer than the guitar sides, and so we must infer that the peripheral bond of the soundboard assumes slightly different characteristics in the two cases (mould and frame). Actually, the stiffness of the peripheral bond influences the mobility of the soundboard, especially in mode ⟨0 0⟩, which presents a *ring nodal line* running close to the bond itself. As a consequence, this mode can be partially affected by this condition, the nodal line slightly shifting towards the centre of the soundboard. The analysis of upper vibration modes is not so much affected, these being influenced by the inherent stiffness of the soundboard, rather than by the stiffness of the bond.

As will be more extensively discussed in Sect. 11.4—dealing with the use of the mould for board analyses—the most relevant data that these periodic controls provide are frequencies and amplitudes of the soundboard fundamental resonances. Discrepancies of a few Hertz in shortfall or excess—as can be the case between results obtained from the soundboard on the mould and those obtained from the soundboard glued to the sides—have actually scarce significance.

Therefore we can claim that this innovative mean brings great advantages, not only in the construction of musical instruments, but in designing them as well, understandably allowing experimentation of new and otherwise impracticable assembly methods.

As for the use, it is very simple indeed. Mounting the soundboard or the back on the mould just takes a few seconds (the time to tighten the 11 hex key bolts). That is why we advise using bolts of reasonable length, in order to avoid wasting time in long screw and unscrew operations.

Finally, an improved version of the mould can be realised by means of a small pipe, inserted into a groove carved along the mould profile and inflated at constant pressure, to ensure correct adherence of the boards. Advantages would be quicker operation and, above all, uniform pressure; this affords invariable sealing conditions and more reliable, reproducible data.

Chapter 11
The Soundboard on the Mould

Abstract This chapter introduces the concept of testing and tuning the top by using the mould, and measuring the resonances at various stages of construction. In order to prevent damages to the soundboard due to the pull of the strings, basic design criteria and a mathematical calculation (moment of inertia, dimensions) are given for braces. The chapter also presents no fewer than seventeen innovative, non traditional bracing patterns and their properties. Then the reader is guided through the progressive construction of two reference guitar—respectively with radial and lattice bracing—from the bare board to gluing the back. Every phase is described by several relevant plots, like response of resonances or thirds of octaves graphs implemented at any step.

In the newly built mould, slots must be carved along the linings for the insertion of the main braces, so as to comfortably lodge any soundboard. We also must take care to have the midline of the soundboard matching with the axis of the *mould*, which is then ready for use. As for the fixing bolts, we advise using hex key ones because these, holding the key, facilitate operation.

By means of this *mould* which— as described in Chap. 10—emulates the guitar frame, we will be able to check the construction of the soundboard, and then of the back, in order to optimize both before gluing them to the final frame.

Now, let us begin inspecting the soundboard of the guitar under construction, clamping it to the mould by means of the upper frame. As pointed out in Sect. 10.1, conformity between data collected from this setting and those obtained from the soundboard glued to the frame is adequate, though not exact. However, these little discrepancies do not invalidate the conspicuous advantages implied in the use of the mould.

Once acknowledged the benefits brought by the *mould* for the analysis of resonances—either obtained through vibrations engendered by the electromagnetic exciter, or by percussion of the pendulum hammer—we indentify passable frequency values in the band comprised between 50 and 1000 Hz, with their relative amplitudes. When the whole bracing is complete and roughly finished, the following resonances should appear:

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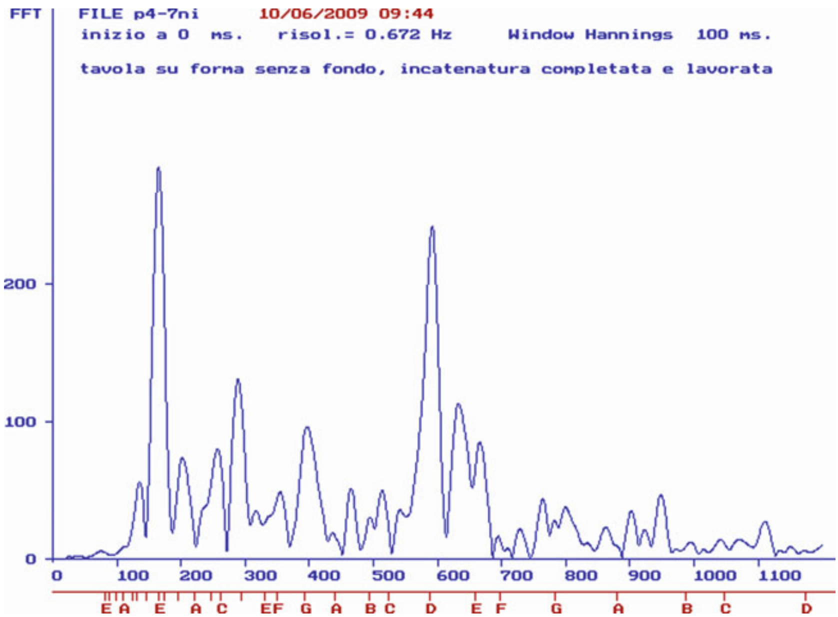
https://doi.org/10.1007/978-3-030-32992-1_11

- at about 150–160 Hz, with amplitude around 300, the resonance of the soundboard in mode ⟨0–0⟩;
- at about 300 Hz, with amplitude around 100, the resonance in mode ⟨1–0⟩;
- at about 400 Hz, with amplitude around 100, the resonance in mode ⟨0–1⟩.

Amplitude values are relative to a full scale of 400 points. Subsequent resonance peaks should occur, whose origin has been described in Chap. 6. Their presence is always crucial, since they define the propensity of the soundboard to vibrate at highest frequencies too.

Should the soundboard manifest its fundamental resonance at a lower frequency than the above mentioned, actions must be taken to further stiffen it, at the same time avoiding overload. Cedar wood, being lighter than spruce, can be profitably employed for the braces. After application of a non-rigid back, the resonance of the air in the body is expected to appear at about 90/95 Hz. The amplitude of this resonance indicates the propensity of the soundboard to set the air inside the resonator into oscillation. A very small amplitude would be detrimental to the resonance of the back, with obvious and undesired consequences. In fact, without vibrations from the back, a balanced and powerful instrument is not possible.

The next is the chart of a guitar soundboard analyzed under the above mentioned conditions, namely complete with bridge and finished braces, mounted on the *mould*, without the back.



In this graph, the resonance of the soundboard appears at 165.5 Hz and the subsequent, relevant resonances at 288.6 Hz (mode ⟨1–0⟩) and 396.3 (mode ⟨0–1⟩). The strong resonance at 591 Hz is among those described in Chap. 6—dealing with upper resonances. This is enough to infer that here certain resonances of the soundboard are coupled with specific vibration modes of the air. Quite understandably, this feature has been found in every acoustically valuable guitar.

Anyway, resonance peaks revealed by the FFT analysis indicate that the soundboard, when hit by the acoustic hammer, positively responds to the mechanical impulse: in fact, the more and larger are the peaks, the louder and more balanced will be the guitar.

The advantage in applying these controls, and the consequent opportunity to adjust elements under construction, is clearly significant in consideration of efforts aimed at improving the instrument. Since the soundboard is mounted on the *mould* with the back open, we will be able to modify its stiffness, to change the arrangement of the braces for a better distribution of sound, to experiment with new structural solutions and develop new manufacturing techniques inspired by the practice.

Before fixing the final back—which may be still unavailable at this stage of construction—a rigid back can be mounted on the mould. As described when talking about its construction process, the rigid back is just a plywood panel, about 18 mm thick, obviously applied on the side of the mould opposite to the soundboard by means of fixing bolts.

The rigid back closes the resonator structure, which is then able to manifest its inner air resonance, called *resonance of the body*. This resonance, because of the stiffness of the back, should appear at about 100 Hz.

Because of its inflexible nature, this kind of back does not go into oscillation when the board is hit or otherwise vibrated, and the typical resonance peaks of the back—normally spanning between 200 and 350 Hz—will not be displayed in the diagram of the soundboard.

11.1 Bracing Design Criteria

The ***bracing*** is the whole set of wood strips (braces) glued inside stringed instrument soundboards. The name originates from the tradition of manufacturers, who used it to define a crucial function: strengthening the soundboard in order for it to withstand the pull of the strings and, therefore, both static and dynamic stress typically occurring in musical instruments.

Making thicker soundboards partly answered the problem in earliest times. Nevertheless, getting acoustically accurate structures was that way impossible. Their sound was in fact very feeble, even though excellent and balanced. However, guitars were played at that time in small locations, where their loudness was enough to satisfy

the audience. A louder sound was later asked from guitars, along with better sustain and overall performance features, especially when composers began including guitar solos in orchestral concerts.

The solution experimented by the great luthiers of the late 1800s and early 1900s was making thinner soundboards, to be subsequently strengthened with strips of the same wood—arranged in a fan bracing pattern, or differently—in order to increase loudness and sound projection.

The adoption of nylon and new polymers for the strings, either single threaded or wound ones, obliged constructors to look for even stronger bracings, able at the same time to bring advantages, rather than inconveniences, to the quality of sound. This led to experiment with very particular bracing patterns—in the effort to strengthen larger areas of the soundboard. Recently new materials—like carbon fibre, fibreglass, and others—have been successfully exploited. Avoiding deeper argumentation, we just point out once again that selected spruce, and a proper bracing, afford optimal results in solidity and—most important—in sound quality.

As formerly stated in the chapter about wood selection, the timber destined to the construction of the soundboard, as well as for the braces, must be properly aged and have regular veining, according to the individual growing features of the tree; the veining should be orthogonal to the soundboard surface, as far as possible, and free from evident flaws, especially for aesthetical reasons. For soundboards, Red spruce or American cedar are normally the favourite choice.

The thickness of the soundboard can be comprised between 2 and 2.5 mm. Slighter thicknesses are not advisable, being the source of different kinds of problem.

Before building a soundboard, we must be conscious that aging and ambience variations affect the behaviour of wood. Figures relative to resistance reported in databases can diverge considerably, when wood undergoes long deformations over the time and does not completely recover its elastic deflection after removal of the load. This hysteresis rapidly grows over time until it reaches a certain steadiness. Lab tests established that the maximum load for elastic deflections is about 50% of the rupture load (UTS: ultimate tensile strength).

The direction of the grain with respect to the axis of the soundboard is also very important, with reference to elastic resistance. For instance, a 20% angle of the grain reduces the elastic resistance by 50%, and so the utmost care must be taken in the selection of wood for soundboards and braces.

As regards the time of application of the load—which in the guitar is virtually constant—we must take into account that a load applied for one minute on a wooden sample, without reaching the breaking point, must be reduced to 62% after a yearlong application. In addition to that, the load must be further reduced to 57% after 10 years, and to 53% after 50 years. Consequently, for practical purposes, we should keep this in mind: any permanent load must be reduced by 56% of what is able to break the board after a 5 min long application.

But determining the rupture load, either by deflection or axial pressure, is very difficult because of intrinsic factors like grain angle, load angle, sample temperature, and others. For that reason, high safety coefficients must be always applied to the wood we are using.

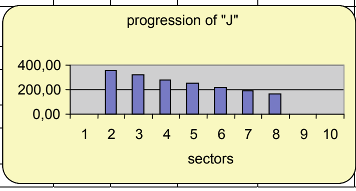
Upon this premise, and going back to the bracing—this time merely considering its static properties—we suggest some formulae to determine the minimum moment of inertia "J" required for the soundboard to withstand the permanent pull of the strings. Initial calculations were executed by means of a FEM model in a university campus we asked for support, aiming to calculate the moment of inertia of an Engelmann guitar soundboard which should not suffer deformation beyond 0.1 mm (at the surface near the bridge) under a string tension of about 47 kg. The simulation gave an outcome of 1342 J/mm^4.

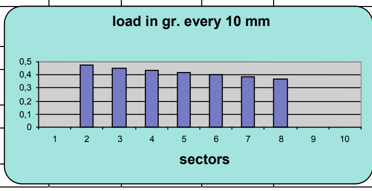
We report hereafter a table that, when transferred into a spreadsheet, automatically provides the correct measures for the braces by just entering the following data established a priori: thickness of the board, number of the braces, extent of the areas covered by the braces. Our practice suggests that the calculated values must be raised by about 50% because of the above mentioned reasons; the resulting values must be regarded as maximum limits for the height of the braces, not to be exceeded when lowering them to adjust their stiffness and reach the proper resonant frequencies.

In order to use this spreadsheet we must first establish, as before stated, the desired soundboard thickness. Supposing a Torres style bracing composed of 7 braces—the central one also working as joint cover—we have seven underlying areas on the soundboard surface. Sectors 1-9-10 are absent when adopting a 7-brace pattern, therefore very little values were entered there, just to let the program work. Values are expressed in mm.

In the first column (Sectors) we have the corresponding numeral of each band; in the second column, the width of these bands which, in this example situation, are all the same size; in the third column, the thickness of the soundboard (mean thickness must be calculated from different measurement points if we do not work the soundboard using a precision calibrating machine). In the fourth column the width thickness of the braces, and in the fifth column their height. Once values in the preceding columns are set, this last input will provide the final outcome. As a consequence, the 11th column displays the resulting values of "J" corresponding to each of the sectors and, at the bottom of the same column, we find the sum of the whole data about the soundboard. It is advisable to increase this value by 50%, considering the specific physical attributes of each soundboard, and the risk of failures due to protracted load.

MOMENT OF INERTIA IN SOUNDBOARDS WITH BRACES OF RECTANGULAR AND TRIANGULAR SECTION										
Sectors	Width	Thickn.	Base of the brace	Height of the brace	Overall height	Xgr axis	X values	Xgt axis	Xgt axis	Jg rectang.
	b	s (h)	b∞	h∞	H	Xgr	X	h1	h2	Jgr
I	0,1	0,10	0,1	0,1	0,2	0,1	0,041667	0,09167	0,00833	0,00
II	35	2,80	4,0	5,0	7,8	2,0610	0,519774	1,91977	0,88023	358,33
III	35	2,70	4,0	4,8	7,5	1,9832	0,498153	1,84815	0,85185	318,68
IV	35	2,60	4,0	4,6	7,2	1,9055	0,476539	1,77654	0,82346	282,07
V centr.	35	2,50	4,0	4,4	6,9	1,8277	0,454932	1,70493	0,79507	248,37
VI	35	2,40	4,0	4,2	6,6	1,75	0,433333	1,63333	0,76667	217,48
VII	35	2,30	4,0	4,0	6,3	1,673	0,411744	1,56174	0,73826	189,26
VIII	35	2,20	4,0	3,8	6	1,5946	0,390166	1,49017	0,70983	163,59
IX	0,1	0,10	0,1	0,1	0,1	0,1	0,041667	0,09167	0,00833	0,00
X	0,1	0,10	0,1	0,1	0,1	0,1	0,041667	0,09167	0,00833	0,00
										1777,78





Total weight
2,94304
every 10 mm

For the reader who patiently considered the directions for usage of the previous table, we also list below the required formulae. In the first five columns, and along the first sector line, namely the 5th line, data are set by the luthier, so no formula is applied here.

On the same line, the following formulae must be entered:
in the 6th column: = C7 + E7
in the 7th column: = S7/T7
in the 8th column: = P7/Q7
in the 9th column: = C7/2 + HZ
in the 10th column: = C7 − I7
in the 11th column: = (B7*G7^3 − (B7 − D7)*(G7 − C7)^3 + D7*(F7 − G7)^3)/3.

The program should automatically apply the formulae into the subsequent lines as well. Columns 8-9-10 contain processing formulae of the worksheet, and their values can be ignored.

In designing the whole structure of the soundboard, braces included, the need for lowest weight and, at the same time, adequate solidity, must be fulfilled. Lightness is essential for the soundboard to vibrate at highest frequencies. Logically, the heavier the soundboard, the stronger its opposition to accelerations due to impulses coming from the strings; this especially occurs at high frequencies, where oscillations are closer in time. Therefore, if the soundboard is heavy, its inertia is high and its motion

start-up slower. Furthermore, greater mass means greater amount of wood set into vibration, resulting in greater losses during transmission of the kinetic energy from the strings to the resonator. Actually, during vibration, wood cells rub against each other, this way reducing the available energy because of the viscous friction inside the vibrating mass. So, the greater the amount of material set into motion, the greater the losses. Friction losses reduce selectivity and amplitude of the resonances as well and, moreover, lower sustain.

The bracing must be designed so as to confer the soundboard all the characteristics required for this crucial element of the guitar to oscillate, at all frequencies involved in the instrument operation, with visible amplitudes and good overall parameters.

The back as well requires an equally important and demanding, though simpler, design. In fact, the traditionally employed transverse braces serve not only to confer the back a correct arching, but also the necessary stiffness to let it resonate at the proper frequencies.

This being a crucial issue in itself, the different bracing patterns for the back will be described in a subsequent and specific chapter.

Now, as regards the soundboard, the target is to distribute to the whole surface the energy stored in the excitation area—i.e. at the bridge location.

In summary, the arrangement of the braces on the soundboard must fulfil the following requisites:

1. Provide adequate reinforcement, to endure static and dynamic stress brought about by the pull of the strings during the life of the instrument. Resistance is measured in terms of moment of inertia "J", which can be calculated by means of the previously provided formulae.
2. Distribute the energy coming from the bridge to every point of the soundboard, including most distant areas.
3. Compensate, through some proper arrangement of the braces, the unequal propagation of sound across the soundboard, in the longitudinal and transverse direction, due to the orthotropic nature of wood.
4. Establish asymmetries in all of the symmetrical vibration modes, to get adequate sound pressure.
5. All of the targets above must be always pursued with the main vibration modes in mind, in order to intervene as required for their optimization. It is highly advisable, at this stage of work, to frequently check the formation of the vibration modes, exciting the soundboard by means of the electromagnetic exciter, and highlighting them with the scattered dust and the Chladni method.
6. Always remember that, along with stiffness, adding wood strips also increases the mass. Growth in stiffness raises the resonant frequency of some fundamental modes, while mass increase brings it down, as seen in the first part of the book. Therefore, we advise making very light braces, reducing their thickness in favour of height, and possibly using cedar wood rather than spruce.

A classical guitar soundboard can be correctly manufactured keeping a minimum thickness of about 2 mm. Lower thicknesses have also been tested but, under 1.8 mm, and despite employing top quality boards, an annoying phenomenon is likely to occur:

with the customary and normally efficient bracing patterns, the areas between the braces are not sufficiently stiffened and, as a consequence, they vibrate independently, this way generating undesired vibrations.

Let us now review the most common, and once most common patterns for the bracing of classical guitars, first of all the traditional Torres style, either with 5 or 7 braces slightly diverging towards the bottom of the soundboard. They can also be enclosed by two transverse wood strips at the bottom—as if to encircle the vibrating area.

In our opinion, on the basis of a lot of analysis we have executed on guitar soundboards, encircling an area of vibration is pointless. Besides, these wood strips are normally placed just a few cm. from the edge where the soundboard is glued to linings. Experimentally we found that there are vibrating areas of some high pitched resonances located near that edge, and therefore these wood strips would not be merely ineffective, but also detrimental.

Another drawback of the 'Torres style' design of the braces, is that actually it emphasises the orthotropic nature of the soundboard rather than opposing to it. Sound velocity in a spruce soundboard goes from 5600 to 5800 m/s in the fibre length, and from 800 to 900 m/s in the transverse direction. Each of the wood strips applied transversely—and departing from the bridge area—dramatically improves the conveyance of vibrations generated by the strings to the lateral areas of the soundboard. It is routine for the luthier to place frequently the tuning fork on the soundboard, and verify sound increase in correspondence with the position of the braces. Upon this reasoning, bracing patterns with transverse reinforcing wood strips (e.g. fan brace patterns diverging from the bridge) are logically more advisable.

The braces placed under the bridge can be broken or unbroken. Unbroken braces, like those of the Torres bracing style, deliver a more tympanic sound. The attack is faster, but so is also the decay of sound. On the contrary braces interrupted at the bridge location favour a softer attack of the oscillation, and better sustain.

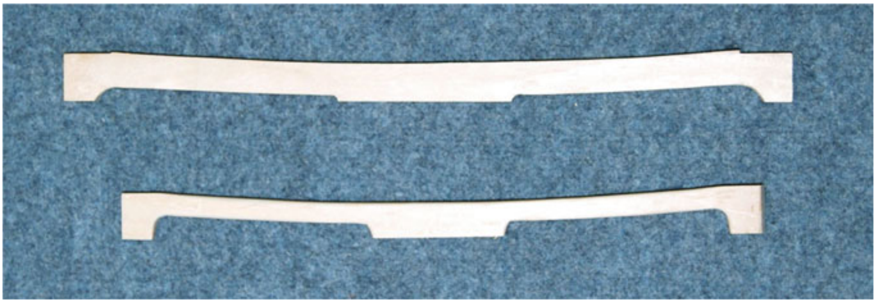
Output can be regulated by varying how much the wood strip ends cover of the surface under the bridge plate. This method is also advantageous to a critical vibration mode of the soundboard, namely mode $\langle 0\ 1\rangle$: this mode appears when the two areas of the soundboard, respectively above and below the bridge, oscillate in antiphase. In this situation, the dipole represented by the two parts of the soundboard must be rendered asymmetric, to prevent the formation pressures equal in amplitude but opposite in phase which, in fact, would reduce sound to nothing. This phenomenon is easily verified by means of a piezoelectric sensor able, when set in contact with one of the two halves of the soundboard, to measure its displacements, and a microphone picking up the incoming sound. The accelerometer will detect a lot of action while the microphone almost none, unless the dipole is sufficiently asymmetric or, in other words, the net vibrating surface is much different from zero.

The same problem arises when the soundboard oscillates according to another important mode, generated around its vertical axis and called $\langle 1\ 0\rangle$. Here again, if perfectly symmetric—for instance braced in a Torres style and with uniform thickness—the two halves of the soundboard engender counterbalancing oscillations that mutually nullify.

The problem can be answered by just varying the thickness in the two halves or, even simpler, applying higher braces in the treble area, to get an asymmetrical dipole. This way, the two vibrating surfaces that define the dipole have different extent and, as a consequence, generate greater sound pressure. This concept applies to any kind of bracing, based on the fact that a contact sensor only measures soundboard displacements occurring in the point of contact, while a microphone placed at a certain distance detects the overall sound pressure of the instrument, including the amount that comes from soundhole, back, and sides.

The main brace under the soundhole represents, just like the linings, part of the soundboard peripheral bond. We advise a bridge-shaped profile for this brace, with wings lifted at the sides of the soundhole and only glued at the ends and the centre.

A 2.3 mm thick soundboard, with a deflection in the middle of 0.75 mm under a 500 g load, seems to afford best results.



An example of bridge-shaped braces, respectively above and below the soundhole.

Deflection can be simply measured by means of a micrometer indicator, the brace ends resting on two sharp supports, and the load weighing down on the centre.

The bridge-shaped brace also allows to extend at least two of the fan braces beyond the soundhole, and even up to the sides of the neck end. For this purpose we suggest to bridge-shape the **brace above the soundhole** as well, with the double target of leaving way for the long braces and preventing cracks, which may occur over time in the soundboard at the gluing with the fingerboard. It must be noticed that spruce shrinkage, due to aging and low humidity ratio, is essentially restricted to the transverse section.

A spruce soundboard stored in damp location for some days, and then moved to a dry location, can shrink by up to 2–3 mm transversely, while imperceptibly in the longitudinal sense. Therefore, the wood strips glued across the vein on flat areas of the soundboard can generate very evident fractures. This is less likely to happen with the wood strips glued on the larger and usually slightly bulging part of the soundboard, where shrinkage is compensated by reduction of the arch.

Spruce strips employed for the braces must be selected among the best material available, with perfectly straight and regular veining, lowest density, and rectangular section. American cedar, lighter than spruce, is excellent for the purpose. The triangular section is actually pointless, though largely employed by luthiers, since maximum

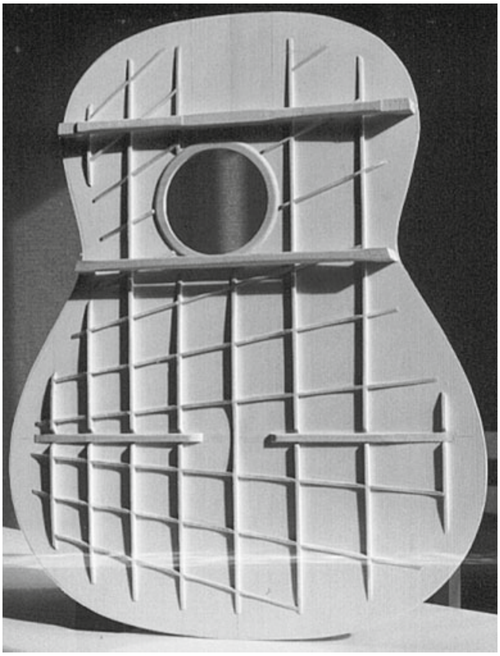
stiffness with lowest weight are achieved by means of a rectangular section, the highest possible with respect to thickness. It is not by chance that metal beams used in masonry have a rectangular profile, profiled in the centre at both sides: they are called double T or H beams. In theory, the same could be done on both sides of the braces to lower weight, though obviously implying damage hazards for the wood strips, which are very light by nature. In practice, for the fan braces the thickness of the wood strips can reduced to 3 mm, with no risk of gluing failure. Initially, the wood strips must be kept much higher than requested by static calculations of the moment of inertia; the stiffness will be regulated through step by step reduction, till achievement of the established resonant frequencies.

As a general rule, the best bracing combines maximum stiffness with minimum weight. Luthiers all over the world have tried innovative materials to pursue this target: for instance, ultra thin carbon fibre sheets inserted between two light wood layers (e.g. from Balsa timber).

A less conventional but undemanding construction method and, at the same time, a highly efficient one, is the *radial* bracing shown ahead, with two of the braces extending beyond the soundhole up to the sides of the neck end, and running under the two main braces (bridge-shaped).

Also very efficient is the *lattice* bracing, where the wood strips intersect with each other, forming a series of irregular, trapezoidal polygons all over the soundboard inner surface. The size of the polygons is not constant, but increasingly larger from the

area of treble tones towards bass tones. This is to render asymmetrical the vibration modes generated during low frequency oscillations.



The next paragraph shows many soundboards, all of which were actually used in completed instruments, exemplifying the most employed pattern categories in classical guitar bracing. Furthermore, the main construction parameters of two very significant examples will be provided.

11.2 Bracing Styles and Characteristics

A set of pictures will be hereafter shown for reference, concerning actually realized soundboards.

The bracing styles displayed here trace the road to our present opinion about the subject. Each of them seemed in turn to be the best ever when first designed, and their acoustic results proved in fact to be satisfying.

But in guitar making, like in all human achievements, one is never fully satisfied. So, large and small differences between the illustrated bracing styles testify the attempt to improve results already achieved, viewing them like a collection of facts to be constantly interpreted and corrected. Obviously, examining these facts and linking results, whether positive or not, to different bracing patterns or different thicknesses of both soundboard and braces, was the most challenging part. Sometimes some

choice was taken, seemingly the best one at that moment. Later on, after adopting specific analysis tools, the final quality of the instruments constantly grew, reaching outstanding standards.

In designing the bracing for a soundboard, we must keep in mind what is requested from it. The soundboard, as already observed, opposes the string tension—on average corresponding to a 47 kg load. At the same time, it also works like a conductor of the energy applied on the bridge by the oscillations of the strings, in order to generate enough sound pressure as to render vibrations audible, as loud and linear as possible. While the static part of the design is actually unproblematic, being based on available formulae for correct calculation of the moment of inertia, the dynamic part is a very tough issue—provided that excellent results are what we expect to obtain.

First of all, the guitar soundboard must be as thin as possible, to keep the induced oscillations from damping too fast because of simultaneous losses, which tend to gradually minimize them. These two kinds of losses intervene: the first, beneficial to sound, is due to *radiation resistance*, i.e. the resistance encountered by the soundboard surface when vibrating against the air; the second is due to inner friction losses, in turn depending on the thickness of the wood. Furthermore, different loss rates are generally found between equally thick soundboards, owing to the influence of their individual quality on this parameter.

Spruce boards used in guitar making are orthotropic (with veins running in the longitudinal sense). This, as commonly known, causes the different sound transmission velocity in transverse and longitudinal direction. The bracing, in our opinion, must help narrowing this difference, in order to maximize the available surface of the soundboard.

As the vibration frequency changes, the soundboard—bound along its border by the gluing with sides and linings—oscillates in many different modes represented by the vibrating (antinodal) areas, which are defined and separate by nodal areas. For instance, at the fundamental frequency (the lowest one), the oscillation of the soundboard forms a vibrating area in the middle, almost as large as the whole lower bout, and encircled by a nodal line that runs close to the border. As frequency grows, the vibrating area splits into two parts that oscillate in antiphase with each other, again and again. So, the bracing must support these oscillations up to the highest frequencies, where the vibrating areas multiply while reducing in size. We recall the meaning of the symbols adopted: vibration modes are identified by two increasing numerals, each respectively representing the number of longitudinal or transverse nodal lines: $\langle 0\ 0 \rangle$, for example, designates the vibration mode of the soundboard fundamental frequency, the mode with no nodal line. On the contrary, the symbols $\langle 1\ 0 \rangle$ indicate the presence of a longitudinal nodal line, and none transverse, while the symbols $\langle 0\ 1 \rangle$ refer to the vibration mode with a transverse nodal line, and none longitudinal.

We recall the scheme of vibration modes to remind that the arrangement of the braces on the soundboard is able to influence, either positively or negatively, the formation of these modes. It is the designer's task to identify any problem arising from the position of the braces and, consequently, search for a solution.

For instance, the traditional Torres style, which is still nowadays often employed, partly impairs the formation of vibration modes with transverse nodal lines, or at least reduces their oscillation amplitude. In fact, when braces run parallel or just slightly angled to the direction of the vein, they enhance the already stiff condition of the soundboard due to the veining itself, so that the oscillations of modes $\langle 0\ 1\rangle$ and $\langle 0\ 2\rangle$ only rely on the potential pliancy of the braces. If this is the kind of bracing pattern we want to employ, we must at least *break* the longitudinal braces at the bridge location, letting them overlap inside its outline to a certain extent. This way we also get a better sustain, whereas unbroken braces bring about a tympanic and short lived sound.

In addition to that, the traditional shape of the bridge hinders vibration modes presenting one or more longitudinal nodal lines: here the oscillation amplitude of the soundboard tends to follow sinusoidal patterns with minimum values along the longitudinal axes; at the same time, peaks that would occur in the oscillation right and left of the transverse axes are hampered by the bridge, which acts like a transverse brace. Further on, we will suggest different bridge shapes designed to minimize this inconvenience.

The above mentioned vibration modes, in the analysis charts obtained through the illustrated tools and software appear as resonance peaks with variable amplitude. It will be the manufacturer's task to get this peaks as high as possible, and occurring at the proper frequency: to do that, first of all the formation of these resonances must be accurately traced, using the electromagnetic exciter in combination with the Chladni method and trying—through rearrangement or calculated size adjustment of some of the braces—to improve their amplitude and position on the diagram.

We remind what the final quality control of an instrument is meant to verify: (1) the presence of a good and constant mean emission level over the whole scale of the fundamental tones; (2) the absence of tones noticeably feeble, or unequal in sound duration with respect to the overall trend of the scale; (3) that natural harmonics are loud and easy to execute, with the most uniform timbre possible between strings; (4) last but not least, the instrument must be efficient when either played very softly or very loudly.

Let us now see which bracing patterns can be employed with best results, beginning from a Torres style. As previously mentioned, this kind of pattern can be improved through little precautions; in any case, we advise to spread the braces apart more than the original model (for a better exploitation of the soundboard surface) and, also, to slim down a little the bridge wings.

On the contrary, we advise against bracing patterns that restrict the vibrating areas, at least in the planning stage: this because, in the high frequency range, every sector of the soundboard manifests little vibrating areas which can be hampered by too many braces—even if small sized—placed in those locations. By the way, we remind that each of the braces affects the formation of the vibration modes, expanding some of their vibrating areas while reducing others. This mechanism allows us to overbalance the symmetric areas of some modes, in order to get higher emission levels.

One of the bracing styles—the radial pattern shown in the previous paragraph—has proved very efficient. Here the role of reinforcing the structure is mainly appointed

to the central braces, running basically in the longitudinal direction. But the radial braces—spreading out from the middle of the bridge base outline—effectively convey oscillations even to the peripheral areas of the soundboard, better exploiting the limited operating surface of the table. In this kind of patterns the main braces are *bridge-shaped*, in order to leave way to a couple of long braces, designed to transfer energy from the bridge to the upper areas of the soundboard, where high-pitched resonances are located.

The elasticity of the (bridge-shaped) main brace under the soundhole must be checked and regulated to a deflection in the middle (obviously measured before gluing) of about 0.75 mm under a central load of 500 g. The radial arrangement can be varied to offer best performance and, also, to fit with the shape of the soundboard. A typical example is the double soundhole soundboard (second picture in the following series) where a fishbone pattern is employed for the bracing.

For particularly thin soundboards (under 2 mm) we advise the lattice pattern shown in Sect. 11.1. Here the arrangement of the braces, all of which initially measuring 4×4 mm, forms a series of increasingly larger polygons, diminishing stiffness in the right (bass) side of the soundboard in order to render asymmetrical the acoustic dipole engendered by the bracing.

The wood strips are slotted into each other for half of their height in the crossing points and, in the fine-tuning stage, will be worked in the tips in order to get more flexibility at the periphery of the soundboard.

Lastly, we call the attention of the reader to a particular bracing pattern (shown in Fig. 1 and 3) that was just realised for study purposes. Here the braces were all arranged in parallel direction, respectively longitudinal the first example, and transverse the second. Although these soundboards never became part of a finished instrument, we mounted them on the mould to verify which best supported the formation of vibration modes. We were surprised to find that the transverse braces drastically improve the FFT diagram, both in number and amplitude of the resonance peaks. We can therefore claim that the transverse arrangement of the braces favours the elastic equilibrium of the soundboard in the two directions, properly compensating the orthotropic nature of wood.

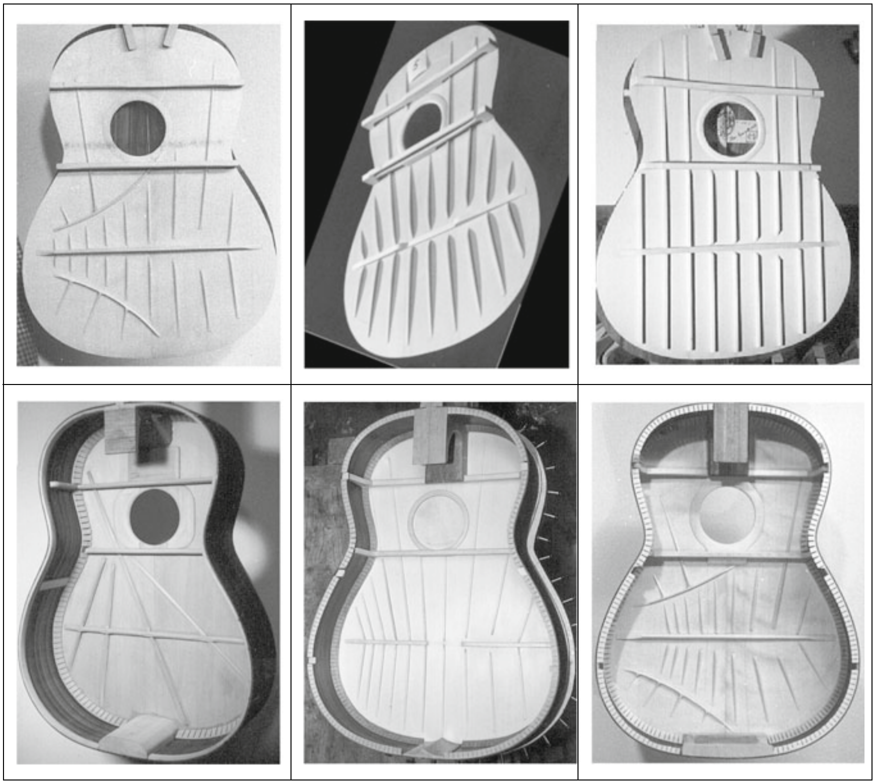
An important target of the bracing is to obtain the proper natural frequency of the soundboard (see Chap. 5 about the global resonator, dealing on the evaluation of this frequency). To achieve this result we must work on two parameters: stiffness and weight. We remind how increase in stiffness of a vibrating area raises its frequency, while an increase of weight brings it down.

Therefore—regardless of the bracing style—braces have to be worked in order to achieve the right stiffness, reducing the mass at the same time. For this purpose we recommend once again using rectangular wood strips, the narrow side glued to the board, taking care not to give them a triangular shape, which would reduce stiffness rather than weight. If required, we also advise using American cedar, which is somewhat lighter than spruce.

Another way to increase stiffness is to confer the soundboard a little arching. A careful adjustment of the outline and length of the transverse braces is necessary to prevent the arch from reaching the rim of the soundboard, where the linings are

glued. This would reduce the vibrating surface of the soundboard and, consequently, its performance at low frequencies. A band of about 3 cm along the soundboard perimeter must be left free to oscillate at low frequencies. This is the reason why in bowed instruments both soundboard and back are tapered along their edge.

The following bracing styles were put to practice: some only for study purposes, others also becoming part of finished instruments.



11.3 Effects of the Brace Arrangement—Examples of Two Reference Guitars

The arrangement of the braces brings about many consequences, not always easy to understand. Except for its function of reinforcing the soundboard against the pull of the strings, dynamic results—i.e. the propensity of the soundboard to start vibrating under the impulse of the strings—are not so intuitive.

First of all we need to understand what mechanisms rule the vibration of some of the guitar soundboard areas which, as commonly known, represents an oscillating system able to couple with the other vibrating elements of the instrument.

In a bare soundboard the main vibration modes can be easily evidenced. Furthermore, these modes have almost perfect geometrical outlines.

In the previous paragraph we mentioned how guitar soundboards made in the early 1800s (with no other braces than the transverse ones—like lutes) had excellent sound quality, though scarce emission power.

But the adoption of wood strips as elements of a bracing pattern, besides the obvious reinforcing function crucially affects vibration modes as well. As a consequence, their arrangement must be carefully planned.

Selecting layout and size of the braces is not an easy task. We know that a brace placed in the centre of a vibrating area restricts the amplitude of its oscillations but, sometimes, both amplitude and vibration frequency increase. Similarly, a wood strip placed on a nodal line has no influence on the related vibration mode, but may affect upper or lower frequency modes.

Search for the best bracing style began with ancestors of the guitar like lute, archlute, rebec, mandora, vihuela, etc. Each of them had a particular bracing style, proven over time as the most efficient solution.

As mentioned in the previous paragraph, one of the most traditional patterns employed in classical guitars was conceived by the Spanish luthier Antonio De Torres, with 5 or 7 braces essentially arranged in the longitudinal sense, almost parallel to the vein of the soundboard, and sometimes finished by two transverse wood strips at the bottom—as if to encircle the identified vibrating area.

The French Robert Bouchet was perhaps the first guitar maker to let some of the longitudinal braces reach beyond the main brace under the soundhole, which was conveniently bridge shaped.

The American physicist Michael Kasha, in cooperation with the luthier Richard Schneider, approached the issue of guitar bracing from a completely different point of view than his forerunners, getting partially rewarding results whose details, also, were never available for verification.

More recently, luthiers from all over the world began to test new bracing styles and, above all, new ways to make soundboards.

If one considers that the soundboard of a classical guitar must effectively respond to tones spanning a very large range of frequencies (HI-FI systems are usually provided with sets of two or three speakers for that purpose), making high quality guitars is clearly a tough challenge. In addition to that, the structure of the soundboard needs to be strong enough as to resist the permanent load exerted by the pull of the strings (about 47 kg).

Among the many patterns I personally carried out—as shown in the previous images—the most controversial results came from those designed to delimit the vibrating areas ideally to separate those appointed to high-frequency oscillations from those assigned to low-frequency sounds. But modestly sized areas that resonate at high-pitch frequencies are present in many points of the soundboard, often at the sides of the soundhole, and always according to a natural partition due to the succession of vibrating areas separated by nodal lines.

Since the guitar soundboard is not a plate with regular sides, like a square or rectangular plate, these areas are not arranged in a regular pattern over the soundboard surface, being affected by the adjoined elements and by the contour of the sides. However, the goal of the guitar maker is to always obtain from the soundboard the maximum performance in terms of acoustic pressure; for that purpose, we need to render most of the soundboard able to vibrate at all frequencies. Therefore,

besides extending the braces of the fan up beyond the main upper brace, all superfluous elements must be avoided, like braces delimiting vibrating areas, redundant reinforcements of the soundhole, unnecessarily oversized tail blocks, and so on.

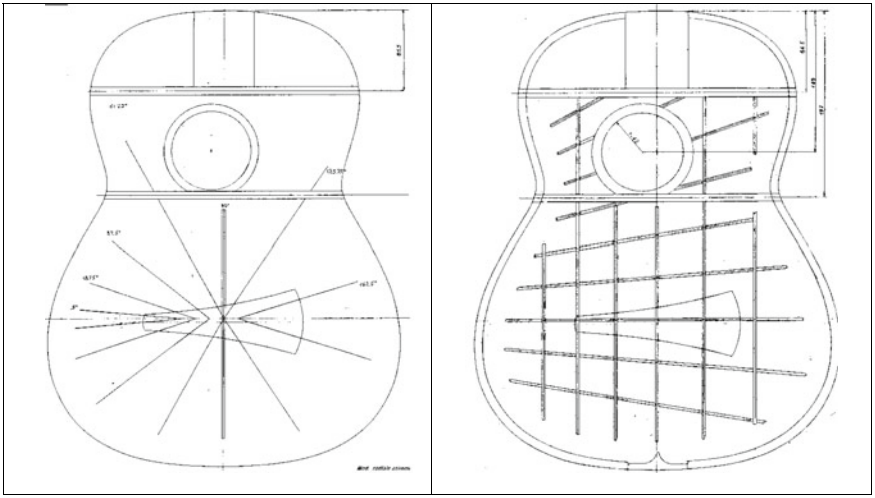
Vibrating areas must not be delimited by any means, to consent a better development of the vibrating areas at high pitched frequencies. Taking into account that the larger the net surface of a vibrating area, the greater its acoustic pressure at equal vibration velocity, applying these transverse wood strips—typical of Torres style bracings—is not advisable.

Just for example of how to answer—at least in part—problems about the bracing design, we illustrate hereafter the plans of two soundboards that yielded excellent results, despite different timbre qualities and different response to attack transients.

The first is a ***radial*** pattern, i.e. with braces departing from the bridge location in all directions, and also unusually extending towards the lateral areas of the soundboard. This method partially reduces the difference in propagation velocity of acoustic waves between parallel and transverse direction (with respect to vein direction) due to the orthotropic nature of spruce. Braces with a large angle to the veining oppose scarce resistance to the pull of the strings but are extremely useful, conveying sound to the sides at a higher velocity than what is due to the bare soundboard. Therefore, with this kind of pattern, the structural solidity of the soundboard will be chiefly assigned to longitudinal braces.

As you can see from the plan, there was a search for asymmetry between the two halves of the soundboard in the transverse direction, providing the left half with more braces—this way increasing its stiffness. This is to prevent the two sectors of the dipole from producing insufficient acoustic pressure when vibrating in mode $\langle 1\ 0 \rangle$ around the vertical nodal line (running approximately on the midline of the soundboard), owing to the counterbalancing oscillation of the two halves of the soundboard.

The second pattern nearly covers the whole soundboard surface, with braces arranged so as to form a lattice of irregular polygons. This structure confers the whole soundboard a considerable and duly distributed stiffness which, if properly regulated through the height of the braces, almost restores the response of the bare soundboard, with no significant interference on the formation of the main vibration modes. Correct regulation of the stiffness is very important with this arrangement of the braces, and the analysis methods proposed in this book will be very helpful for the purpose. The braces must be patiently slot into each other at every crossing point, and directly glued one by one. In this pattern too, the asymmetry of the dipole generated by mode $\langle 1\ 0 \rangle$ is obtained through differently sized polygons, which become progressively larger towards the area of the bass strings. This pattern also supports a natural development of mode $\langle 0\ 1 \rangle$, thanks to the limited height of the braces, and to the absence of conventional, big longitudinal braces.



In both patterns you can notice that the soundhole is placed a few mm off-centre (to the right hand of the outside view) in order to leave a larger area for low frequency vibrations. This, at least, was the purpose; in practice, it has not been verified yet, because of difficulties in comparing the two solutions (centred and off-centred soundhole) under analogous situations.

Notice that the main braces are bridge-shaped in both plans, to leave way for some of the fan braces. We must point out that the brace under the bridge cannot be abolished, unless we excessively stiffen the soundboard. In fact, as we have seen, it represents an equally important bond as the sides and, if absent, the soundboard would flutter, with undesirable effects on sound.

The brace above the soundhole is bridge-shaped too, for the previously mentioned reasons and, also, to let it follow expansions and contractions of the soundboard under shifting ambience conditions. Otherwise, since the fibres of wood strip and soundboard cross each other, these natural adjustments would be impaired, and cracks may occur in the soundboard over time.

In the left, radial pattern, proper angles for the arrangement of the braces are indicated. Obviously, these angles can be modified according to needs, provided that a certain asymmetry between the two sectors of the dipole is obtained, as explained previously.

As for the right pattern, we call instead your attention to the particular shape of the tail block, made from solid and vein-free wood, and the linings which, in this example, were unbroken—i.e. composed of some thin strips of spruce, glued to each other on a mould with the same profile as the sides. We also point out that the main upper brace is slotted into the soundboard surface and, after gluing, forms with the fingerboard a very solid structure.

11.4 Examples of Soundboard Analyses with the Mould

After collecting all physical data of the soundboard we are about to manufacture, it is convenient to begin by inserting the rosette, the reinforcement to the soundhole and the two main braces. Now the soundboard can be mounted on the mould, to execute initial analyses with the Chladni method, and start getting acquainted with this technique. The Chladni method is an excellent and fascinating inquiry method to begin with, and gain first impressions on how the soundboard moves at different excitation frequencies.

Mould and loudspeaker

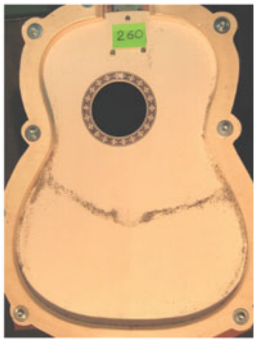
Mould with soundboard

The first picture shows the mould with the loudspeaker placed in the middle of the *lower bout*, while the second shows the mould with a soundboard mounted for analysis.

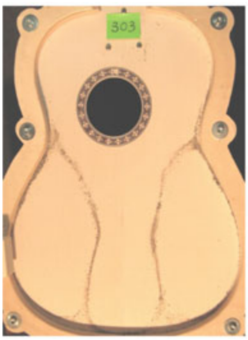
With the soundboard clamped to the mould, the loudspeaker placed beneath and connected to the low frequency generator, we can begin to set the soundboard into vibration, obviously starting from lowest frequencies. Sand, or other thinly grained material, is scattered over the soundboard (begin with a very little amount of it, uniformly scattered all over the surface). If the soundboard, as advised, is still free of fan braces, the first pattern relative to mode ⟨0 0⟩ will take form around 90/100 Hz (on a 2.3 mm thick soundboard). As we increase the excitation frequency, subsequent vibration modes will be highlighted; in order to make them more evident, the speaker underneath must be moved to the centre of each vibrating area that appears. When two or more vibrating areas become visible, the speaker can be just placed under one of them. The following photos represent the first vibration modes. The green labels above the soundhole indicate frequencies, in Hz, identified by the analysis.

Mode <00>

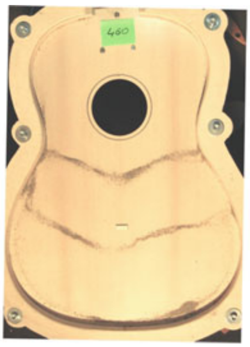
Mode <10>



Mode <01>



Mode <20>



Mode <02>

Mode <40>

With the same setting, examination through the acoustic hammer can be subsequently carried out. The analysis will result in an FFT diagram, showing all the resonance peaks generated by percussion, which the FFT software highlights through a Cartesian diagram. By careful observation, we will be able to recognize the same resonance peaks identified by the Chladni method.

In this initial stage, the most important resonance to be identified is that of mode ⟨0 0⟩, corresponding to the fundamental vibration mode of the soundboard. The first picture illustrates this mode and its vibration frequency, as shown by the Chladni pattern analysis.

Its outline suggests that mode ⟨0 0⟩ can be influenced by the shape of the bridge (presenting, in its traditional shape, two rather large "wings"). This may hinder the development of subsequent modes (especially mode ⟨1 0⟩).

Mode ⟨0 0⟩ might also be disturbed by an excessively stiff bridge. For this purpose, we made some completely "wing-free" models of bridge, getting excellent results. Anyway, if the soundboard has been selected according to the previously mentioned guidelines, this resonance will be large enough and will occur at the right frequency.

The first analysis is therefore meant to identify the fundamental resonance, its frequency and amplitude. In addition to that, the chart should display several resonance peaks in the high frequency band, though their amplitudes—at the moment—are small.

Now the braces of the selected pattern can be glued, but we recommend to begin by applying the bridge in the correct position, according to the scale length of the guitar.

When all elements are assembled and the glue has hardened enough we can proceed with further analyses, aimed at determining what to do to maximize the potential of the structure. We advise leaving the fan braces oversized in the beginning, to allow proper adjustment of their height and, accordingly, of the overall soundboard stiffness.

Steps must be taken to regulate the fundamental resonance of the soundboard at about 145–150 Hz, with bridge and all fan braces glued. This frequency level favourably couples with the Helmholtz resonance, which in a normally sized guitar appears at about 130 Hz. This coupling is indispensable, when the back is closed, to allow the manifestation of the first fundamental resonance of the body, i.e. the oscillation of the inner air. As suggested at the beginning of this chapter, the rigid back can be mounted on the mould (the two must be manufactured in conjunction).

With the rigid back mounted on the mould we repeat the pendulum test; this will give a very different result from the previous analysis. In fact, in the initial part of the diagram, a resonance with intermediate amplitude value will be clearly visible at about 100 Hz, corresponding to the resonance of the air set in motion by the stroke of the hammer on the soundboard. The soundboard resonance will in turn shift to a higher position—hopefully beyond 200 Hz—gaining considerable amplitude, too. The resonance peaks of the back will be visible afterwards, being now too high pitched because of the stiffness of this kind of back.

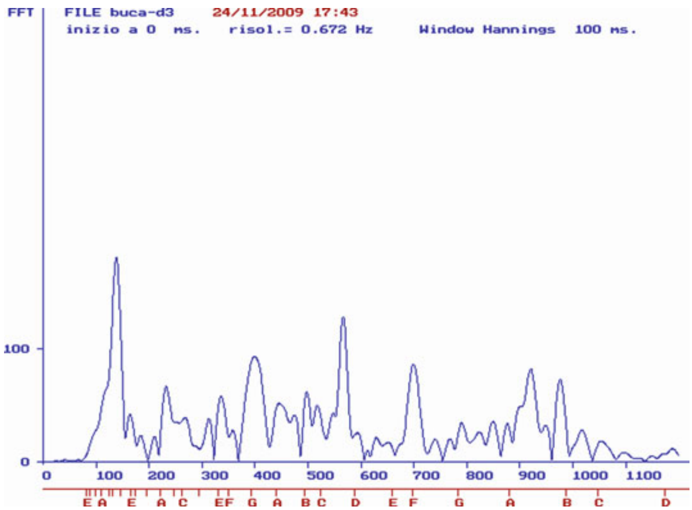
If the final back of the guitar is already available, it can be provisionally mounted in place of the rigid back to verify its resonance frequencies. Fine-tuning of these frequencies is the subject of a further chapter but, if the back has already been properly tuned, its resonances should be a little higher than the soundboard fundamental resonance.

The next resonance to be identified will be visible at about 400 Hz, with a medium-high peak corresponding to vibration mode ⟨0 1⟩.

As a factual example, we consider now a real soundboard that has been provided with all the necessary elements—beginning, as suggested, with the bridge.

The braces underneath the high-pitched strings have been left somewhat stiffer than required, to be regulated as desired through the simplest method we have, namely reducing their height.

The soundboard was firstly examined through hammer percussion and FFT software. The back of the mould was open. This is the resulting diagram:



The natural resonance of the soundboard is clearly visible at 137.9 Hz, which is a rather low value; this despite the precaution to leave the braces in the high-pitched area a little higher than required. This means that the soundboard is intrinsically scarce in stiffness and, therefore, its fine-tuning will require a couple of additional braces, or the substitution of the current braces with higher ones.

Photo 1

Photo 2

Photo 1 shows vibration mode ⟨0 0⟩ at 133 Hz, measured with the Chladni method, and corresponding to the resonance identified on the chart of the pendulum test at 137.9 Hz; these few Hz are simply due to the little additional load that the exciter applies on the point of contact with the soundboard, whose resonance frequency results slightly diminished.

The nodal line highlighted at the soundboard perimeter is blurred in the high-pitch area, probably owing to excessive stiffness in this location, which hampers free vibration at that frequency. Although a certain stiffness asymmetry is required between the two areas, care must be taken in order to keep the soundboard perimeter from being excessively rigid.

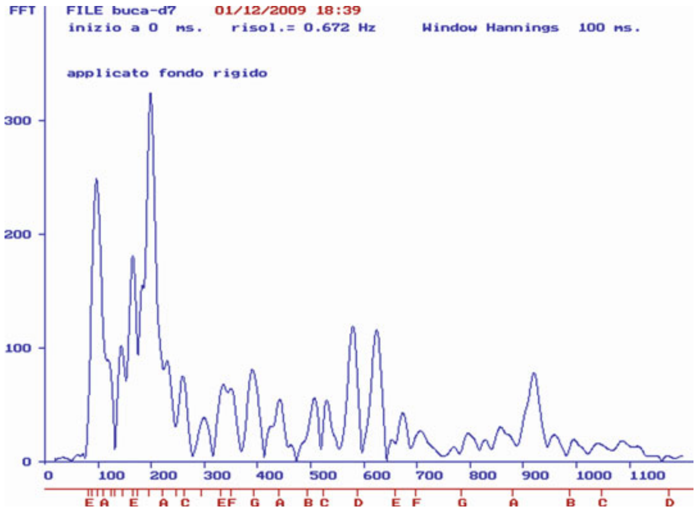
As an example of how to adjust the stiffness of a soundboard, photo 2 illustrates the same soundboard after slight reduction of the braces in the high-pitch area. The bottom right nodal line—and so the outline of mode ⟨0 0⟩—is more distinct, signifying that this mode takes place more naturally now.

We will then proceed by reducing a little the height of the braces on that area and, above all, by further thinning the soundboard in the peripheral band.

Practice with these methods also brings to light some small but significant differences, that may otherwise pass unnoticed. In fact, as we can see in photo 2, the nodal line has shifted a few mm towards the perimeter, equally enlarging the vibrating area.

After this little correction of the soundboard stiffness in the high-pitch area, a new analysis was executed with the pendulum: the frequency of mode ⟨0 0⟩ was unaffected but, on the other hand, its amplitude had suitably grown from 179.9 to 219.3 in the scale of the graph.

With these frequency and amplitude values, mode ⟨0 0⟩ is reasonably efficient. From the FFT analysis we also notice a good distribution of the resonances, up to the highest frequencies of the band involved, proving the soundboard performance to be quite appropriate. Now, before gluing the soundboard to the linings, we can mount the rigid back on the mould to verify the development of the resonance of the body and its frequency position.



The graph obtained with the rigid back on the mould clearly shows the presence of a substantial peak at the frequency of 96.89 Hz, corresponding to the first fundamental resonance of the air in the body. The amplitude of this peak—generated by the acoustic pressure due to the soundboard oscillation—proves that the soundboard, vibrating near the Helmholtz frequency, is profitably coupled with the latter, this way setting the inner air into vibration.

Further on in the graph, another marked resonance peak of the soundboard is visible at 198.5 Hz, which before the application of the rigid back occurred at a much lower frequency. This growth means that the soundboard operates now on a constricted air cushion due to the presence of the rigid back, consequently raising its resonance frequency.

At higher frequency levels we can see many more peaks, some of which cannot be easily interpreted, especially at highest frequencies. Around 400 Hz occurs a resonance of mode ⟨0 1⟩, illustrated afterwards, and two strong resonances next to 600 Hz, one of which relative to mode ⟨2 0⟩: this is the mode with two vertical nodal lines that divide the soundboard into three sectors which, oscillating in antiphase with each other, yield a good acoustic output.

Let us now review this kind of bracing, introduced in the previous paragraph. This soundboard was provided with a new kind of bridge, practically free of wings. In

designing this bridge, the absence of the wings as a gluing area has been compensated by extending the bridge down, this way virtually granting the same gluing surface.

As shown in the picture, the braces here are arranged in a *radial* pattern, i.e. departing from about the middle of the base outline of the bridge in all directions.

This pattern, as we already pointed out, is meant to compensate the orthotropic nature of wood, conveying vibrations generated by the strings to all directions of the soundboard and, above all, to areas that, normally, standard bracing patterns do not involve.

The following photos were taken during Chladni method analysis of the soundboard we are talking about: here are highlighted the three most important vibration modes, starting from the firsts in the frequency scale.

photo 1
mode <00>

photo 2
mode <10>

photo 3
mode <01>

The bridge-shaped brace below the soundhole lets the nodal line of modes $\langle 0\ 0 \rangle$ and $\langle 1\ 0 \rangle$ extend beyond the soundhole itself, with a consequent expansion of the vibrating area. This is not possible with the traditional main braces.

We proceed with our survey about soundboard vibrations, as revealed through the suggested analysis methods, by following partial results obtained during some construction stages of a guitar (identified as *Rica* from the initials of the consignee). The attributes of this guitar soundboard were:

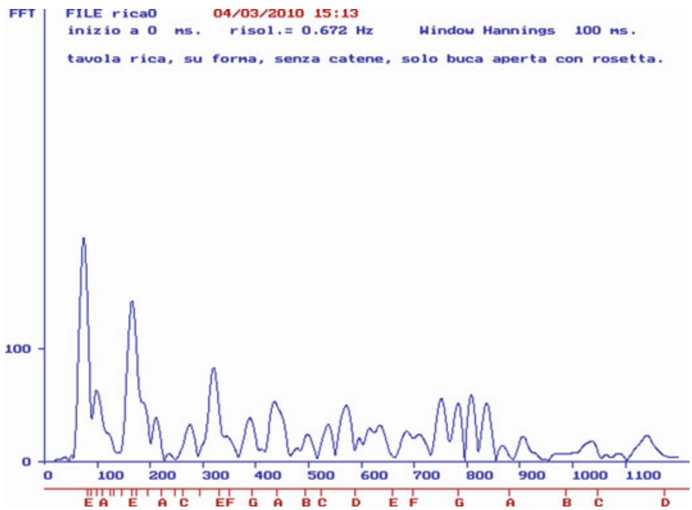
1. density = 372.7 kg/m^3.
2. mass = 113 gr.
3. mean thickness = 2.1 mm.
4. main axis length = 486 mm.
5. width of the upper bout = 282 mm
6. width of the waist = 244 mm.
7. width of the lower bout = 372 mm.

The analysis illustrated hereafter was executed on the soundboard with no brace applied, but just soundhole and rosette.

The first resonance on the left of the graph corresponds to the fundamental oscillation of the soundboard, located at 74 Hz with an amplitude of 198.4 points.

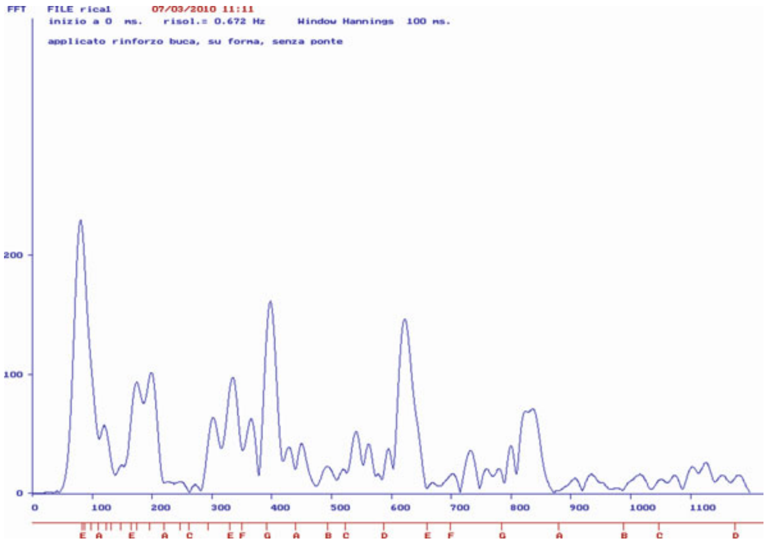
Subsequent resonances are scarcely relevant at this stage of construction. However, we can say that the second important resonance, occurring at 165.5 Hz, concerns vibration mode $\langle 1\ 0 \rangle$ while the following, at 320 Hz, may be the resonance of mode $\langle 0\ 1 \rangle$ which, in a finished guitar, is generally located at about 400 Hz.

The resonance of the body is obviously absent, being due to the air inside the guitar—or the Helmholtz resonator which, without the back, does not exist yet.



All of the subsequent resonances show progressively smaller amplitudes, and are relative to upper vibration modes which, especially with this setting, have no relevance.

The next step was gluing the reinforcement of the soundhole, consisting of a lens-shaped rosette made from spruce, 1.5 mm thick and 8 mm large. Then a new pendulum test was executed, resulting in the following diagram.



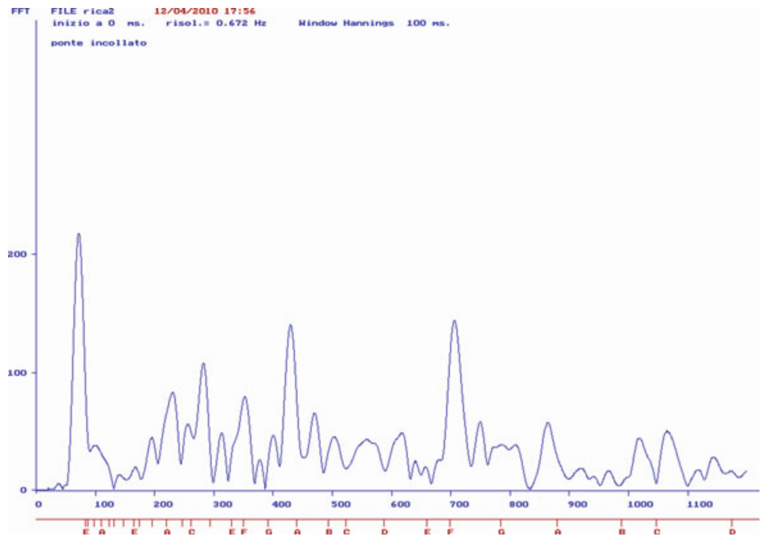
What comes immediately to notice is a substantial increase in both amplitude and frequency of the main resonances. In fact, the first peak due to the resonance of the soundboard has reached up to 80.7 Hz, and its amplitude has also grown from 198 to 212. This enhancement of the soundboard overall performance, simply owing to the reinforcement of the soundhole, means that the soundboard was initially too flexible to resonate at highest frequencies. In fact, larger differences are found above 300 Hz, rather than in the low frequency band.

The bridge will be subsequently applied according to former advice. This structural element affects the soundboard vibration modes in two ways: first, increasing stiffness, and second, adding mass onto the middle of the vibrating area of mode ⟨0 0⟩.

The additional stiffness due to the bridge tends to raise the resonant frequency of mode ⟨0 0⟩, i.e. the main resonance of the soundboard; on the other hand, increase in mass at the centre of the vibrating area tends to reduce it.

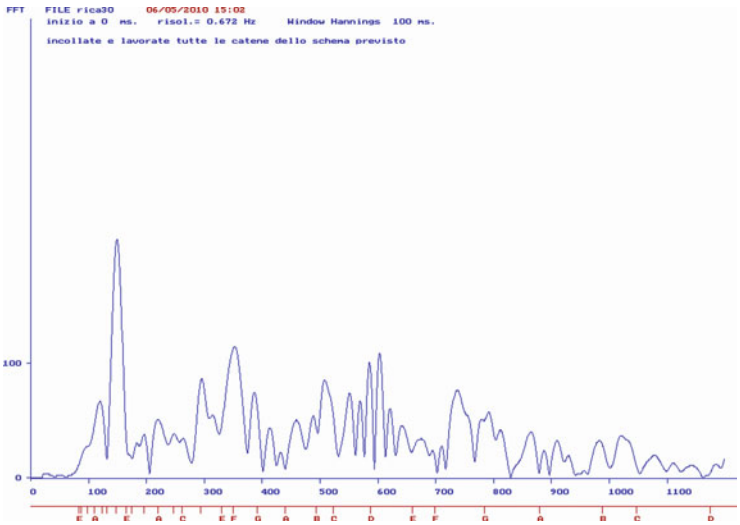
These consequences affect the overall performance of the soundboard; therefore, the weight of the bridge must be reduced, as far as possible, to prevent the fundamental resonance of the soundboard from falling short in frequency. The bridge must also be flexible enough—in the transverse direction—in order not to damage some of the upper vibration modes, like modes ⟨1 0⟩ and ⟨2 0⟩.

The following chart resulted after gluing the bridge.



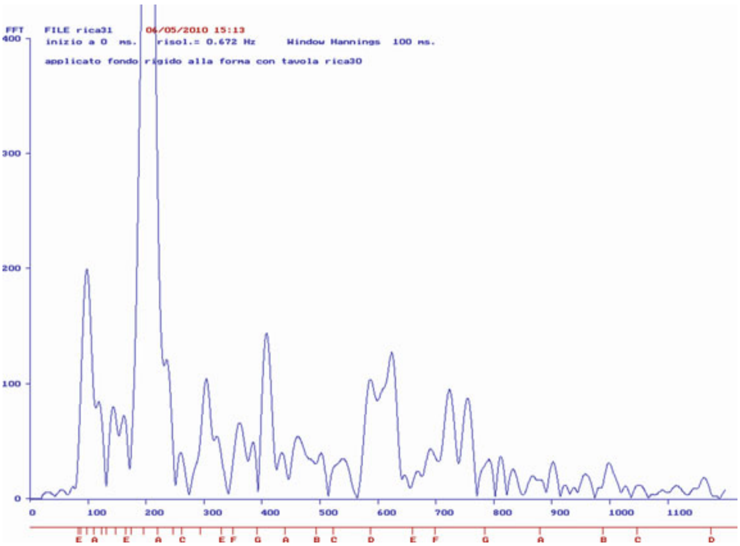
The additional stiffness brought about by the bridge and a better, wider distribution of vibrations over the soundboard, resulted in improved resonances throughout the frequency band, especially evident in the high-pitch portion of the graph—beyond 800 Hz—where resonances were rather weak before. Nevertheless, it is worth noticing that the bridge also increased the mass at the centre of the vibrating area of mode ⟨0 0⟩, corresponding to the first peak of the diagram, bringing down its frequency from 80 Hz (in the previous graph) to 72 Hz. But we must point out that none of the fan braces had been applied yet.

After that, all the braces were glued according to the pattern adopted for this guitar. The soundboard was first examined with all the braces intact, i.e. before any reduction, in order to determine its initial overall stiffness. Through repeated, regular controls, the braces were then adjusted, finally resulting in the following graph that represents the whole set of fundamental resonances of the soundboard by itself, i.e. mounted on the mould without a back.



The first important peak displayed on the graph, reaching about the middle of the screen, represents the fundamental resonance of the soundboard which, thanks to the bracing, has grown from 72 to 149.3 Hz, our target frequency.

Still with the soundboard on the mould, we coupled it with a rigid back to complete the resonator provisionally, and let the resonance of the body appear. This resonance is due to coupling the soundboard to air in body, and the result of the test with the back applied is the following:

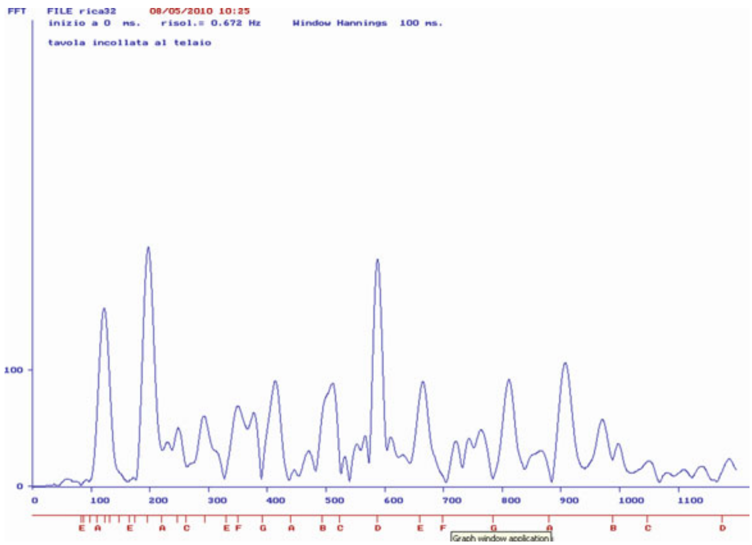


What comes immediately to notice from the graph is the fundamental resonance of the soundboard, considerably increased in frequency (205.2 Hz) and, above all, in

amplitude—so far as to reach out of the scale. Moreover, coming first in the diagram, the resonance of the body is clearly represented by the peak at 97.6 Hz—a suitable level.

Required targets for the soundboard being accomplished at this stage of work, i.e. the resonances complying with expected frequencies (with just a few Hertz difference) we can glue the soundboard onto the frame that has been prepared beforehand.

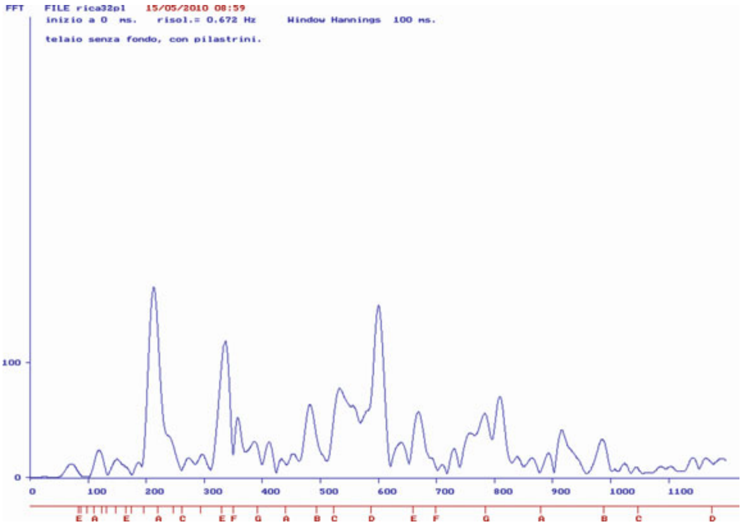
After a couple of days required for complete hardening of the glue, a new analysis with the pendulum gave the following result:



Looking at the graph, the back is clearly absent; in fact, in the initial part of the diagram, the typical resonance of the air is missing, being generally found at about 100 Hz, or just a little lower.

The resonance of the soundboard is correctly placed at 197 Hz, and subsequent oscillations of upper modes are also well distributed. At this stage of construction, the guitar output is therefore compliant with expectations. Note the “bell” outline of the first resonance at 122 Hz, resulting from combined oscillation of soundboard and sides.

As will be illustrated hereafter, even apparently trivial details can drastically affect the diagram. The next step—gluing the reinforcing struts to the main braces of the soundboard—has actually stiffened the side and the bond with the soundboard, turning the chart into the following:

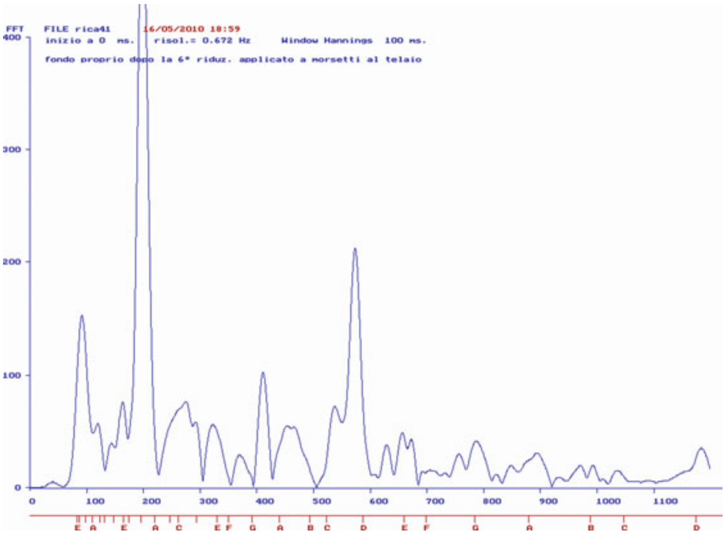


The struts have increased the overall stiffness of the soundboard, this way also preventing the sides from oscillating as shown in the previous graph, and the “bell resonance” has practically disappeared. The soundboard resonance, on the contrary, has grown a little in frequency, shifting from 197 to 212.6 Hz.

With these outcomes, we resolved it was time to work on the back. The three curved braces, worked into a long-tested shape, were glued and kept slightly higher than required, to facilitate the tuning of the back, as we will see.

The mould, on the side opposed to the soundboard, is provided with linings like a guitar frame; in the appointed locations slots are carved to lodge the tips of the three braces of the back. This way, the back can be checked and tuned in the same way as the soundboard.

The whole tuning process of the back through modal analyses will be fully described in a subsequent, specific chapter. Here we assume that the back has been already tuned at the right resonant frequencies, and fastened to the frame of the “Rica” guitar by means of clamps.



At the beginning of the chart a resonance is visible at 91.5 Hz, corresponding to the oscillation of the air inside the body, followed by a strong peak at 198.5 Hz, representing the oscillation of the soundboard. Other resonances, generated by the back, come next in the sequence.

At 411 Hz is a medium high peak relative to oscillation mode ⟨0 1⟩, and then at 573.3 Hz another very evident resonance corresponding to mode ⟩20⟩, the one with two vertical nodal lines that separate the soundboard into three sectors, vibrating in antiphase with each other.

It is worth reminding that the highest resonances (at 411 and 573 Hz) cannot be ascribed to the soundboard only, but they are likewise due to the coupling soundboard—air—back. This issue has been discussed in the section about upper resonances development.

The next procedure is gluing the back to the frame. This procedure is not the subject of the book, so we leave it to the skilled luthier.

For the reader's convenience, we give some elucidation on advantages afforded by these analysis methods.

To begin with, readers who has attentively followed the progress of the guitar we have selected as a model for investigation, have gained consciousness of how enjoyable, as well as encouraging, is the opportunity to adjust the guitar construction at any time, knowing what and how to do in order to improve its response.

Furthermore, collecting data on amplitudes and frequency levels of resonances produced by different models is very useful, building up an archive where each construction plan is associated to specific acoustic results. In effect, differently from the traditional manufacture method—aiming to reproduce all components with the same mechanical features as those of a good reference guitar—this method and these analysis systems allow us to identify the right resonance frequencies for the three fundamental oscillators that compose the guitar structure, and so to replicate their

performance in spite of very different conditions, like different quality and different sorts of woods, or even completely innovative bracing patterns.

Moreover, records acquired by means of simulations—with the mould and the provisional fastening through clamps—are highly reliable in comparison to the information that is normally obtained from back and soundboard glued to the frame.

The following table reports data relative to frequency and amplitude obtained during construction of a guitar. The record shows how resonances and relative amplitudes do not significantly change in the two conditions (fastening the back with clamps or by gluing).

Just the amplitude of the resonances is affected, being generally greater in the finished instrument, because the weight of the 26 clamps and their stiffening action on the sides hold them from vibrating naturally with the soundboard.

Fastening	***Frequencies in Hertz***								
	1st resonance Body	2nd resonance Body	**Percussion: Soundboard**	1st right side	2nd right side	3rd right side	**Percussion: Back**	2nd right side	3rd right side
with clamps: amplitudes	96.89	417.2	197.8	251.6	285.3	355.2	228.7	520.8	594.1
	220.7	117.0	230.7	103.8	122.0	83.31	143.2	69.64	41.26
by gluing: amplitudes	95.55	421.9	203.2	261.0	288.6	353.2	241.5	534.2	598.8
	318.5	261.9	325.6	1126.5	115.1	82.77	96.42	74.44	100.2

Chapter 12
The Soundboard on the Frame

Abstract This chapter details the effect of gluing the top to the real sides, a situation where adjustments on the dimensions of the braces are still possible. The result of the optimisation process is shown for the same two reference guitars already examined. This step ends with gluing the back when the relevant response plots show an appropriate behaviour of the top fastened to sides.

Once the soundboard on the mould has proved compliant with expected parameters, it can be glued to the frame. The frame, according to the luthier's preference, can be provided with kerfed or unbroken linings.

If the soundboard results slightly low-pitched from the analyses, unbroken braces may be helpful, increasing its stiffness. Just the opposite, in case of exceeding frequency.

Experience gained through extensive testing, as shown in the table at paragraph 11.4, tells us that frequency grows significantly when the soundboard is glued to the frame—especially after gluing reinforcing struts to the main braces. Values can rise by as much as 25–30 Hz and, as a consequence, the natural frequency of a fully braced soundboard being about 150 Hz when mounted on the mould, might reach up to 180–185 Hz.

The amplitude of the main resonances also tends to grow. The rise in frequency is mainly due to a stiffer bond, basically owing to sides and soundboard when the glue is hardened. As for the increase in amplitude, the reason is probably a different mobility—much greater for the sides of the guitar than those of the mould; the soundboard, being less constrained, drags the sides into its own oscillation.

Actually, in the diagrams obtained from the soundboard glued to the frame, a resonance occurs quite regularly at 122–125 Hz: this is the resonance of the structure composed by sides and soundboard, going into vibration like a bell.

We must however point out that, even when glued to the sides, the soundboard can still be adjusted, modified and improved until achievement of the expected quality.

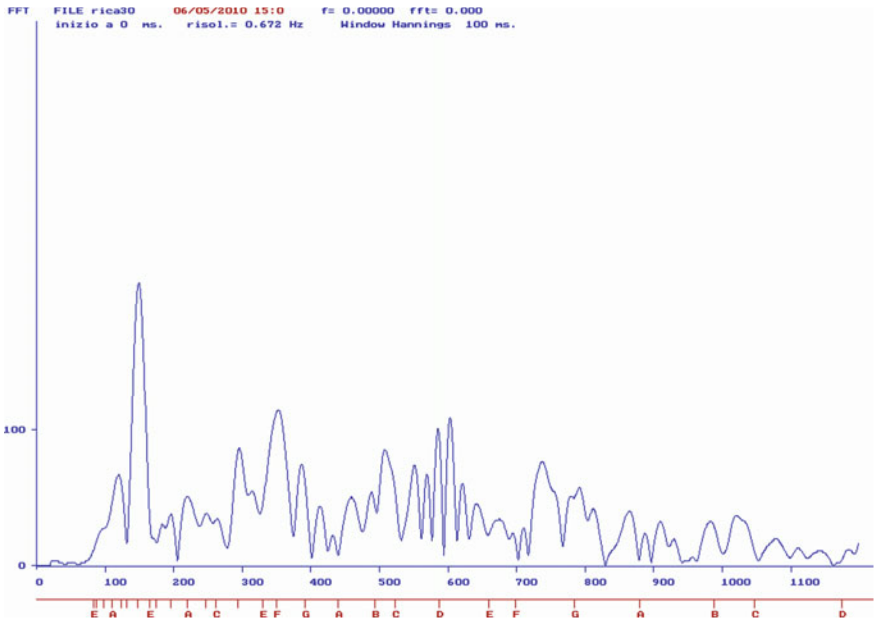
G. Cuzzucoli and M. Garrone, *Classical Guitar Design*,
https://doi.org/10.1007/978-3-030-32992-1_12

Gluing the soundboard to the sides is a customary practice for the luthier, so we give no advice on the subject. We just remark that the soundboard, having the bridge already applied, needs a niche in the work surface to lodge this element and facilitate operation.

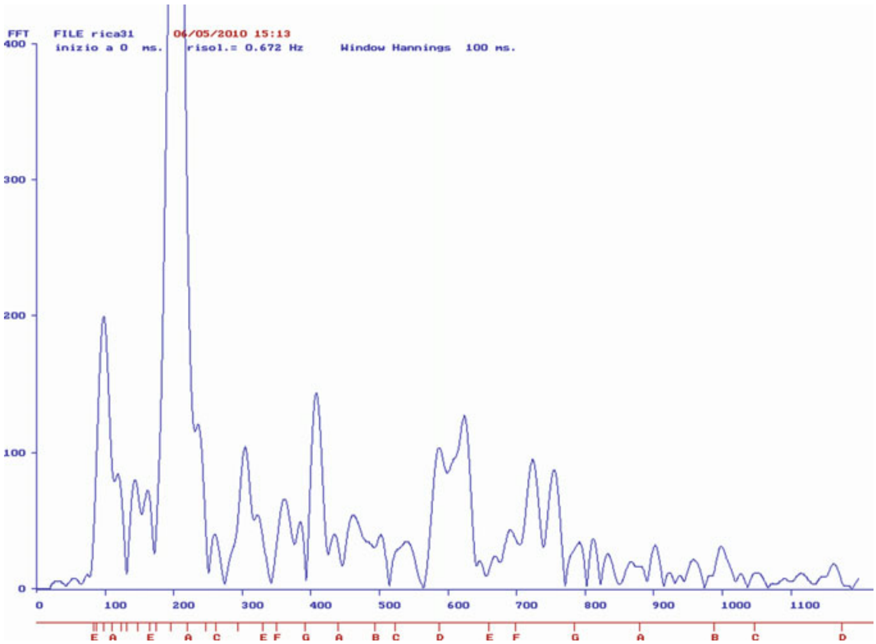
12.1 Examples on Reference Guitars

We turn again to the "Rica" guitar as a reference for the analyses executed before and after gluing the soundboard to the frame.

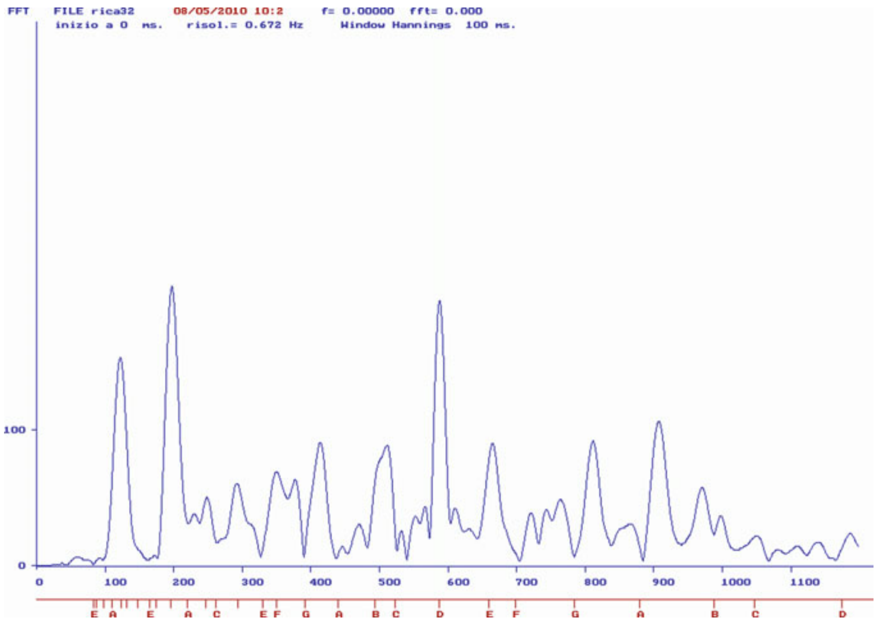
The graph below represents the soundboard when still mounted on the mould and just before gluing to the sides. Here the fundamental resonance of the soundboard appears at 149.3 Hz.



In order to determine if the soundboard is able to couple properly with the air and, consequently, generate the resonance of the body, a rigid back was mounted on the mould; the subsequent analysis revealed in fact a resonance of the body at 97.57 Hz, as expected. The resonance of the soundboard had grown in frequency, reaching 205.2 Hz.



The following chart represents the soundboard glued to the sides, but still without the reinforcing struts of the two main braces.



Some important resonances are visible, like the resonance of the soundboard at 197 Hz, the resonance of mode ⟨0 1⟩ at 413 Hz, a very high peak at 588 Hz corresponding to the resonance of mode ⟨2 0⟩, and an initial resonance at 122 Hz that will be absent in the subsequent diagrams, representing the vibration of the sides in combination with the soundboard ("bell resonance").

The previous diagram is an excellent example of resonances with good frequency and amplitude values.

12.2 Clamping a Back to the Frame

We have seen how a rigid back on the frame generated a resonance due to the oscillation of the air in the body; we can therefore try with a flexible back, i.e. a standard back, which will be the final one.

The procedure requires about 30 clamps, illustrated below in the form we have designed them.

The stem is a hexagonal profile made from a very hard aluminium alloy (Recidal); the two wooden heads are covered with cork, and the operating span is 5–12 cm. The blocking knob holds a 6 MA threaded brass bearing. The total weight of the clamp is about 35 g.

Slots will be carved in the linings to lodge the ends of the braces of the back, which will be fastened with 26–28 clamps. A perfect closure is vital, otherwise air may filter through and impair the resonance of the body. Three traditional braces, kept high enough to consent later adjustments of the frequency, will be glued to the back in order to confer it the right arching.

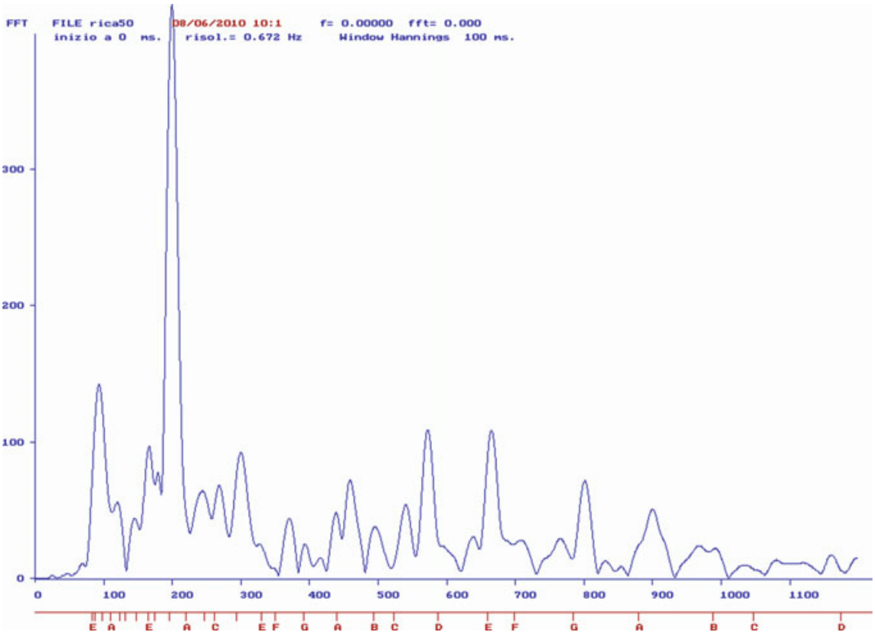
Like the rigid back on the mould, the flexible back—though not tuned yet—will likewise bring about the resonance of the body. If not, or in case of very small resonance amplitude, one of the following can be the reason: air has infiltrated through the closure of the back—which is easy to recognize—or the soundboard is not able to couple with the air, owing to its scarce vibrating surface. This can be due to excessive stiffness of the soundboard perimeter, or to excessive weight of the vibrating mass (too much high weight/vibrating surface ratio).

This condition can be easily highlighted through a Chladni examination of the soundboard mode ⟨0 0⟩. As repeatedly stated, the nodal line of this crucial mode must be very close to the sides, i.e. to the soundboard edge.

Before gluing it to the frame, the back was tuned through the modal analysis we are going to describe afterwards. Tuning is meant to bring the three key frequencies of the back just above the soundboard resonance. The back, tuned and mounted on the mould, yielded the following results:

- Mode ⟨0 0⟩ resonance = 210 Hz
- Mode ⟨0 1⟩ resonance = 285 Hz
- Mode ⟨0 2⟩ resonance = 336 Hz

When applied onto its own guitar (our reference guitar), the tuned back gave the following outcome:



Therefore, the resonant frequencies obtained with this new setting are:

- Resonance of the body = 93.5 Hz
- Resonance of the soundboard = 199.1 Hz
- Resonances of the back = 243.5–267.8–300.7 Hz

Resonance peaks at higher frequencies complete the diagram, being a sign of good performance throughout the frequency band of the instrument. The resonances of the guitar own back have right positions in frequency; in fact, the three resonances of the back occur just after the resonance of the soundboard, filling out the diagram in that zone.

These resonances naturally result from coupling between the resonances of the back—as manifested on the mould—and the guitar soundboard, through vibration of the air in the body. This coupling, as we have seen above, modified the initial frequencies.

12.3 Correct Behavior of the Soundboard on the Frame

The behaviour of the soundboard fastened to the frame is reasonably appropriate when the above mentioned conditions are met, namely:

- presence of the fundamental frequencies of air, soundboard, and back (though provisionally applied), identified through analysis of the harmonics;
- adequate amplitude values for the above mentioned;
- resonances with good amplitude values throughout the scale of the graph, despite a tendency to decrease at high frequencies.

The frequency of the air resonance generally is located about half of the value found in the soundboard resonance. The amplitude of the soundboard resonance sometimes reaches much greater levels.

Numerical data provided by the software we have employed, also include amplitude values for thirds of octave. These values define the power balance of the instrument with great accuracy. Furthermore, the distribution of the amplitudes is divided up into bands: a significant indication about the instrument overall performance is actually the mean value of resonance amplitudes in the frequency band between 80 and 1000 Hz.

Chapter 13
The Back

Abstract The height adjustment of the back braces serves the twofold purpose of tuning the frequencies of its main modes compared to the soundboard resonances, and of adjusting the amplitudes and the shape of the modal areas. The target is to reach an optimum coupling top to back via the air in the cavity. As this chapter shows, the process is done firstly with the back on the mould, both looking to the Chladni patters obtained via the electromagnetic exciter (to check the regularity of the vibrating areas), and using the acoustic hammer (to obtain optimal amplitudes and shapes of the modes). Once the result from the mould is appropriate, the back is fixed to sides, at first provisionally and then definitely. The impact of adding a fourth brace is discussed as well.

The back of the classical guitar is one of the three combined oscillators that compose the so-called "body". It is therefore a very crucial element, almost as crucial as the soundboard.

Nevertheless, many authors consider it just as a reflecting element for the acoustic waves produced by the soundboard, someone going so far as to make it very thick and shaped like a violin back. Sometimes it has been even connected to the soundboard through a sound post, once again emulating bowed instruments. On the contrary, the back of the classical guitar must be free to oscillate independently—following the laws of fastened plates vibration—and must resonate at frequencies higher than the fundamental of the soundboard, in order to couple with it through the air contained in the body.

The standard design of the back in the classical guitar foresees three transverse braces, similarly but not equally arched, to give it a proper curvature. The curvature essentially fulfils two requirements. The first, and most immediate, is for aesthetic purposes: a properly arched back looks better, showing a smoother angle with the sides. The second is due to an acoustic requisite: the back needs considerable stiffness in order to resonate within a range of frequencies comprised between 200 and 400 Hz.

The back is normally made of the same wood as the sides, with a standard thickness of about 2 mm. Thickness can vary by one or two tenths of mm, depending on the quality of the timber. For instance, a back made of maple wood can be 2.2 mm thick,

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https://doi.org/10.1007/978-3-030-32992-1_13

while Brazilian Red Wood just requires 1.8/1.9 mm. Also, with woods particularly hard thickness must be reduced. The three braces need not to be regularly spaced, but just need to follow the shape of the sides: specifically, the longest brace can be applied a little below the lower bout; the middle brace, 4–5 cm below the waist; the upper brace, in the middle of the upper bout bend. The distance from the bottom of the three braces, as we actually arranged them, is respectively 125 mm for the first, 254 mm for the second, and 395 mm for the third.

The same precautions must be taken in the building process of the back as of the soundboard, i.e. oversized braces must be applied with respect to designed final dimensions, to consent tuning of the back in the optimization stage. Initially, the height of the three braces can be, respectively, 20-19-18 mm; their thickness, 8 mm each. Just like those of the soundboard, the braces of the back must be light and strong—i.e. high and thin—for proper tuning. Otherwise, their weight would diminish the resonant frequency of the entire back. The strut usually attached in the centre of the back to reinforce the joint of the two halves, must be thin and flexible, in order not to compromise the ability of the back to vibrate.

At this stage of construction, the back can be mounted on the mould, this being provided with slightly inclined linings to suit the bend of the braces. During calibration of the braces, the mould must be open on the soundboard side, in order to distinguish the natural frequency of the back from that resulting by its coupling with the soundboard.

13.1 Modal Analysis

With the mould placed in front of the acoustic hammer the first test will be executed, taking care that the position of the pendulum is precise and invariable. The pendulum must obviously hit the back in the centre of its horizontal axis, while a midpoint between the lower braces can be appropriate on the vertical axis. From now on we will refer to the three braces using the letters A—B—C, beginning from the shorter one, near the neck.

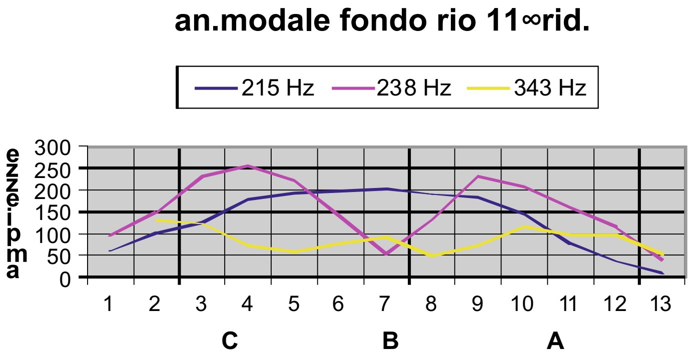
After the first test, frequencies showing the largest amplitude at the start of the band will probably be about 220, 260, and 300 Hz.

Now a patient tuning stage begins, following this procedure: through modal analysis, we obtain the frequencies of the vibration modes generated by the back, before any reduction of the braces.

13.1.1 Procedure

We mark with chalk a line of points on the outer face of the back, about 30 mm apart from each other, along the longitudinal axis. Then we mount the back on the mould, and test with the pendulum all of the points, writing down the amplitudes of the three

fundamental resonances in each point. These data can be subsequently entered into a spreadsheet, which will automatically generate the relative diagram. This is the outcome of the modal analysis executed on a back made of Brazilian Red Wood.



The three fundamental resonances, respectively coloured blue, magenta and yellow represent a single pole, a dipole, and a three-pole vibration area. The single pole is the lowest pitched vibration mode, clearly covering the central area of the back; this, as shown at the indicated frequency, mostly oscillates with a well-centred vibrating area. The dipole consists of two vibrating areas located next to the braces A and C, while the three-pole consists of three sectors that partially vibrate in antiphase with the dipole. Amplitudes are obviously lower in the sectors of the three-pole, since high frequencies cannot bring the back to larger deformations.

The figure represents a back whose braces had already been reduced to a certain extent—operation requiring much sensitivity, and patience. Sometimes, reducing one of the braces brings a negative result, like a decrease instead of an expected increase in the amplitude of a resonant sector. We must remember here that stiffness variation does not intervene in a single brace independently from the others. For instance, reducing the height of either brace A or brace C, the other sector of the dipole is consequently affected, being connected to the first by the same resonant frequency.

A limited amplitude in the monopole may indicate that brace B—placed in the centre of its vibrating area—is excessively rigid. In fact, if reducing this brace surely increases the amplitude of this resonance, at the same time it also brings its frequency down. So, a proper tuning of the back requires a good deal of practice and tenacity.

Proceeding step by step with reductions of even just 0.5 mm, the modal analysis will finally show a broad and well-centred monopole, with suitably increased amplitude too.

The final frequency of the single pole should be 10–20 Hz higher than that of the soundboard mounted on the mould or frame, without the back.

For instance, if the fundamental frequency of the soundboard appeared at 210–215 Hz, the resonance of the single pole of the back can be at 220–225 Hz. These values, obtained from the two boards not yet fastened, will change by a few

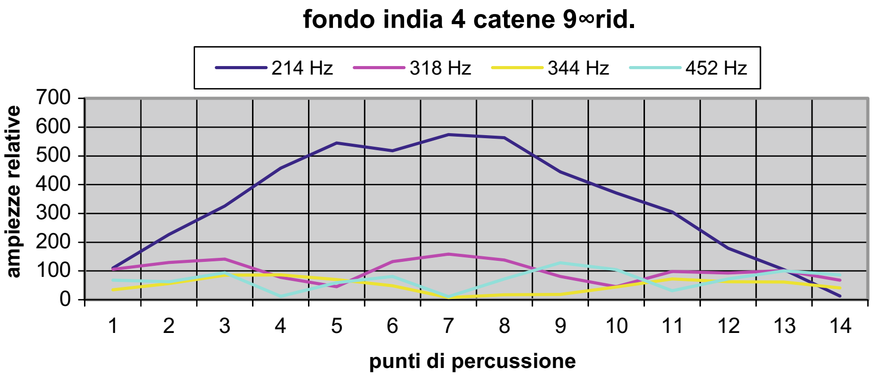
Hz when back and soundboard will be coupled, diverging from each other: a few Hz loss in the soundboard, and an approximately equal growth in the back.

13.2 Sample Graphs and Chladni Patterns

The resonance amplitudes of the monopole, as reported in the previous graph, are too small: some additional work is necessary to achieve the right tuning of the back. Despite a good balance between the single pole, the dipole and the three-pole area, their amplitude levels are still too low—this parameter being an important symptom of the propensity of the back to vibration.

Construction and tuning of the back by means of the modal analysis can be greatly simplified adding a brace, for instance in the centre of the back, as if to double the middle brace of the standard three-brace pattern.

The following graph resulted from the modal analysis of a back made from East Indian rosewood, which at the ninth reduction of the four braces yielded an amplitude of the monopole next to 600 points, against 200 points reported by the previous graph.



Important information about amplitude and shape of the vibration modes is provided by Chladni patterns which, as explained in Sect. 11.4, take form on the surface of the boards vibrated through the electromagnetic exciter or, simply, through a loudspeaker placed underneath and connected to a sinusoidal signal generator of adequate power (10 W is enough).

Photo 1

Photo 2

Photo 3

The three pictures illustrate a back made of East Indian rosewood with the three-brace pattern, mounted on the mould and vibrated at the appointed resonance frequencies through the electromagnetic exciter.

- Photo 1 shows the Chladni pattern relative to mode ⟨0 0⟩—the monopole.
- Photo 2 illustrates mode ⟨0 1⟩—the dipole.
- Photo 3 shows mode ⟨0 2⟩—the three-pole area.

These three patterns present satisfying amplitude and shape of the poles.

13.3 Provisional Assembly of Back and Frame. Optimization Procedure

As seen in Sect. 13.1, the purpose in tuning the back—and so the adjustment of the height of the braces—is to get broad and regular Chladni patterns at proper resonant frequencies. We also pointed out that the resonant frequency of the single pole, corresponding to mode ⟨0 0⟩, must be about 10–20 Hz higher than the fundamental resonance of the uncoupled soundboard—that is to say, mounted on the mould or glued to the frame, without any back affecting its vibration.

Also, the fundamental resonant frequencies of back and soundboard, when coupled with each other through the air inside the instrument body, have a tendency to shift apart by a few Hz. This variation can be negligible or reach as much as 10–20 Hz, depending on the coupling coefficient between the boards, in turn affected by variables like extent and mass of the vibrating surfaces, volume of the inner air, and mass/surface ratio.

Therefore, it is not always possible to determine beforehand the right resonant frequency of the back for a proper coupling with its counterpart in the soundboard. For that reason, it is very useful to couple the back with the frame by means of clamps—as shown at Sect. 12.2. In fact, clamps suitably emulate the final gluing of the back.

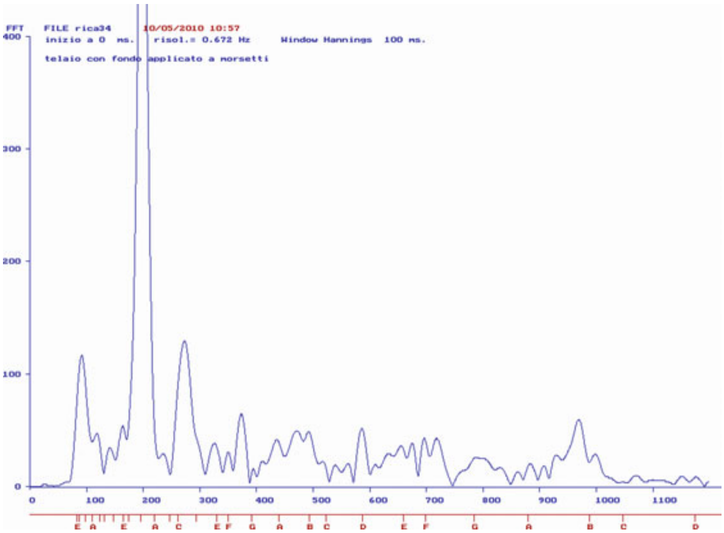
Differences found between analyses executed with both the clamped and the glued back are rather narrow, proving it very useful to examine a provisionally fastened back. This way, variations in the resonant frequencies of both back and soundboard can be identified and, since the vibrations of the back rebound onto the soundboard that first excited it, we are in the position to regulate the frequency of the back, so as to possibly fill up incomplete areas—normally occurring between 250 and 350 Hz—with the missing resonances.

The increase in weight represented—normally—by 30 clamps, may just bring about a decline in amplitude of the resonances displayed by diagrams, leaving their frequency levels unaffected.

Chapter 14
Closing the Instrument. Final Tuning

Abstract This chapter shows the measurements to be done on the instrument to check its quality: spectrograms, thirds of octave plots, decay times. Minor adjustments to be done on the nut, fingerboard, saddle are presented. The Appendix reports an experiment performed on six different saddles made of different materials (bone, carbon, ivory, PT). The amplitude and the decay time were measured at the fundamental, first and second harmonic of the first string at two different plucking position. It came out that noteworthy differences exist between saddles made of different materials (ivory being the worst).

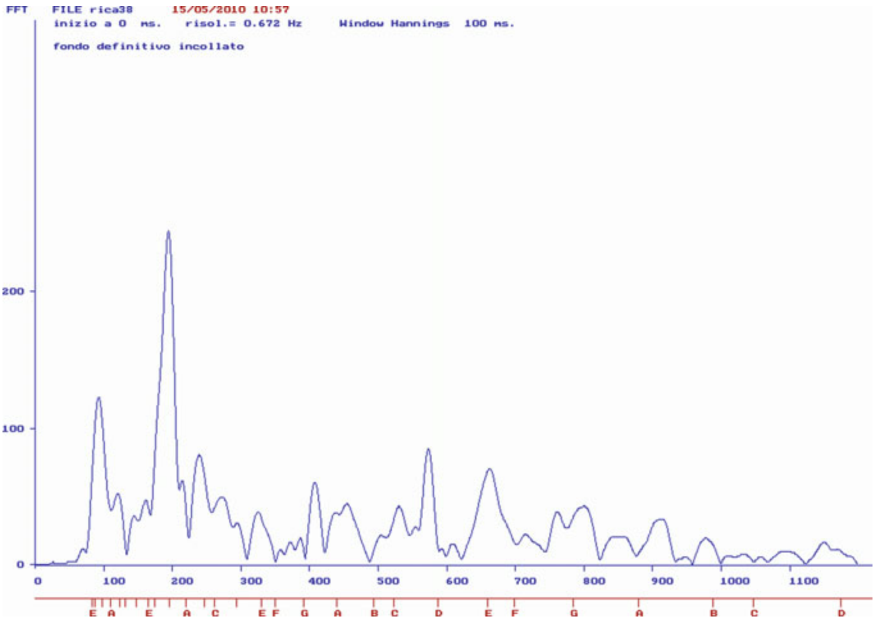
The analysis executed after fastening with clamps the finished back of our reference guitar to its soundboard and frame glued together—and complete of all details included reinforcing struts—gave an excellent result, displayed in the following graph.



G. Cuzzucoli and M. Garrone, *Classical Guitar Design*,
https://doi.org/10.1007/978-3-030-32992-1_14

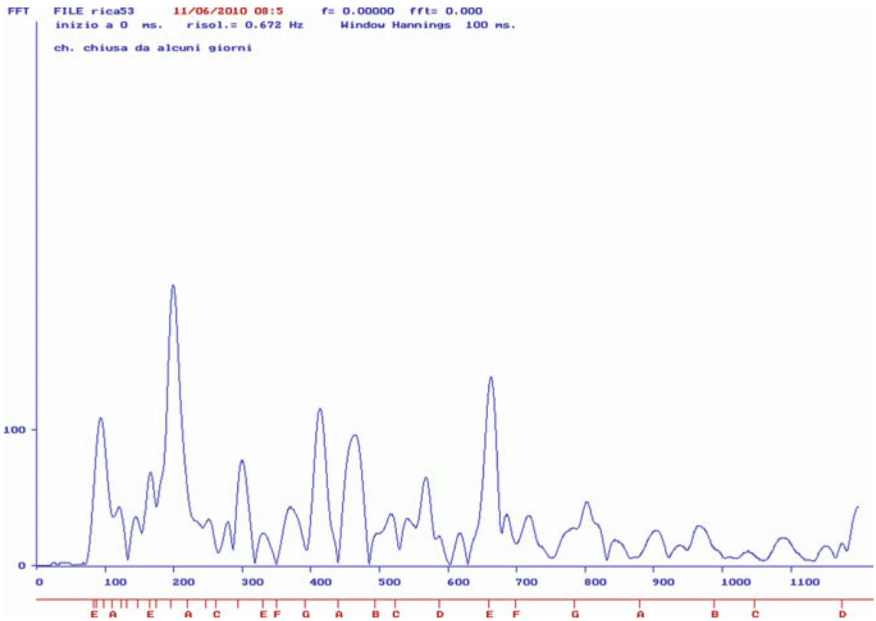
The first resonance of significant amplitude, shown by the diagram at 91.5 Hz, results from the oscillation of the air inside the guitar body. The second peak represents the fundamental resonance of the soundboard, found at 198.5 Hz, and next is the fundamental resonance of the back, at 273.2 Hz. The sequence of smaller resonances following up to about 1000 Hz depicts a quite dynamic response of the guitar to pendulum percussion, this being presage of a good quality instrument.

We therefore proceeded to the final fixing of the back and, after a few days required for the glue to be completely hardened, we carried out again the pendulum test, resulting in the next chart.



The resonance of the air has grown a little, reaching 92.8 Hz, while the fundamental resonance of the soundboard has lost about 3 Hz. All other resonances up to almost 1000 Hz show a slight increase in amplitude.

The instrument has been then completed with bindings, fingerboard, and shaping of the neck. After varnishing, the instrument was tested once again with the pendulum, yielding the following diagrams.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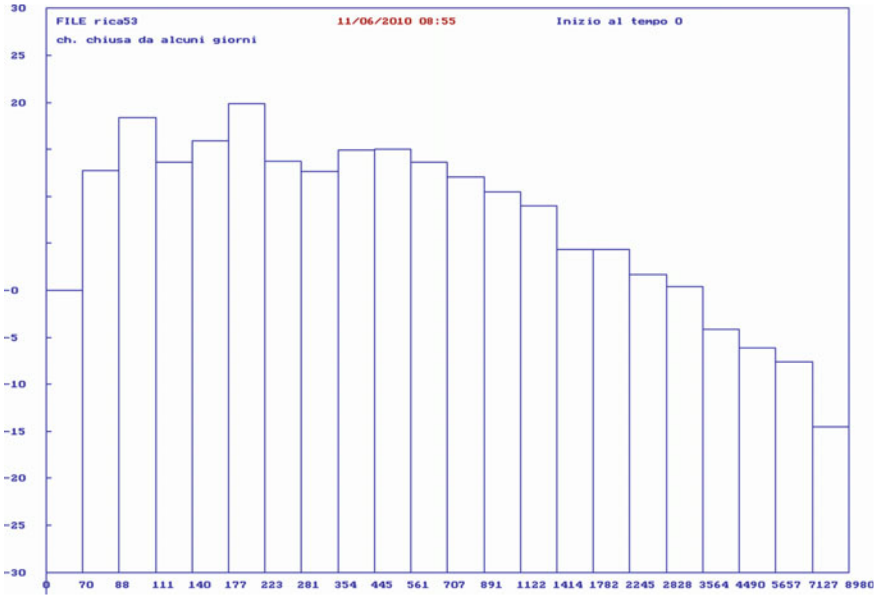
0 100 200 300 400 500 600 700 800 900 1000 1100
E A E A C E F G A B C D E F G A B C D


FILE rica53 11/06/2010 08:55 Inizio al tempo 0
ch. chiusa da alcuni giorni
30 25 20 -0 -5 -10 -15 -20 -25 -30
0 70 88 111 140 177 223 281 354 445 561 707 891 1122 1414 1782 2245 2828 3564 4490 5657 7127 8980

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File   rica53      11/06/2010 08:55

ch. chiusa da alcuni giorni


valore delle terze tra 80 Hz e 8000 Hz              7.642
valore delle terze tra 80 Hz e 1000 Hz             14.42
valore delle terze tra 1260 Hz e 3175 Hz            3.956
diff. di livello tra terze 80 -1000 e 1260-3175    10.46
valore delle terze tra 4000 Hz e 8000 Hz           -8.098
valore delle terze tra 80 Hz e 125 Hz              14.91
valore delle terze tra 160 Hz e 250 Hz             16.49
valore delle terze tra 250 Hz e 400 Hz             13.79
valore delle terze tra 315 Hz e 500 Hz             14.22
valore della terza a 630 Hz                        13.66
valore delle terze tra 800 Hz e 1260 Hz            10.50

Inizio al tempo                              0   msec
Finestra di osservazione (Hannings)        100   msec
```

RISONANZA (Hz)	AMPIEZZA	FATTORE Q	
93.53	100.9	6.029	
120.4	39.72	9.416	stima a destra
144.6	33.27	10.90	
165.5	63.74	13.40	
199.1	191.1	12.85	
250.9	31.58	19.05	stima a destra
278.5	29.62	27.73	
299.4	71.90	21.09	
330.4	22.43	22.94	
370.1	39.75	20.39	stima a sin.
413.1	107.4	27.11	
464.3	89.47	19.60	
502.6	22.55	20.31	stima a sin.
516.7	35.33	31.00	
540.3	32.38	40.06	stima a sin.
567.2	60.14	40.88	
586.7	20.22	55.27	stima a destra
617.0	22.18	52.82	
662.8	128.5	44.63	
685.0	34.84	59.64	
717.3	34.28	41.28	
783.2	25.58	non calcolabile	
802.1	43.41	44.19	
814.2	28.85	39.15	stima a destra
843.1	17.50	41.96	
903.7	24.13	40.46	
938.0	13.72	non calcolabile	

The second diagram illustrates the instrument response for thirds of octaves, showing a quite balanced output between 70 and 900 Hz, while the value between 80 and 1000 Hz—reported in the second line of the list—tells us that the instrument is very powerful (14.48 in our own established scale). The list reports some significant third of octave values and the resonances identified by the FFT software, where we can see that the resonance of the air finally stabilized at 93.53 Hz, and the fundamental resonance of the soundboard at 199.1 Hz.

Final Fittings

Now the instrument is ready to perform, and the final elements can be applied: nut, saddle of the bridge, tuning machines, and strings. Then, all of the adjoined components will be checked once again for optimal results.

The **nut** is generally made from bone, but we had excellent results using brass or zinc. Metal nuts, because of their mass greater than that of the bone, behave as an acoustic impedance to the waves travelling along the plucked strings, this way reducing energy losses into neck and head.

The nut controls the height of the strings at the first fret and their spacing. It must be made so as to leave the first string just flickering the first fret when the string

is pressed at the second fret; this tiny gap must be progressively increased up to 0.3/0.4 mm at the sixth string, in order to prevent undesired vibrations of the wound strings when playing certain notes.

The standard spacing between strings is 9 mm, which can be brought down to 8 mm on particularly narrow necks. Slots are made using specific nut files; the section of the files must be semicircular, and the depth of the slot should not exceed half of the strings diameter, to fit them properly.

The **saddle** of the bridge, too, is generally made from bone nowadays, after the rightful banning of ivory for wildlife preservation. Based on extensive testing of alternative materials, we now prefer using very thickly textured carbon fibre.

In Appendix, two diagrams resulting from a study on saddles made of different materials show their different sound transmission, related to the different harmonic component generated by the string and to different plucking positions on the scale length.

However, regardless of the material, the saddle calls for a very careful and verified manufacture. In fact, this element transmits sound to the bridge and then, in turn, to the soundboard; therefore, it must perfectly fit into its slot. For that purpose, the lower edges of the saddle must be rounded off, and the slot made with a little backward inclination, opposite to the pull of the strings, which will consequently tend to push it in.

The **fingerboard**, generally flat in the longitudinal sense, must be checked with a metal ruler for possibly though slightly protruding frets, which must be levelled using an abrasive stone or specifically shaped files that respect the contour of the fret, in order to keep the instrument performance from being blemished by *buzzing* or otherwise disagreeable sounds.

The height of the strings is measured at the 12th fret: a regular clearance (height from fret top to bottom of string) is 3 mm for the first string, which can be reduced to 2.5 mm if the player appropriately controls the force applied on it. The sixth string clearance must be 4.5 mm, but can also be lower with the above mentioned advice.

Appendix

A friend of ours, the guitarist Giuseppe Brancaccio from Torre del Greco (Naples), once told us he had perceived a change in his guitar timbre after replacing the original bone saddle with a carbon fibre one. This information, coming from a specially sensitive performer, prompted us to study the matter in depth: we wondered if the material from which a saddle is made could actually affect sound quality to the extent of being noticed by the player, and if the same hardware/software technologies we were familiar with, would be applicable to this kind of inquiry.

The last and most practical question is: *does it exist an indisputably superior material for building the saddle?*

To answer these questions we recorded the sound of E on the first open string (whose fundamental resonance is found at 329.7 Hz. The string was plucked on three

different positions: at 1/8, 1/4 and 1/2 of the scale length, measured from the saddle (results relative to 1/2 plucking will be omitted, having scarce practical relevance). The performer obviously tried to pluck the string as uniformly as possible.

So we considered the point of view of a player which checks the sound quality at different plucking positions and with saddles of different nature. As explained in the first two chapters, timbre depends on the ‘recipe’ of sound, i.e. the character of the string fundamental tone, the amplitude and decay rate (or sustain), the nature of the harmonics (for convenience we do not go beyond the second harmonic at about 989 Hz), and finally the presence (or absence) of the tuning note (the resonance of the air) and of the soundboard fundamental resonance. These parameters needed to be evaluated, in order to try and associate acoustic performance to different materials.

Moreover, as explained in the first two chapters of the book, the string acts on the resonator conveying periodic force impulses to the bridge, which follow one another at the frequency of the fundamental. These force impulses incorporate in their shape, duration and amplitude the whole information contained in the string motion: frequency and amplitude of the fundamental, as well as the information concerning the whole sequence of related harmonics; as a consequence, also information about the position and mode of plucking. In other words, force impulses summarize what we have called the ‘recipe’ of the string motion.

But between strings and bridge (ideally considered as a part of the soundboard) is interposed the saddle, playing its crucial role as *interface between the string and bridge-soundboard structure*.

Going directly to results, we will see that the saddle works like a sort of *damper* between string and bridge-soundboard structure; in other words, like an element connected in series between the string and the rest of the resonator which, moving at the same speed as the soundboard, modifies the characteristics of the force impulse according to its own composition. Therefore the saddle, too, behaves like an oscillator mass—spring—damping.

Based on these theoretical assumptions, we examined saddles made from six different materials: carbon, two different kinds of bone, two recycled pieces of ivory, a bar of sintered material available on the market that we will call PT. Some of these materials are ‘natural’ (bone and ivory), others are the product of chemical synthesis (carbon and PT).

We consider unnecessary to report results of all the numerous analysis executed: we will just mention hereafter the most significant examples.

The two basic resonances (of air and soundboard) are present in the recipe of the sound, though the fundamental of the string motion—at 329.7 Hz—is very distant from both. As pointed out in Chapter 1, the resonance of the air is always present in the recipe of the guitar sound, at least in medium-low registers, and represents a sort of *basic colour* on which the sound texture is interwoven. Analyses highlight a difference, however small, between different materials: on the whole, the best response comes from carbon, which provides greater amplitudes at both 1/4 and 1/8. As for the decay time, within the normal span of action—plucking position between 1/8 and 1/4—the range of variation between materials is rather narrow, and we believe

that divergences measured in sound duration at basic resonances are not relevant for the instrument response.

On the contrary, the components of the acoustic spectrum of the string (the fundamental and the harmonics) manifest a much more significant response with respect to amplitude and sustain, when the string is plucked at 1/8 or 1/4 of the length. As an example, the following graphs illustrate the response in terms of amplitude and decay time of the fundamental and of the first two harmonics when the string is pressed at 1/4.

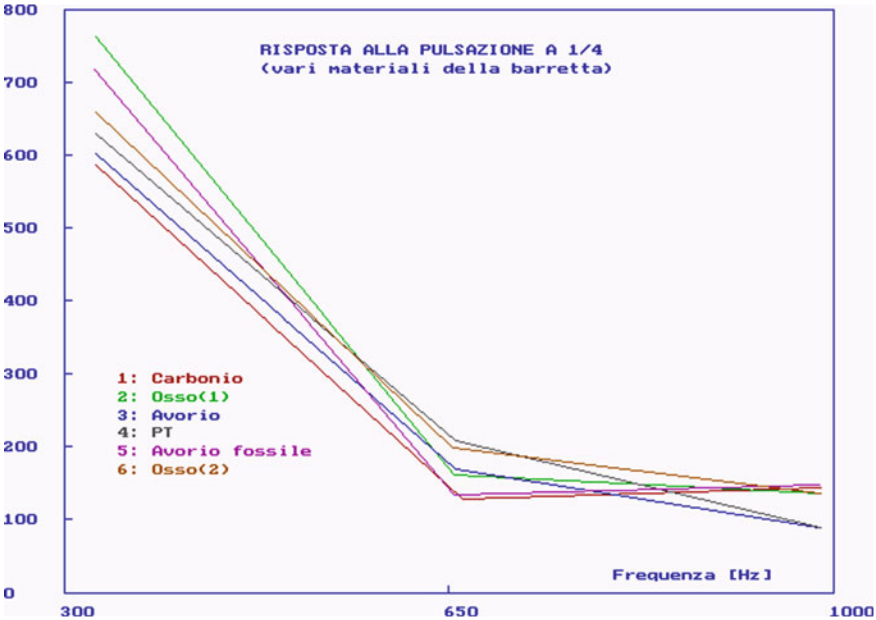
Obviously, the values of amplitude and decay time depend on the amplitude of the spectral components of the impulses that the string conveys to the resonator through saddle and bridge. These attributes are in turn affected by the plucking position. Discrepancies between theoretical and measured values tell us that *only part of the available energy passes into the resonator, the remaining part being absorbed by the saddle, according to the characteristics of the material.*

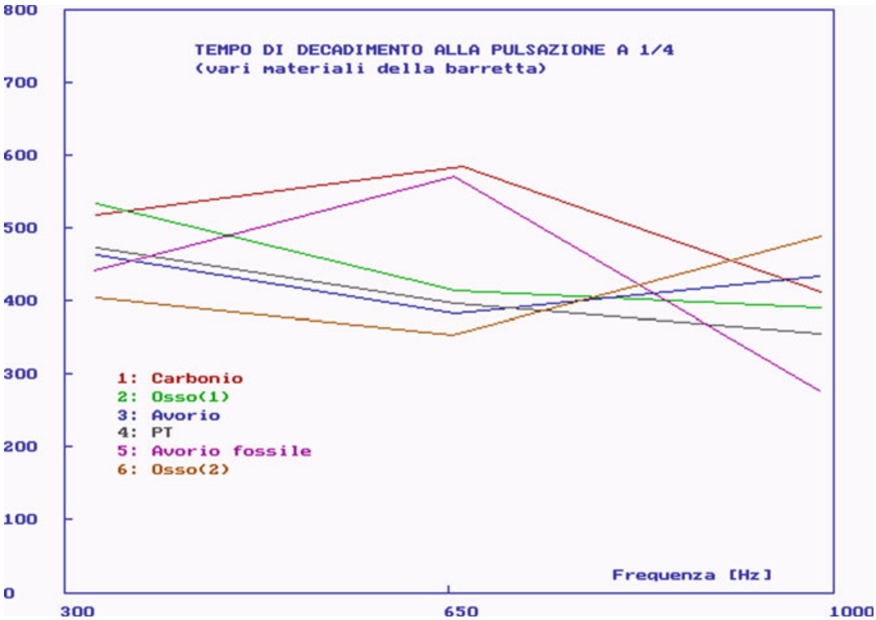
Similarly, decay rates depend on losses due to viscous friction within the material, but they also depend on the force applied to the saddle: this is a *non-linear* phenomenon, especially noticeable in 'natural' materials like bone or ivory.

From this set of measurements we deduce that noteworthy differences exist between saddles made from distinct materials but, at the same time, *there is no material superior to others under all testing conditions and playing technique*

Nevertheless, we were able to determine a *mean global performance* of the different saddle compositions. We classified materials by their performance on different plucking positions, with relation to parameters that affect sound quality. The best global results come from carbon and bone, and this for different reasons:

- **Carbon** confers excellent <u>amplitude</u> to high pitched sound components—the response is therefore very 'brilliant'—but slightly inferior to bone at the fundamental and the first harmonic. <u>Sustain</u> is outstanding at the level of both the fundamental and the first harmonic—namely, in the middle frequency band—while inferior at higher frequencies. It is worth pointing out that this is true on both plucking positions.
- **Bone** manifests excellent <u>amplitude</u> response relative to the fundamental and the first harmonic, but inferior to carbon at the second harmonic. <u>Sustain</u> is generally shorter with respect to carbon at mid-range frequencies, therefore implying greater losses due to friction within the material.
- **Ivory** behaves poorly under all situations, without any remarkable performance. **PT** is even inferior.





The overall evaluation of bone is very good, which justifies its traditional employ in guitar making. Nevertheless, we must consider that bone is a natural material, whose features vary on its provenance: of the two bone samples we have analysed, one was

especially good while the other was very poor in front of our global evaluation criteria. Carbon—unlike bone—manifests similar characteristics in the two conditions of percussion, as well as between various samples. In addition to that, the following attributes, as shown by the analysis, are worth noticing: optimal amplitude response at high frequencies and optimal sustain at mid-low frequencies. This is why we advise using them in preference to traditional materials.

Now, the question may arise: do these conclusions apply to the nut as well? The answer is no, since the saddle needs to transmit most of the force received from the string to the resonator, while the nut must reflect the incoming wave, absorbing the least amount of energy. Therefore, a good material for the saddle is not quite so for the nut.

Our friend Giuseppe Brancaccio, the guitarist who—as previously mentioned—promoted this investigation, has long experimented with different materials, finding out that **brass** and **zinc** are the best materials for the nut. Garrone guitars normally feature them in this component, which in fact is far from insignificant—as it may at first appear.

Furthermore, many of these instruments have a 250 g lead mass applied under the nut, which increases the overall reflection at the nut of the incoming wave.

Chapter 15
Analysis of Historic and Modern Guitars

Abstract This last chapter analyses and compares six different guitars—1921 Fleta, 1931 Simplicio, 1974 Gallinotti, 1982 Ramirez, 2011 Garrone and a replica (by Zontini) of a 1862 Torres. Measurements on the instruments and comments on relevant parameters were in line with the methods presented in the previous chapters, and a summary shows all the results in one table. The purpose of this chapter is twofold: at first to demonstrate how the key parameters discussed in the book can proficiently be used to assess an existing instrument, eventually to replicate its sounding characteristic; secondly to gain an understanding of the design criteria that guided a luthier in designing an existing instrument, and how certain criteria (and the underlying physical and geometrical parameters) evolved in the course of the years.

In Chaps. 5 and 6 we talked about the distinctive parameters of the guitar resonator, how they can be measured, and their significance in determining the characteristics of an instrument. This analysis helps optimizing the design during construction while, if applied on a finished guitar, it offers an insight into the designing criteria followed by the manufacturer. This *reverse engineering* investigation, often mentioned in the book, will be illustrated in this chapter through some example instruments that hold a significant place in the history of classical guitar, from Torres to present times, offering a general though partial survey of its evolution.

These instruments are:

- **Torres** style guitar, by the luthier **Fabio Zontini**. This is a replica of the famous guitar Torres made in 1862 with cardboard back and sides.
- **Fleta (1921)** This is one of the first—if not the first—guitar produced by Fleta.
- **Simplicio (1931)—model A**
- **Gallinotti (1974)**
- **Ramirez (1982)**
- **Garrone 92 (2011)**.

Each of them is reviewed through a record featuring:

- preliminary data about the most relevant parameters (either measured or estimated);

G. Cuzzucoli and M. Garrone, *Classical Guitar Design*,
https://doi.org/10.1007/978-3-030-32992-1_15

– some of the most significant response diagrams;
– a brief commentary.

Parameters have already been illustrated in detail, but we will summarize their significance here for the reader's convenience.

Preliminary Data

- The '*Basic resonances*' entry reports the two fundamental resonances (of the air $\mathbf{F_1}$ and of the soundboard $\mathbf{F_2}$) that determine the behaviour of the resonator at low frequencies. Their importance and meaning was discussed in general terms throughout Chap. 5, and in detail in Sect. 5.2. We have seen how their values depend on some parameters, like the frequency of the *Helmholtz resonator* $\mathbf{F_h}$, represented by volume of the air inside the body and dimensions of the soundhole. The value of the Helmholtz resonance reported in this introductory paragraph was measured with the method described in Sect. 4.1.
  Basic resonances are also connected to the natural frequency of the soundboard in mode ⟨0 0⟩, i.e. the main vibration mode of the soundboard when fastened to a rigid frame—but not to the sides of the guitar. Obviously, this parameter cannot be directly measured on a finished guitar, but just esteemed. Therefore, it will be reported under a subsequent entry.
  This entry also reports the values of the two 'covered soundhole' resonances $\mathbf{F_p}$ and $\mathbf{F_{pm}}$ (with an additional mass). These values—presented in Sects. 5.2 and 5.5—help determining the relevant parameters of the resonator. Summarising, the procedure for the suggested measurements includes assessment of the following parameters:

  – $\mathbf{F_1}$ (resonance of the air)
  – $\mathbf{F_2}$ (resonance of the soundboard)
  – $\mathbf{F_h}$ (Helmholtz resonance)
  – $\mathbf{F_p}$ (covered soundhole resonance)
  – $\mathbf{F_{pm}}$ (covered soundhole resonance with additional mass).

These are the measurements carried out on the examined guitars. In addition to that, the diameter of the soundhole, which is used to estimate the volume of the air inside the body. Making reference to Sect. 5.5, we remind that the additional mass is a 20 g scale weight, provisionally attached with double sided tape to the bridge plate.

- The entry relative to '*Resonances of the back*' reports their values measured on the back in the three most significant modes (mode ⟨0 0⟩, mode ⟨0 1⟩, mode ⟨0 2 ⟩). Beside, in brackets, we report the corresponding values measured on the soundboard which, as you can see, differ by just a few Hz with respect to measurements made on the back.
  We refer the reader to Sect. 5.6 for a detailed analysis of the resonances due to the back (specifically its fundamental modes), and to Sect. 5.7 as regards the contribution of the back to the instrument 'global' response.

- The entry concerning 'Characteristic parameters' reports a set of results estimated on the base of measurements. In particular:
  - Natural frequency of the soundboard $\mathbf{F_{p0}}$, presented in Sect. 4.2. As we have seen, this frequency plays a crucial role in determining the basic resonances of the guitar, in combination with the Helmholtz resonator (see Sect. 5.3 about coupled oscillators functioning). In Sect. 5.5 we have seen how the natural frequency of the soundboard can be estimated on a finished guitar, starting from the values of basic resonances and Helmholtz resonance.
  - Natural frequencies of the back in its three fundamental modes. In Sect. 5.5 we pointed out the significance of the natural frequencies of the back. Their value was estimated by means of the formula presented in Sect. 5.7.
  - Other resonator parameters, estimated according to the model presented in Chap. 5. Their meaning, how to measure them on a finished guitar, and their importance in the evaluation of an instrument quality are discussed in Sect. 5.5.

Diagrams
Some especially meaningful graphs are shown concerning the examined guitars:

- Global response of the instrument, excited on the soundboard at the bridge location (see Sect. 5.2).
- Thirds of octave chart, introduced in Sect. 1.6. Each of the sectors in the diagram tells us how powerful is sound emission within its frequency range. By adding the mean value between various intervals, we can assess the sound emission value relative to registers that are significant for the quality and attributes of an instrument. After the third of octave diagram, we report the mean values relative to the range of frequency involved in these significant registers, whose limits were defined in Sect. 1.6. The third of octave diagram offers an overview on the global balance of the instrument, highlighting intervals with noticeably higher or lower sound emission with respect to adjacent ones.
- Decay time diagram, presented in Sect. 1.4, whose mathematical description is shown in Appendix 1.3. To simplify, the decay time represents and is proportional to sound duration (sustain) at a given frequency. Especially worth of notice is the decay time relative to the resonance of the air (the tuning note). The frequency of this resonance gives a basic colour to sound (see Sect. 1.4), while the decay time specifies its duration in the sound 'recipe'

Comments
Each instrument data record is followed by a short commentary. We would not express opinions about an instrument being better or worse than others, but just highlight designing criteria revealed by analyses, and resulting acoustic features. Following this method, the reader will be able to find out the concept underlying an instrument construction, far beyond 'conventional' assessment of geometries, wood thicknesses, bracing patterns, etc.

15.1 Fabio Zontini—Replica of the 1862 Torres Guitar

♦ <u>*Basic resonances*</u>

$\mathbf{F_1}$	(Resonance of the air)	84.5 Hz
$\mathbf{F_h}$	(Helmholtz resonance)	143 Hz
$\mathbf{F_2}$	(Basic resonance of the soundboard)	182.6 Hz
$\mathbf{F_p}$	(Covered soundhole resonance)	155 Hz
$\mathbf{F_{pm}}$	(Covered soundhole resonance with additional mass)	136 Hz
Diameter of the soundhole		78.6 mm

♦ <u>*Resonances of the Back*</u> *(measured)*

		on the back	*on the soundboard*
F<00>	(Mode <00> resonance)	300 Hz	(299 Hz)
F<01>	(Mode <01> resonance)	340 Hz	(343 Hz)
F<02>	(Mode <02> resonance)	426 Hz	(420 Hz)

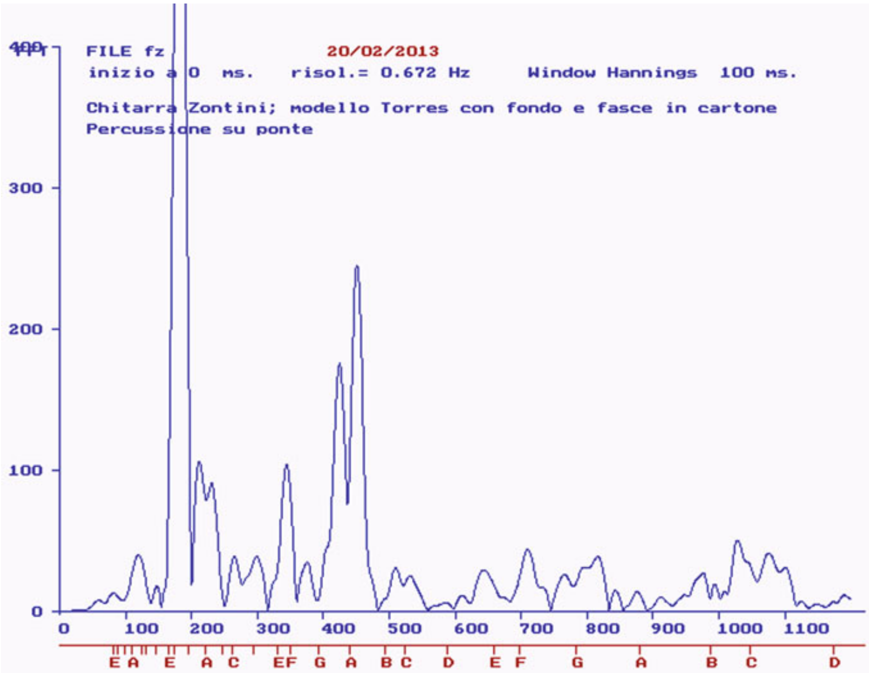
♦ <u>*Characteristic Parameters*</u> *(estimated)*

Inner air volume 10.7 litres

$\mathbf{F_{p0}}$ (Natural frequency of the soundboard) 108 Hz

Natural frequency of the back (mode <00>)	301 Hz
Natural frequency of the back (mode <01>)	341 Hz
Natural frequency of the back (mode <02>)	426 Hz

Vibrating mass	69 g
Vibrating surface	416 cm^2
Soundboard stiffness	31650 N/m
Surface/mass ratio	6.03 cm^2/g
Coupling coefficient	0.80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Chitarra Zontini; modello Torres con fondo e fasce in cartone
Percussione su ponte
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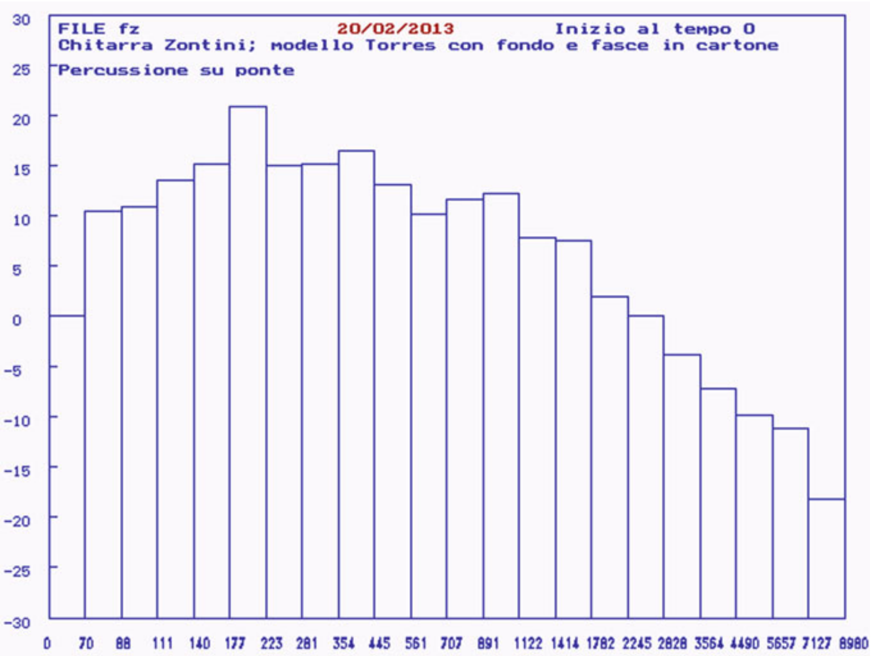
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Chitarra Zontini; modello Torres con fondo e fasce in cartone
Percussione su ponte
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0 70 88 111 140 177 223 281 354 445 561 707 891 1122 1414 1782 2245 2828 3564 4490 5657 7127 8980

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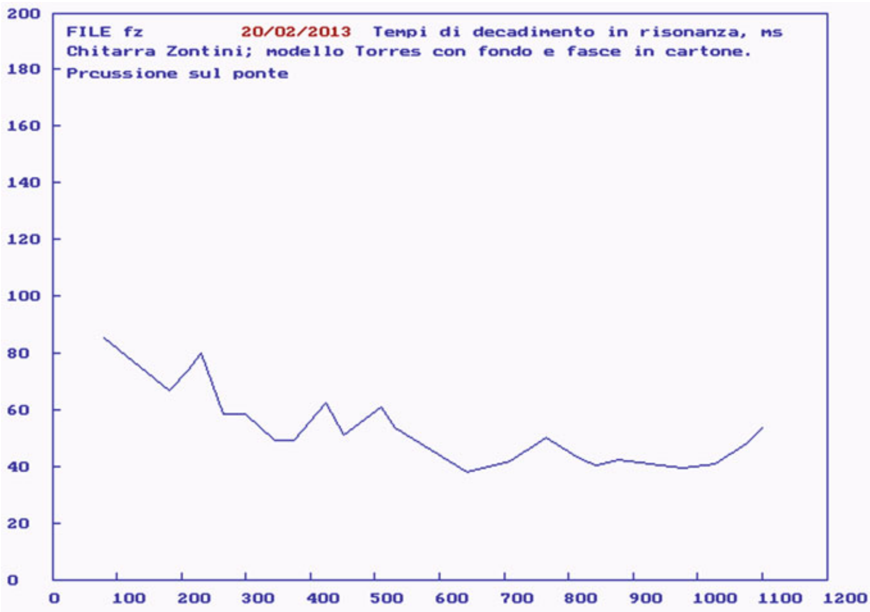
Chitarra Zontini; modello Torres con fondo e fasce in cartone
Percussione su ponte

valore delle terze tra 80 Hz e 8000 Hz	6.257
valore delle terze tra 80 Hz e 1000 Hz	13.68
valore delle terze tra 1260 Hz e 3175 Hz	2.725
diff. di livello tra terze 80 -1000 e 1260-3175	10.95
valore delle terze tra 4000 Hz e 8000 Hz	-11.60
valore delle terze tra 80 Hz e 125 Hz	11.58
valore delle terze tra 160 Hz e 250 Hz	16.98
valore delle terze tra 250 Hz e 400 Hz	15.44
valore delle terze tra 315 Hz e 500 Hz	14.81
valore della terza a 630 Hz	10.17
valore delle terze tra 800 Hz e 1260 Hz	10.56

Inizio al tempo 0 msec
Finestra di Hannings 100 msec.



Comments
Notice first of all the resonance of the air (the ‘tuning note’), occurring at 84.5 Hz (therefore between E at 82.407 Hz and F at 87.307 Hz on the sixth string). We would expect a higher frequency of the air from an undersized instrument like this; despite the limited volume, this resonance was brought to very low values by reducing the natural frequency of the soundboard to 108 Hz, i.e. lower than the Helmholtz resonance (found at 143 Hz). This setup, unusual in more recent guitars, is however

foreseen by the model shown at Sect. 5.5 where, under the subheading '*From soundboard natural frequency to basic resonances*', we provided a set of curves allowing calculation of the basic resonances from the natural frequency of the soundboard and the Helmholtz frequency. Setting 108 Hz as the natural frequency of the soundboard and 143 Hz for the Helmholtz resonance, the basic resonances result to be at 84.6 and 182.6 Hz, matching the measurements carried out on the instrument.

Such a low natural frequency of the soundboard is basically due to its low global stiffness (31650 N/m), while the vibrating mass is rather considerable (69 g). Lacking further information, we may assume the soundboard to be very thin (hence very flexible) and quite heavily braced—typical of old-fashioned guitars. Therefore, we should not be surprised that its surface/mass ratio (6.03) is lower than it is in more recent instruments, by reason of a limited vibrating surface (416 cm^2) as well as a limited total surface of the soundboard.

Looking at the graph obtained from soundboard percussion, we notice a quite 'lively' response approximately between 150 and 500 Hz, the frequency band including the resonance of the soundboard (182.6 Hz), the resonance of the soundboard in mode $\langle 1\ 0 \rangle$, the resonances of the back (examined hereafter), and the substantial resonance of mode $\langle 0\ 1 \rangle$ at about 451 Hz. This last resonance, its importance and meaning are discussed in Sect. 6.1. We just point out that the resonance in mode $\langle 0\ 1 \rangle$ appears in this guitar at a higher frequency, if compared with more recent instruments, because of a smaller volume of the body. The response is instead noticeably flat beyond 500 Hz, i.e. beyond B on the first string, where the instrument performance is presumably poor. This because the upper resonances of the soundboard do not couple profitably with the resonances of the air in the body: the particular structure of this soundboard tends to support the instrument performance in mid-low registers, while the limited volume of the body tends to favour the manifestation of air resonances at comparatively high frequencies (see Chap. 6 about upper resonances). Moreover, the amplitude of the tuning note at 84.5 Hz is quite small with respect to the resonance of the soundboard at 182.6 Hz.

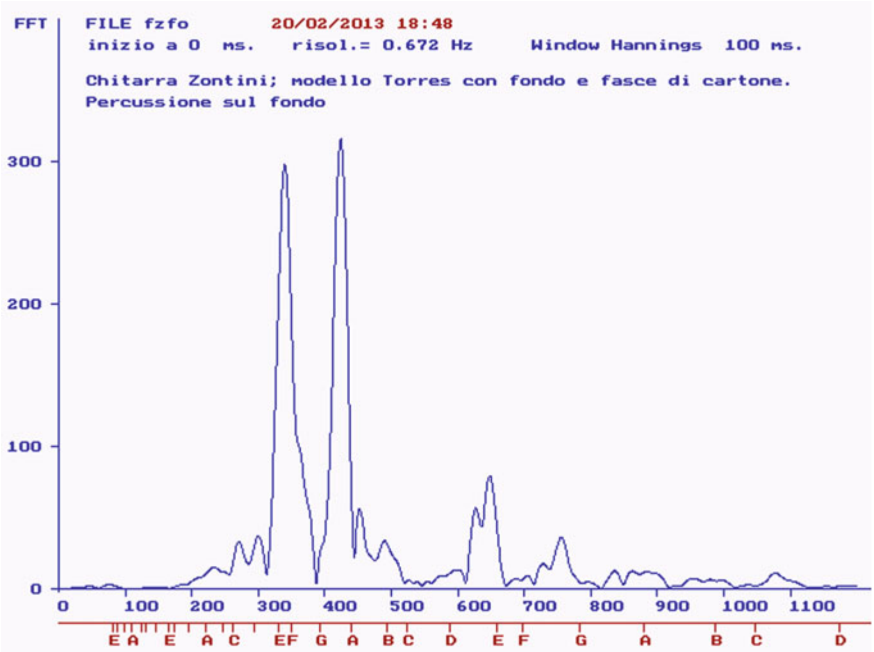
On the response diagram we also notice an important double resonance, with two contiguous peaks at 211 and 231 Hz. It is due to mode $\langle 1\ 0 \rangle$ of the soundboard, and corresponds to the second harmonic of the soundboard natural frequency $\mathbf{Fp_0}$ at 108 Hz, coupled (weakly) with the Helmholtz resonance.

We find these results in the third of octave diagram too, where we can see that mean values are rather balanced in the intervals between 160 and 500 Hz (central values—Appendix 1.5), while decreasing in the lower or upper intervals.

In the decay time diagram we notice that sustain at the tuning note frequency is quite scarce, in comparison with what we will find in more recent instruments. This is due to losses in the resonator components and to the level of the soundboard natural resonance (108 Hz) with respect to the Helmholtz resonance (143 Hz). Conceivably, the tuning note is strongly present in the recipe of sound (giving the acoustic texture a 'dark' colour) but its relatively short duration does not compromise the balance between harmonic components of the sounds. Sustain (in terms of decay time) is good from about 150–500 Hz, with a rather slow decay with respect to frequency.

This confirms the previous observation about the liveliness of the response in this range of frequency where sustain is presumably good, while it is poor above 500 Hz.

In order to facilitate the comprehension of the subsequent observations about the back, and its contribution to the attributes of sound in this instrument, we report the following graph obtained by percussion of the back at the bridge location.



First of all we can see that, on percussion of the back at the bridge location, two big resonances occur respectively at 340 and 426 Hz. A resonance with much smaller amplitude is manifested at about 300 Hz: this is one of the resonances due to the back, and it will be included in the 'global' response of the instrument (see Sects. 5.6 and 5.7). Normally, percussion of the back at the bridge location of a finished guitar mainly excites mode ⟨0 1⟩ of the back itself, determining a single, high resonance peak response. Here instead, besides mode ⟨0 1⟩, also mode ⟨0 2⟩ is excited along with a feeble appearance of mode ⟨0 0⟩. This probably implies a different distribution of the vibrating (antinodal) areas in comparison to the traditional one, illustrated in Sects. 4.3 and 13.2. Lacking additional data, we can claim that the vibrating area of mode ⟨0 0⟩ is off-centred in a higher position (being therefore scarcely excited by percussion at the bridge location) while the two vibrating areas of both mode ⟨0 1⟩ and ⟨0 2⟩ are off-centred in a lower position (and vigorously excited). We remind that the back, in this instrument, is made from cardboard (an isotropic material), while the traditional woods for back are orthotropic.

The natural frequencies of the back appear at quite high levels: the natural (estimated) frequency of mode ⟨0 0⟩ comes at 301 Hz, and the natural frequencies of

mode ⟨0 1⟩ and mode ⟨0 2⟩ are high-pitched as well (see preliminary data). Without more information, we would assume that the back is rigid and light. In conclusion: this back couples poorly with the inner air and with the soundboard; the natural frequencies of the back are very similar to the global resonances measured on the soundboard, and they effectively contribute to the overall response, especially in the band comprised between 300 and 400 Hz.

15.2 Fleta (1921)

♦ *Basic resonances*

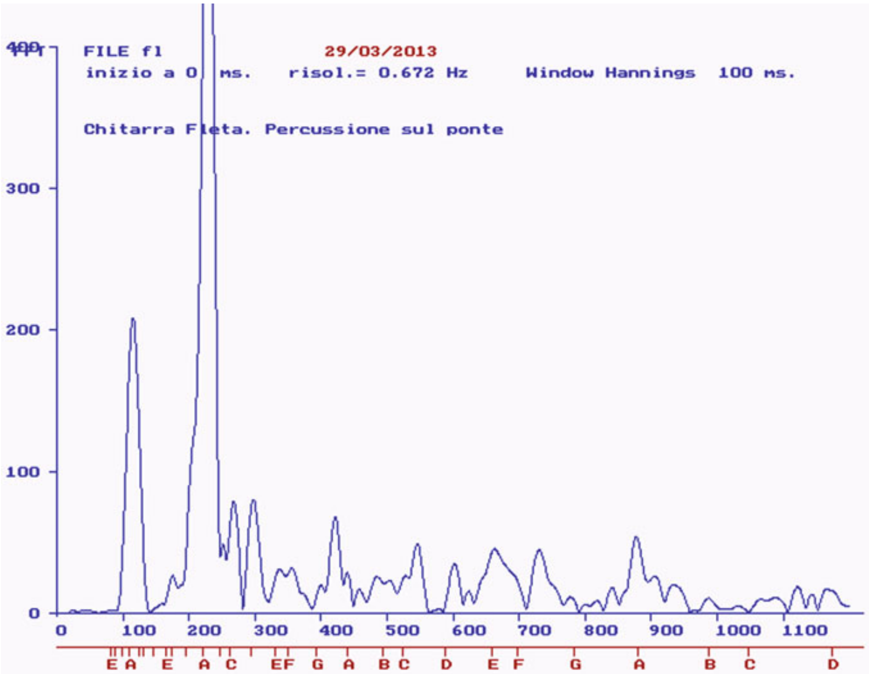
$\mathbf{F_1}$	(Resonance of the air)	115 Hz
$\mathbf{F_h}$	(Helmholtz resonance)	143 Hz
$\mathbf{F_2}$	(Basic resonance of the soundboard)	230 Hz
$\mathbf{F_p}$	(Covered soundhole resonance)	207 Hz
$\mathbf{F_{pm}}$	(Covered soundhole resonance with additional mass)	189 Hz
Diameter of the soundhole		87.3 mm

♦ *Resonances of the back* *(measured)*

		on the back	*on the soundboard*
F<00>	(Mode <00> resonance)	247 Hz	(244 Hz)
F<01>	(Mode <01> resonance)	272 Hz	(265 Hz)
F<02>	(Mode <02> resonance)	296 Hz	(296 Hz)

♦ *Characteristic parameters* *(estimated)*

Inner air volume	12 litres
$\mathbf{F_{p0}}$ (Natural frequency of the soundboard)	186 Hz
Natural frequency of the back (mode <00>)	189 Hz
Natural frequency of the back (mode <01>)	218 Hz
Natural frequency of the back (mode <02>)	251 Hz
Vibrating mass	101 g
Vibrating surface	617 cm^2
Soundboard stiffness	137229 N/m
Surface/mass ratio	6.12 cm^2/g
Coupling coefficient	0.86


FILE f1
29/03/2013
inizio a 0 ms. risol.= 0.672 Hz Window Hannings 100 ms.
Chitarra Fleta. Percussione sul ponte
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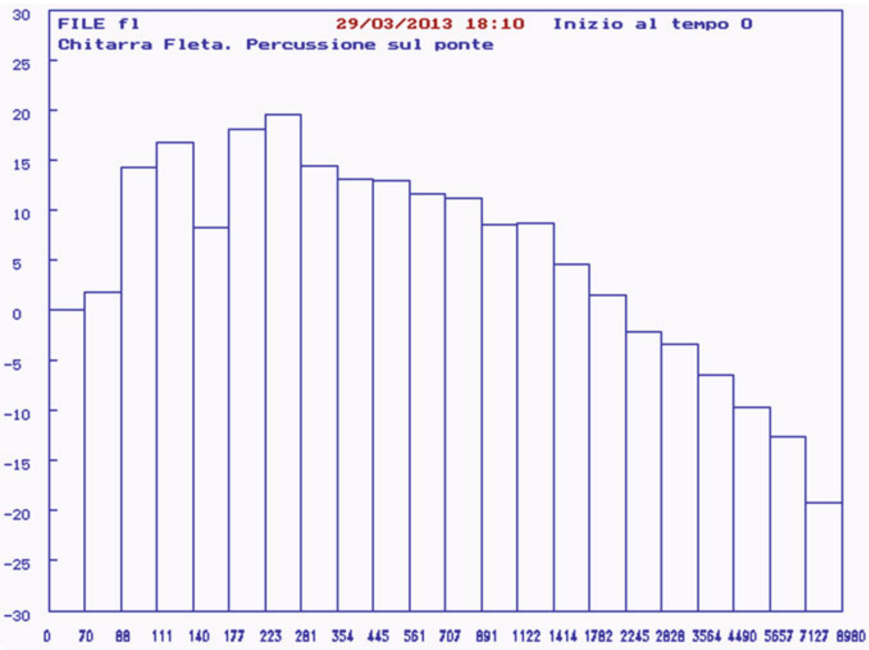
E A E A C EF G A B C D E F G A B C D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Chitarra Fleta. Percussione sul ponte
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-15
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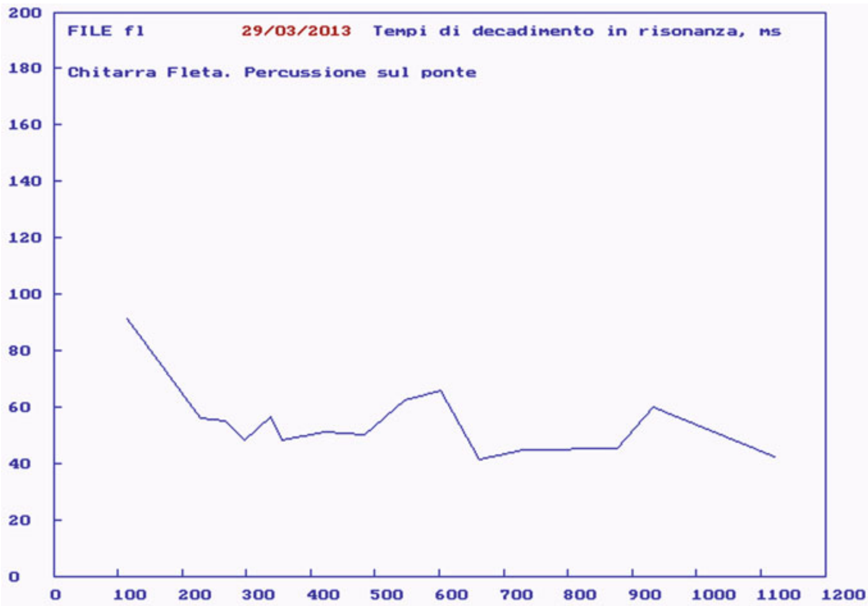
-25
-30
0 70 88 111 140 177 223 281 354 445 561 707 891 1122 1414 1782 2245 2828 3564 4490 5657 7127 8980

File fl 29/03/2013 18:10 Inizio al tempo 0 msec

Chitarra Fleta. Percussione sul ponte

valore delle terze tra 80 Hz e 8000 Hz	5.304
valore delle terze tra 80 Hz e 1000 Hz	12.51
valore delle terze tra 1260 Hz e 3175 Hz	1.817
diff. di livello tra terze 80 -1000 e 1260-3175	10.69
valore delle terze tra 4000 Hz e 8000 Hz	-11.96
valore delle terze tra 80 Hz e 125 Hz	10.88
valore delle terze tra 160 Hz e 250 Hz	15.26
valore delle terze tra 250 Hz e 400 Hz	15.64
valore delle terze tra 315 Hz e 500 Hz	13.46
valore della terza a 630 Hz	11.55
valore delle terze tra 800 Hz e 1260 Hz	9.461

Inizio al tempo 0 msec
Finestra di Hannings 100 msec.



15.2.1 Comments

About sixty years separate this instrument from the 1862 Torres guitar (replicated by the luthier Zontini). Designing criteria revealed by analyses are very different, even if actual results do not show any significant progress in performance.

The estimated volume of the air inside the body (12 L) is slightly greater than that of the Torres-Zontini guitar (10.7 L), but the Helmholtz resonance appears at approximately the same frequency (143 Hz), because now the diameter of the soundhole is larger (see Sect. 4.1).

The most noticeable characteristic highlighted by the examination of this instrument is the considerable stiffness (137229 N/m) and weight (101 g) of the soundboard, with respect to the average stiffness and vibrating mass found in other instruments. Despite a large vibrating surface, the surface/mass ratio (6.12) is small.

Lacking additional information, we may infer that the ‘bare’ soundboard is particularly thick. As a consequence, the natural frequency of the soundboard is equally high ($\mathbf{F_{p0}} = 186$ Hz). This implies that basic resonances are also rather high if compared with the standards of more recent guitars, but also in comparison with the Torres-Zontini guitar, too. Now the resonance of the air $\mathbf{F_1}$ (the tuning note) appears at 115 Hz (corresponding to A#) while the resonance of the soundboard $\mathbf{F_2}$ occurs at 230 Hz. These values are too high to provide a good performance in the low register.

In the response graph obtained from percussion of the soundboard, we notice the considerable amplitude of the basic resonances. This information needs to be interpreted. The decay time diagram shows how, at these frequency levels, amplitudes rapidly decrease; in other words, sustain is poor and the tuning note is present, but short lived in the recipe of sound. This is typical of a ‘tympanic’ sound, and probably due to friction losses in the soundboard. In short, the graph reveals that decay times rapidly diminish within the whole mid-low frequency band, up to 400 Hz.

Beyond the resonance of the soundboard at 230 Hz, no other resonance presents significant amplitude; the resonance in mode $\langle 0\ 1 \rangle$ appears at about 420 Hz, with rather scarce amplitude. In the mid-high register, several resonance peaks have intermediate amplitude values up to about 900 Hz. Some of the resonances due to the coupling between soundboard, air and back (relative to upper vibration modes discussed in Sect. 6.1) are visible, but none of them stands out clearly. This means that, in the mid-high register, the coupling of the soundboard with the air in the body is not really efficient.

These observations are confirmed by the third of octave response graph: a ‘gap’ is visible in the interval at 160 Hz (central value), owing to the pitch of the basic resonance of the soundboard (230 Hz) in comparison with the resonance of the air (115 Hz). Also, the response lacks overall balance, up to at least 300 Hz.

Interesting to notice, the estimated natural frequency of the back in mode $\langle 0\ 0 \rangle$ (189 Hz) is virtually the same as the soundboard natural frequency (186 Hz). This result, though it may have been intentionally pursued, brings the resonances due to the back to an excessively low frequency within the global response of the instrument (see diagram). The resonance of the back in mode $\langle 0\ 0 \rangle$ is practically absent, mingled with the basic resonance of the soundboard; besides, the resonances of the back in mode $\langle 0\ 1 \rangle$ and $\langle 0\ 2 \rangle$ are also too low in frequency to ‘fill up’ the mid-low register.

Finally, a double resonance is visible around 350 Hz (intermediate level between the two peaks), resulting from the soundboard resonance in mode $\langle 1\ 0 \rangle$, weakly coupled with the air in the body.

15.3 Simplicio (1931)

♦ *Basic resonances*

$\mathbf{F_1}$	(Resonance of the air)	96	Hz
$\mathbf{F_h}$	(Helmholtz resonance)	129	Hz
$\mathbf{F_2}$	(Basic resonance of the soundboard)	193	Hz
$\mathbf{F_p}$	(Covered soundhole resonance)	170	Hz
$\mathbf{F_{pm}}$	(Covered soundhole resonance with additional mass)	143	Hz
Diameter of the soundhole		85.5	mm

♦ *Resonances of the back* *(measured)* *on the back* *on the soundboard*

		on the back	*on the soundboard*
F<00>	(Mode <00> resonance)	256 Hz	(258 Hz)
F<01>	(Mode <01> resonance)	297 Hz	(299 Hz)
F<02>	(Mode <02> resonance)	352 Hz	(359Hz)

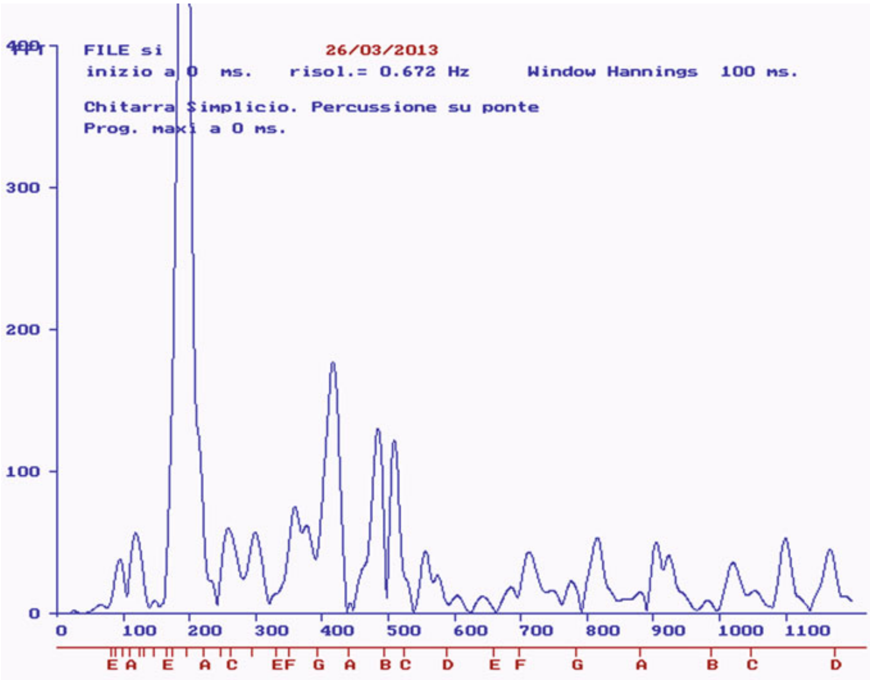
♦ *Characteristic parameters* *(estimated)*

Inner air volume 14 litres

$\mathbf{F_{p0}}$ (Natural frequency of the soundboard) 144 Hz

Natural frequency of the back (mode <00>)	240 Hz
Natural frequency of the back (mode <01>)	280 Hz
Natural frequency of the back (mode <02>)	320 Hz

Vibrating mass	49 g
Vibrating surface	417 cm^2
Soundboard stiffness	39884 N/m
Surface/mass ratio	8.56 cm^2/g
Coupling coefficient	0.86


FILE si
26/03/2013
inizio a 0 ms. risol.= 0.672 Hz Window Hannings 100 ms.
Chitarra Simplicio. Percussione su ponte
Prog. maxi a 0 ms.
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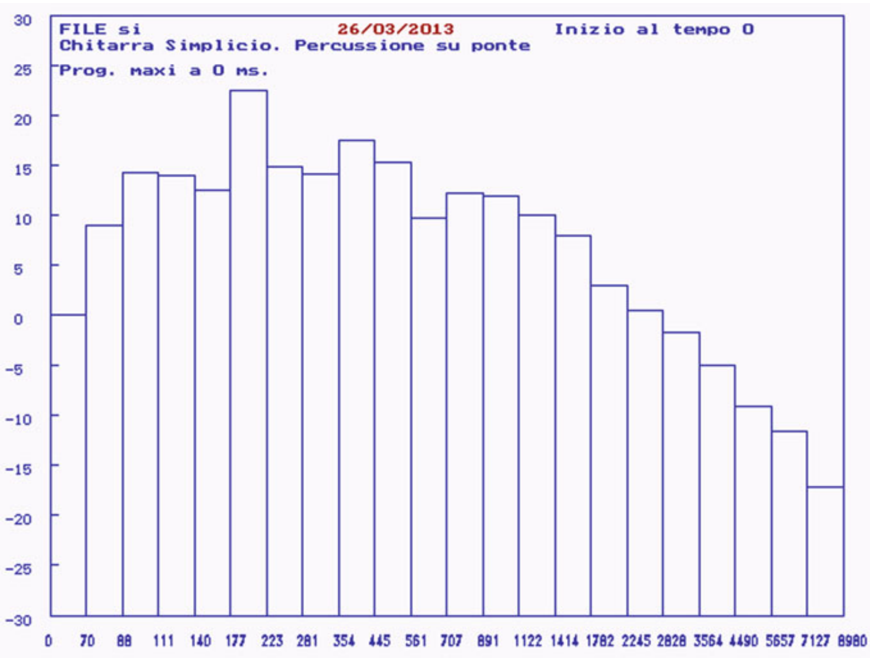
0 100 200 300 400 500 600 700 800 900 1000 1100
E A E A C EF G A B C D E F G A B C D


FILE si
26/03/2013
Inizio al tempo 0
Chitarra Simplicio. Percussione su ponte
Prog. maxi a 0 ms.
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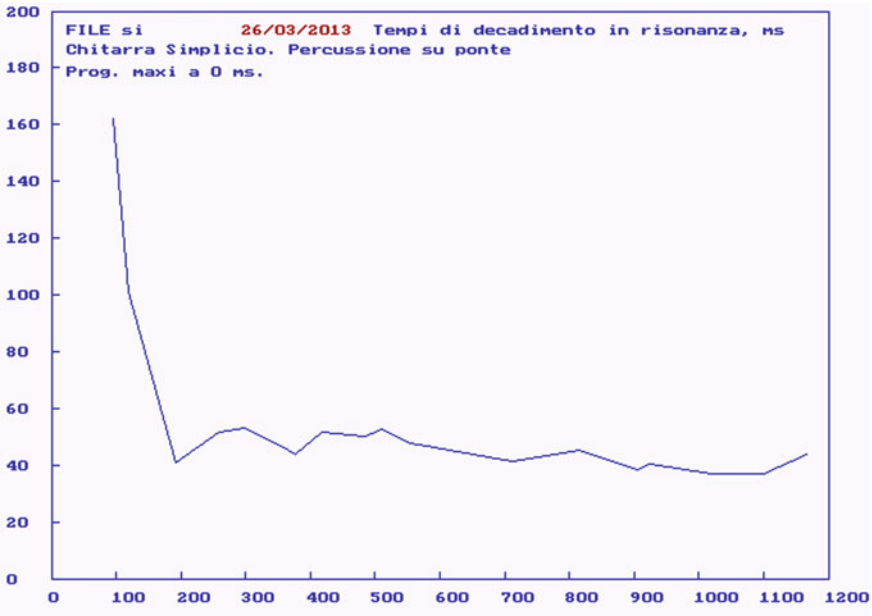
0 70 88 111 140 177 223 281 354 445 561 707 891 1122 1414 1782 2245 2828 3564 4490 5657 7127 8980

File si 26/03/2013 Inizio al tempo 0 msec

Chitarra Simplicio. Percussione sul ponte
Prog. maxi

valore delle terze tra 80 Hz e 8000 Hz	6.862
valore delle terze tra 80 Hz e 1000 Hz	13.94
valore delle terze tra 1260 Hz e 3175 Hz	3.897
diff. di livello tra terze 80 -1000 e 1260-3175	10.05
valore delle terze tra 4000 Hz e 8000 Hz	-10.69
valore delle terze tra 80 Hz e 125 Hz	12.37
valore delle terze tra 160 Hz e 250 Hz	16.56
valore delle terze tra 250 Hz e 400 Hz	15.49
valore delle terze tra 315 Hz e 500 Hz	15.62
valore della terza a 630 Hz	9.665
valore delle terze tra 800 Hz e 1260 Hz	11.32

Inizio al tempo 0 msec
Finestra di Hannings 100 msec.



15.3.1 Comments

Just ten years separate this guitar from the Fleta examined above, but the analysis highlights a turning point in designing criteria. The structure of soundboard, body and back has changed and, accordingly, the characteristic parameters of the acoustic response have also changed. This is finally a 'modern' instrument, whose good performances are highlighted by the analysis.

The greater volume of the body (about 14 litres) and the diameter of the soundhole (85.5 mm) establish the Helmholtz resonance $\mathbf{F_h}$ at 129 Hz, considerably lower than that of older guitars. The natural frequency of the soundboard is $\mathbf{F_{p0}} = 144$ Hz, as a consequence of comparatively low vibrating mass (49 g) and limited soundboard stiffness (39884 N/m). These parameters adequately approach the optimal values indicated in Chap. 5: so we may suppose a flexible and rather lightly braced soundboard. The vibrating surface is quite small (417 cm^2), owing perhaps to the bracing style which prevents the soundboard from fully 'breathing', up beyond the soundhole, but the surface/mass ratio (8.56 cm^2/g) is however high enough, due to the limited vibrating mass. Sound pressure—as seen in Sect. 5.5—actually depends on this ratio.

The first basic resonance of the air (the tuning note), found at the frequency $\mathbf{F_1} = 96$ Hz, lies between F# and G on the sixth string; the frequency of the second basic resonance of the soundboard is $\mathbf{F_2} = 193$ Hz, therefore exactly an octave above the resonance of the air. These are optimal values, according to quality standards illustrated in Chap. 5. We must always consider these basic resonances not just with reference to their frequency, but to their decay time as well (see diagram). In fact, decay times determine sustain (their permanence in the recipe of sound); on the other hand, the frequency of the basic resonances (especially of the air) determines the 'colour' of the sound—whether brighter or darker—according to pitch. Notice that the decay time is rather long at the resonance of the air $\mathbf{F_1} = 96$ Hz, but it rapidly diminishes at the resonance of the soundboard $\mathbf{F_2} = 193$ Hz.

In the response diagram, a large resonance due to vibration mode $\langle 0\ 1 \rangle$ of the resonator stands out at 416 Hz. This resonance has been examined in Sect. 6.1. A couple of important resonances is present at a slightly higher frequency (with peaks at 484 and 509 Hz), resulting from the coupling between the soundboard and the air in the body. Beyond 550 Hz (namely C# on the first string) the response in mid-high registers tends to be flattened, and resonances are present but limited to quite low levels. This because the upper resonances of the soundboard do not favourably couple with those of the air in the body, owing to the bracing of the soundboard which, on one hand, tends to support the action of the vibrating areas in the mid-low register while, on the other hand, it does not allow a proper development of upper vibration areas (we refer the reader to Chap. 6 for the analysis of upper resonances). Accordingly, the decay time also tends to diminish beyond the limit of 550 Hz, probably implying short sustain of the instrument mid-high tones.

Proceeding with the analysis of the response obtained by percussion on the bridge, between the first basic resonance of the soundboard (at 193 Hz) and the resonance

in mode $\langle 0\ 1 \rangle$ (at 416 Hz) we find a set of significant resonances due to the back and to the soundboard resonance in mode $\langle 1\ 0 \rangle$. The resonances due to the back appear at 258 Hz (mode $\langle 0\ 0 \rangle$), 299 Hz (mode $\langle 0\ 1 \rangle$) and 359 Hz (mode $\langle 0\ 2 \rangle$)—see above table. The estimated natural frequencies of the back are 240 Hz (mode $\langle 0\ 0 \rangle$), 280 Hz (mode $\langle 0\ 1 \rangle$) and 320 Hz (mode $\langle 0\ 2 \rangle$), therefore well above the basic resonance of the soundboard $\mathbf{F_2} = 193$ Hz. Presumably, the back of this instrument is rather stiff, but its resonances are well visible in the response graph (see Sect. 5.7, dealing with the resonances of the back). In addition to that, the response diagram by percussion on the bridge highlights a resonance at 376 Hz, due to the soundboard oscillation mode $\langle 1\ 0 \rangle$ in combination with the air in the body.

The observations above are validated by the third of octave graph. Sound emission looks balanced in the intervals between 100 and 500 Hz (mid values), while decreasing in the subsequent intervals. Moreover, the mean emission value (13.94) in the range between 80 and 1000 Hz is quite considerable: as seen in Sect. 1.6, it represents the mean sound emission value in the frequency range involving the fundamentals and the harmonics relevant for a good sound. Therefore, this value characterizes the instrument *yield* within a large frequency band: the higher this value, the more efficiently the instrument turns the available energy into sound emission level. The reader may compare the mean values of this and the other thirds with the same parameters measured on other guitars, linking them to specific response features.

15.4 Gallinotti (1974)

♦ *Basic resonances*

$\mathbf{F_1}$	(Resonance of the air)	104 Hz
$\mathbf{F_h}$	(Helmholtz resonance)	129 Hz
$\mathbf{F_2}$	(Basic resonance of the soundboard)	221 Hz
$\mathbf{F_p}$	(Covered soundhole resonance)	209 Hz
$\mathbf{F_{pm}}$	(Covered soundhole resonance with additional mass)	185 Hz
Diameter of the soundhole		84 mm

♦ *Resonances of the back* *(measured)*

		on the back	*on the soundboard*
F<00>	(Mode <00> resonance)	244 Hz	(239 Hz)
F<01>	(Mode <01> resonance)	280 Hz	(278 Hz)
F<02>	(Mode <02> resonance)	339 Hz	(340 Hz)

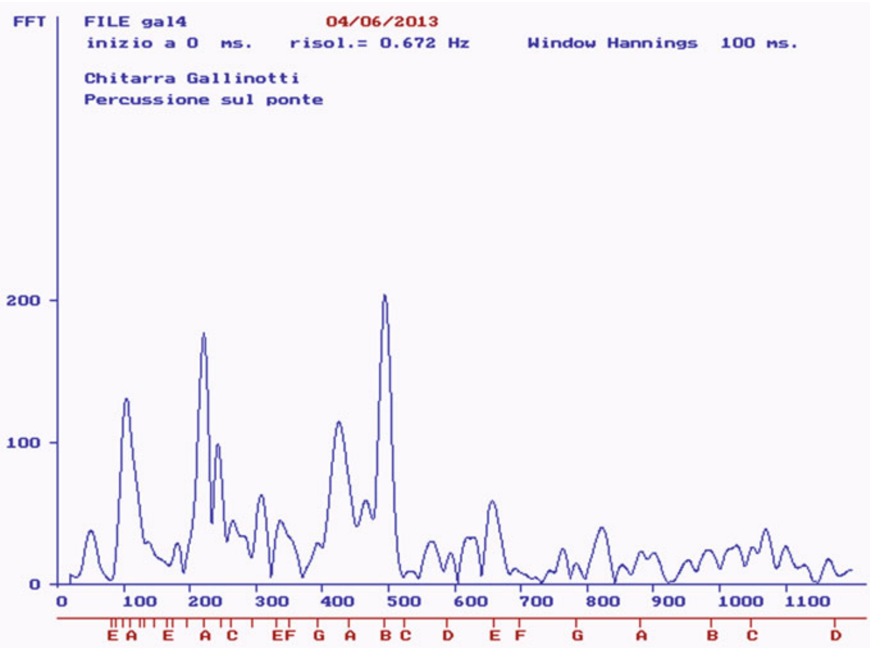
♦ *Characteristic parameters (estimated)*

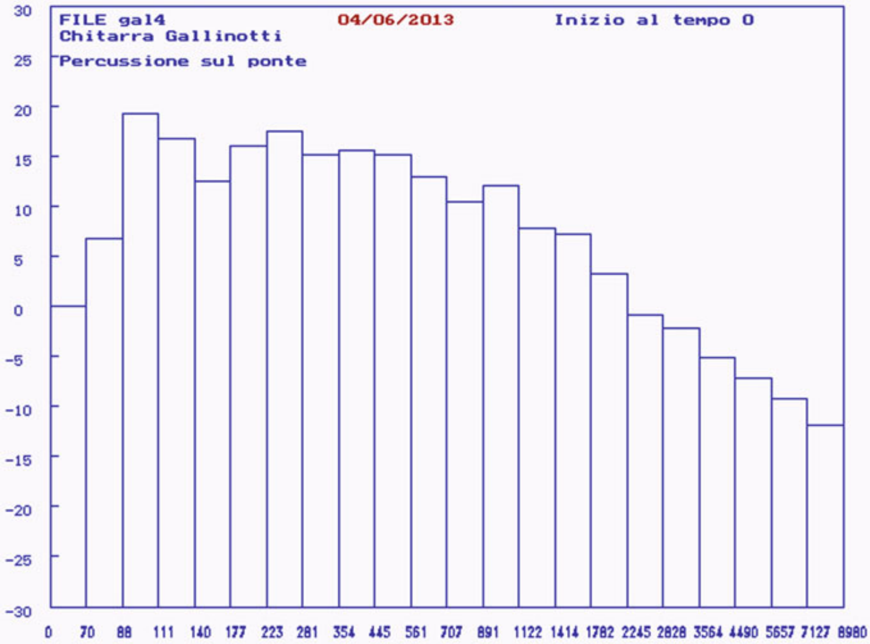
Inner air resonance 14 litri

F_{p0} (Natural frequency of the soundboard) 177 Hz

Natural frequency of the back (mode <00>) 215 Hz
Natural frequency of the back (mode <01>) 231 Hz
Natural frequency of the back (mode <02>) 316 Hz

Vibrating mass	74 g
Vibrating surface	571 cm^2
Soundboard stiffness	91400 N/m
Surface/mass ratio	7.75 cm^2/g
Coupling coefficient	0.91



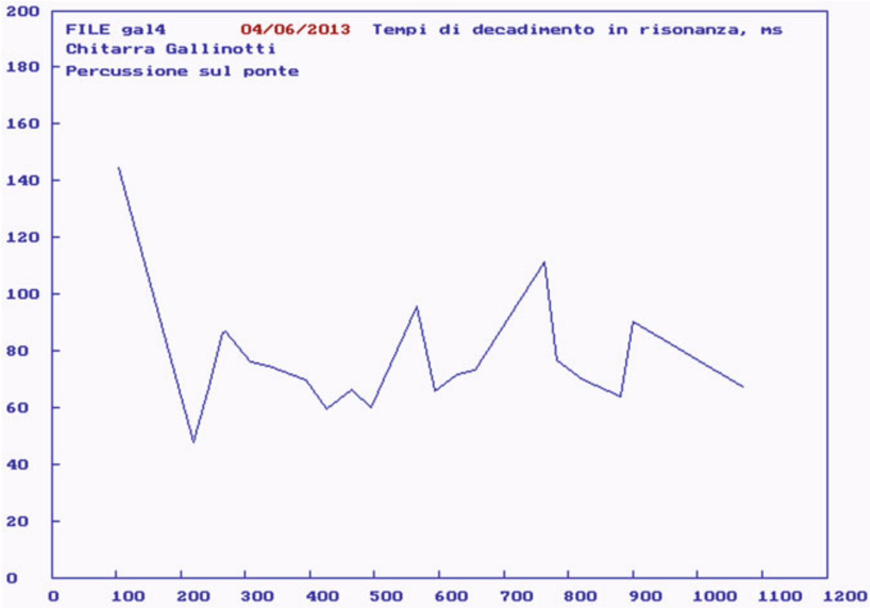


File gal4 04/06/2013 Inizio al tempo 0 msec

Chitarra Gallinotti
Percussione sul ponte

valore delle terze tra 80 Hz e 8000 Hz	7.208
valore delle terze tra 80 Hz e 1000 Hz	14.15
valore delle terze tra 1260 Hz e 3175 Hz	3.013
diff. di livello tra terze 80 -1000 e 1260-3175	11.14
valore delle terze tra 4000 Hz e 8000 Hz	-8.394
valore delle terze tra 80 Hz e 125 Hz	14.19
valore delle terze tra 160 Hz e 250 Hz	15.33
valore delle terze tra 250 Hz e 400 Hz	16.06
valore delle terze tra 315 Hz e 500 Hz	15.28
valore della terza a 630 Hz	12.88
valore delle terze tra 800 Hz e 1260 Hz	10.08

Inizio al tempo 0 msec
Finestra di Hannings 100 msec.



15.4.1 Comments

The first basic resonance of the air—the tuning note—at the frequency $\mathbf{F_1} = 104$ Hz corresponds exactly to the pitch of G# on the sixth string; the frequency of the second basic resonance of the soundboard is $\mathbf{F_2} = 221$ Hz, corresponding to A on the fifth open string, i.e. more than an octave above the resonance of the air. According to standards indicated in Chap. 5, these values are a little too high for an optimal response of low tones.

The natural frequency of the soundboard is $\mathbf{F_{p0}} = 177$ Hz—a rather considerable value; this resonance results from a quite high vibrating mass (74 g), and a considerable stiffness of the soundboard (91400 N/m). We may say that the soundboard is rigid and, at the same time, overloaded by the bracing. The vibrating surface is very large (571 cm^2) because of the excellent coupling coefficient (0.91), but the surface/mass ratio (7.75 cm^2/g) is comparatively scarce, owing to the conspicuous vibrating mass. We remind again that the sound pressure level depends on this parameter (see Sect. 5.5).

Looking at the decay time diagram we notice a rather short sustain, both at the basic resonance of the air (144 ms) and at the basic resonance of the soundboard (48 ms). The permanence of these tones in the recipe of sound is not particularly long, which confers a ‘sweet’ and ‘softer’ character to the acoustic texture—at least in our opinion. In the decay time graph we notice a ‘gap’ at the level of the soundboard basic resonance $\mathbf{F_2} = 221$ Hz. We recall the reason: amplitude is large at the resonance

level, but the energy of the string is rapidly absorbed by the resonator, because of its low impedance at resonance levels; this, in turn, is due to highly selective resonances (displayed on the graph in the form of sharp peaks). Beyond the resonance of the soundboard, decay times are generally very long and, at certain frequencies (270, 565, 764, 900 Hz), they assume exceptionally high values. In comparison, the equivalent graph of the Simplicio guitar reports generally short values beyond the resonance of the soundboard.

However, this graph needs correct interpretation. Long decay time at a given frequency does not imply equally strong response of the resonator at that frequency. For instance, at 764 Hz (between F# and G on the first string) we find a resonance whose amplitude (25) is quite small in our conventional scale, while its decay time (111 ms) is actually very long. We deduce that the harmonics of the tones close to that frequency are not particularly enhanced by the characteristics of the resonator response, but they slowly damp down manifesting a very long sustain.

If we extend these observations to registers comprised between the resonance of the soundboard $\mathbf{F_2} = 221$ Hz (A on the fifth open string) and 980 Hz (C at the 20$^{\text{th}}$ fret on the first string) we can assert that the sustain is generally high with no relevant 'drop' in sound emission at specific tones, although amplitude is not remarkably large beyond 500 Hz (close to C on the first string).

The curve of the response in the mid-high register features a considerable resonance at 425 Hz, due to the oscillation of the resonator in mode $\langle 0\ 1\rangle$—see Sect. 6.1. At 495 Hz occurs a very wide resonance of mode $\langle 2\ 0\rangle$, due to the coupling between soundboard and air in the body (see Chap. 6). Beyond this limit the response is quite poor, except for a resonance of medium amplitude at 650 Hz.

Between the resonance of the soundboard (at 221 Hz) and that of mode $\langle 0\ 1\rangle$ (at 425 Hz) we find a group of resonances due to the back. On percussion of the soundboard they appear at 239 Hz (mode $\langle 0\ 0\rangle$), 278 Hz (mode $\langle 0\ 1\rangle$) and 340 Hz (mode $\langle 0\ 2\rangle$)—see the above table. The distinctive frequencies of the back (estimated) are 215 Hz (mode $\langle 0\ 0\rangle$), 231 Hz (mode $\langle 0\ 1\rangle$) and 316 Hz (mode $\langle 0\ 2\rangle$). The resonances due to the back stand out clearly in the diagram of the instrument response, at a suitable level with respect to the frequency of the soundboard. Within the same frequency range appears another noteworthy resonance at 308 Hz, due to oscillation mode $\langle 1\ 0\rangle$.

As seen in previous examples, the observations above are confirmed by the third of octave graph. Note that sound emission is balanced in the intervals between 100 and 630 Hz (central values), while decaying within higher frequency intervals. The 'gap' in the interval at 160 Hz is due to the frequency of the soundboard basic resonance $\mathbf{F_2} = 221$ Hz, if compared to the resonance of the air $\mathbf{F_1} = 104$ Hz.

We must also notice the considerable mean emission value (14.15 dB) in the range from 80 to 1000 Hz. As observed in Sect. 1.6, this is the mean value of the sound pressure level in the range of frequencies involving fundamentals and harmonics that are relevant to sound quality.

15.5 Ramirez (1982)

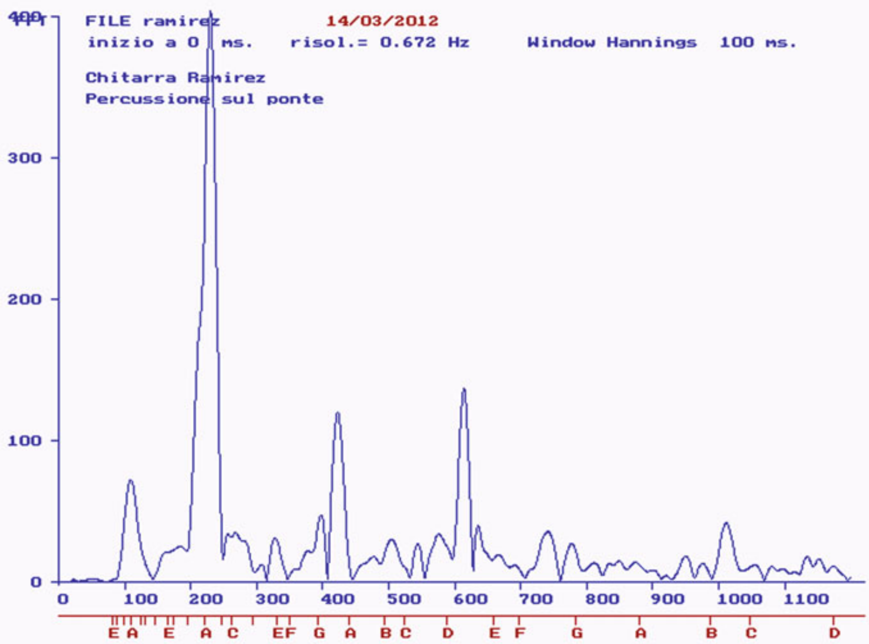
♦ *Basic resonances*

F_1	(Resonance of the air)	109	Hz
F_h	(Helmholtz resonance)	121	Hz
F_2	(Basic resonance of the soundboard)	230	Hz
F_p	(Covered soundhole resonance)	217	Hz
F_{pm}	(Covered soundhole resonance with additional mass)	192	Hz
Diameter of the soundhole		83	mm

♦ *Resonances of the back*	*(measured)*	*on the back*	*on the soundboard*
F<00>	(Mode <00> resonance)	236 Hz	(244 Hz)
F<01>	(Mode <01> resonance)	262 Hz	(265 Hz)
F<02>	(Mode <02> resonance)	304 Hz	(325 Hz)

♦ *Characteristic parameters (estimated)*

Inner air volume	16 litres
F_{p0} (Natural frequency of the soundboard)	207 Hz
Natural frequency of the back (mode <00>)	200 Hz
Natural frequency of the back (mode <01>)	240 Hz
Natural frequency of the back (mode <02>)	292 Hz
Vibrating mass	70 g
Vibrating surface	468 cm^2
Soundboard stiffness	120000 N/m
Surface/mass ratio	6.7 cm^2/g
Coupling coefficient	0.83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14/03/2012
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Window Hannings 100 ms.
Chitarra Ramirez
Percussione sul ponte
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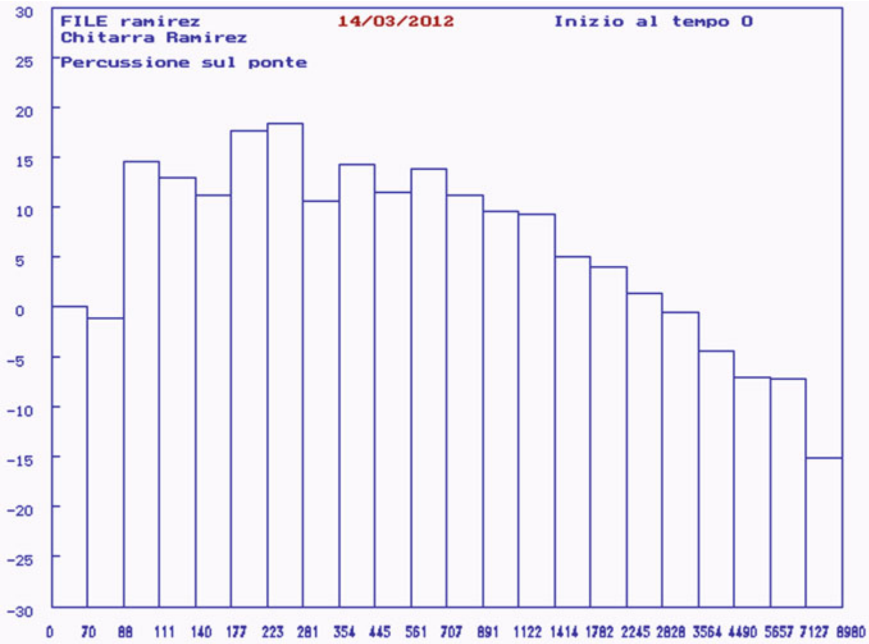
FILE ramirez
14/03/2012
Inizio al tempo 0
Chitarra Ramirez
Percussione sul ponte
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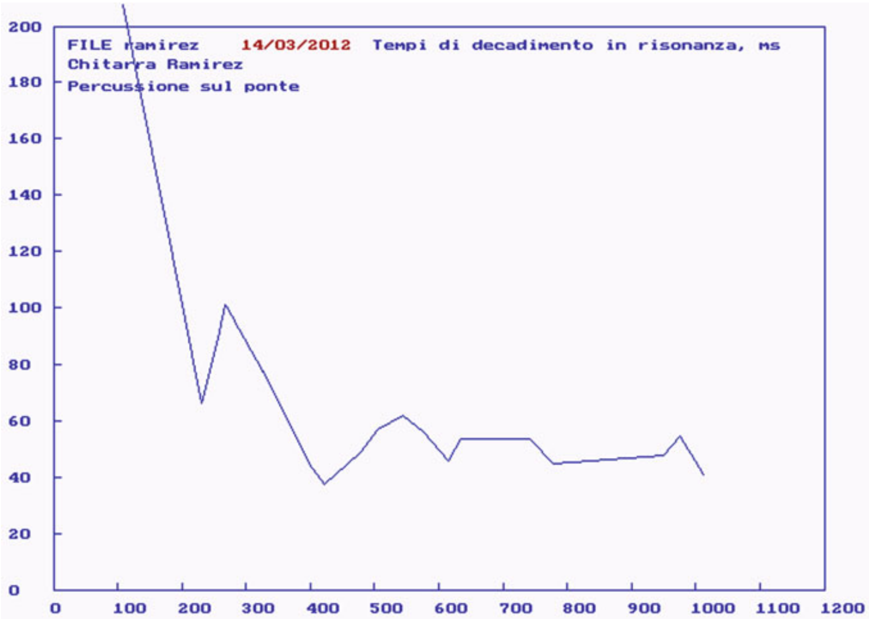
0 70 88 111 140 177 223 281 354 445 561 707 891 1122 1414 1782 2245 2828 3564 4490 5657 7127 8980

File ramirez 14/03/2012 Inizio al tempo 0 msec

Chitarra Ramirez
Percussione sul ponte

valore delle terze tra 80 Hz e 8000 Hz	6.134
valore delle terze tra 80 Hz e 1000 Hz	11.99
valore delle terze tra 1260 Hz e 3175 Hz	3.761
diff. di livello tra terze 80 -1000 e 1260-3175	8.228
valore delle terze tra 4000 Hz e 8000 Hz	-8.468
valore delle terze tra 80 Hz e 125 Hz	8.726
valore delle terze tra 160 Hz e 250 Hz	15.71
valore delle terze tra 250 Hz e 400 Hz	14.39
valore delle terze tra 315 Hz e 500 Hz	12.05
valore della terza a 630 Hz	13.75
valore delle terze tra 800 Hz e 1260 Hz	9.934

Inizio al tempo 0 msec
Finestra di Hannings 100 msec.



15.5.1 Comments

It is worth noticing that the tuning note is quite high pitched, in this instrument, despite a very voluminous body. The measured Helmholtz resonance is $\mathbf{F_h} = 121$ Hz, and

16 litres the estimated volume. The ‘tuning note’ (the basic frequency of the air) is manifested at $\mathbf{F_1}$ = 109 Hz (close to A on the fifth open string), while the basic frequency of the soundboard is $\mathbf{F_2}$ = 230 Hz (around A# on the third string—therefore not exactly an octave above the ‘tuning note’). This is an unexpected result; we would suppose lower values for the basic resonances, but the analysis proves this behaviour to be the result of a particular soundboard structure: the natural frequency of the soundboard (estimated) is $\mathbf{F_{p0}}$ = 207 Hz; this is due to an exceptional stiffness of the soundboard (120000 N/M), while the vibrating mass is limited to 70 g. In absence of additional information, we can claim that the bracing is very rigid and the soundboard thin. The vibrating surface of the soundboard is just 470 cm^2, quite a small value if we consider that, in this instrument, the whole surface of the soundboard is higher than usual. As a consequence, the surface/mass ratio is also poor (6.7 cm^2/g). We can assume that the bracing does not allow free vibration of the soundboard above the soundhole.

Based on analyses, we can say that the instrument does not respond at best in the low register (relative to basic resonances). But if we observe the decay time graph, we see that sustain at the basic resonance of the air is very long (208 ms), meaning a long permanence of the tuning note in the sound texture. Decay time levels remain considerable high up to at least 550 Hz (C# on the first string); this in turn implies good sustain in the mid-high register, too. The decay time chart presents some ‘gaps’ in correspondence with three important resonances (visible in the response diagram at 230, 422 and 635 Hz). The phenomenon has been already discussed in the text: amplitude is large in coincidence with resonances, but the energy of the string is rapidly absorbed by the resonator (its impedance being low at resonance levels); therefore, the tones associated with these frequencies quickly damp down (and the decay time is accordingly short). This is due to highly selective—or ‘sharp’—resonances: at these frequency levels, sound assumes a percussive character.

The response diagram shows a strong resonance at 422 Hz, associated to vibration mode ⟨0 1⟩ of the resonator, whose origin has been previously explained. Between the second basic resonance of the soundboard at $\mathbf{F_2}$ = 230 Hz and the resonance in mode ⟨0 1⟩ at 422 Hz we find a set of significant resonances, due to the back and to the soundboard resonance in mode ⟨1 0⟩. The latter appears at 398 Hz, i.e. about twice the natural frequency of the soundboard $\mathbf{F_{p0}}$ = 207 **Hz.**

The resonances due to the back occur at 244 Hz (mode ⟨0 0⟩), 265 Hz (mode ⟨0 1⟩) and 325 Hz (mode ⟨0 2⟩)—see table. The two lower resonances are close to the large basic resonance of the soundboard, and they do not stand out noticeably. This because the natural resonance of the back in mode ⟨0 0⟩ (estimated) occurs at 200 Hz, therefore virtually matching the natural resonance of the soundboard $\mathbf{F_{p0}}$ = 207 **Hz**. This probably being the constructor’s purpose, results on the other hand detrimental to the manifestation of important resonances due to the back.

Besides mode ⟨0 1⟩ resonance at 422 Hz, a relevant resonance is present at 614 Hz, due to the coupling between the soundboard and the air in the body. Nevertheless, between these two resonances, response is rather flat. Beyond 614 Hz, the resonances at 741 Hz and 1011 Hz stand out in the response diagram. In Chap. 6 we dealt with the meaning of these upper resonances due to the coupling between soundboard and air in the body.

These observations are condensed in the third of octave graph. Response is not very balanced even in intermediate registers, but drops significantly only beyond the interval at 1000 Hz (mid frequency level). The mean emission value in the band between 80 and 1000 Hz is the mean level of sound in the frequency band relative to fundamentals and harmonics relevant to sound quality. Based on analyses, from the measurement carried out on this instrument we obtain a value of about 12 dB—not an outstanding result.

15.6 Garrone 92 (2011)

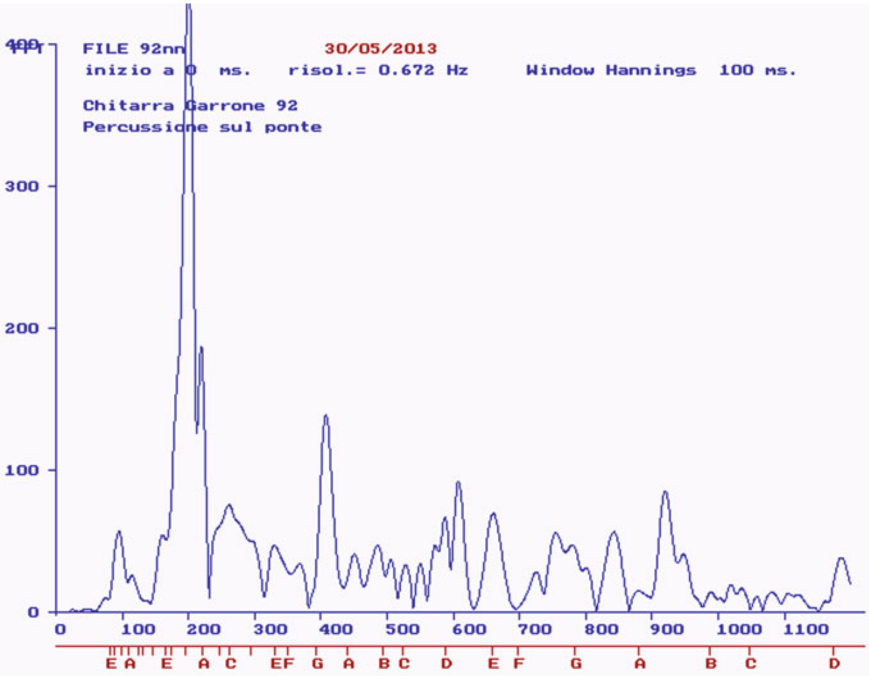
♦ <u>*Basic resonances*</u>

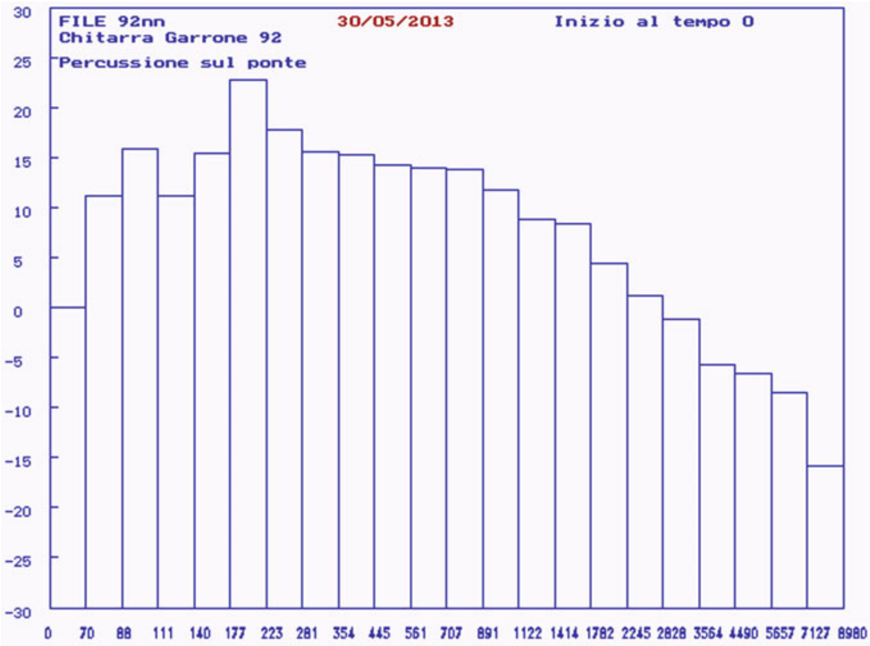
$\mathbf{F_1}$ (Resonance of the air)	95 Hz
$\mathbf{F_h}$ (Helmholtz resonance)	129 Hz
$\mathbf{F_2}$ (Basic resonance of the soundboard)	200 Hz
$\mathbf{F_p}$ (Covered soundhole resonance)	189 Hz
$\mathbf{F_{pm}}$ (Covered soundhole resonance with additional mass)	162 Hz
Diameter of the soundhole	84 mm

♦ <u>*Resonances of the back*</u>	*(measured)*	*on the back*	*on the soundboard*
F<00>	(Mode <00> resonance)	237 Hz	(244 Hz)
F<01>	(Mode <01> resonance)	259 Hz	(261 Hz)
F<02>	(Mode <02> resonance)	336 Hz	(330 Hz)

♦ <u>*Characteristic parameters*</u> *(estimated)*

Inner air volume	14 litres
$\mathbf{F_{p0}}$ (Natural frequency of the soundboard)	147 Hz
Natural frequency of the back (mode <00>)	202 Hz
Natural frequency of the back (mode <01>)	231 Hz
Natural frequency of the back (mode <02>)	310 Hz
Vibrating mass	58 g
Vibrating surface	489 cm^2
Soundboard stiffness	49000 N/m
Surface/mass ratio	8.5 cm^2/g
Coupling coefficient	0.895


FILE 92nn
30/05/2013
inizio a 0 ms. risol.= 0.672 Hz Window Hannings 100 ms.
Chitarra Garrone 92
Percussione sul ponte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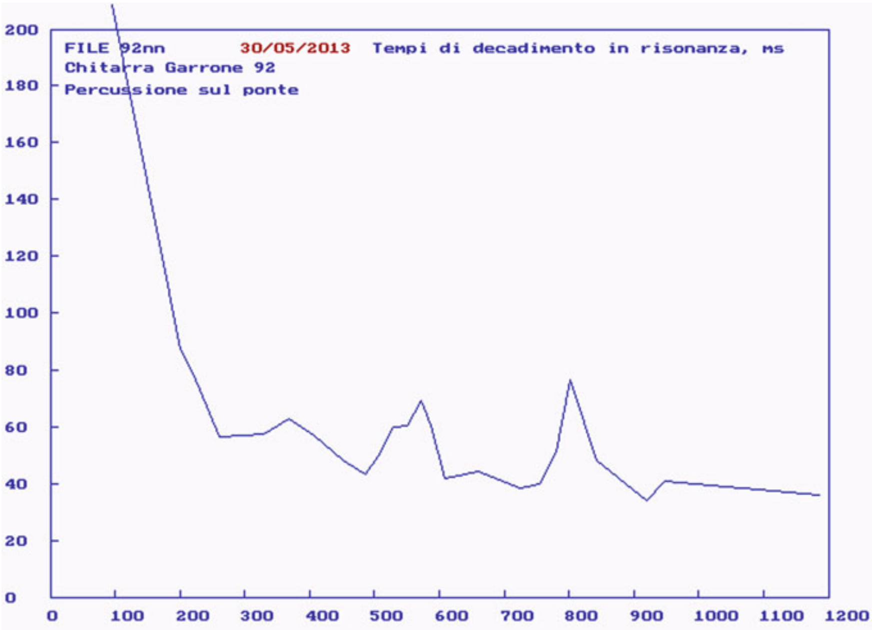
30/05/2013
Inizio al tempo 0
Chitarra Garrone 92
Percussione sul ponte

File 92nn 30/05/2013 Inizio al tempo 0 msec

Chitarra Garrone 92
Percussione sul ponte

valore delle terze tra 80 Hz e 8000 Hz	7.777
valore delle terze tra 80 Hz e 1000 Hz	14.86
valore delle terze tra 1260 Hz e 3175 Hz	4.322
diff. di livello tra terze 80 -1000 e 1260-3175	10.54
valore delle terze tra 4000 Hz e 8000 Hz	-9.163
valore delle terze tra 80 Hz e 125 Hz	12.67
valore delle terze tra 160 Hz e 250 Hz	18.62
valore delle terze tra 250 Hz e 400 Hz	16.19
valore delle terze tra 315 Hz e 500 Hz	15.00
valore della terza a 630 Hz	13.91
valore delle terze tra 800 Hz e 1260 Hz	11.46

Inizio al tempo 0 msec
Finestra di Hannings 100 msec.



15.6.1 Comments

In this instrument the first basic resonance of the air—the tuning note—occurs at $\mathbf{F_1} =$ 95 Hz, hence between F# and G on the sixth string; the frequency of the second basic resonance of the soundboard is $\mathbf{F_2} = 200$ Hz—slightly more than an octave above the resonance of the air. According to quality standards presented in Chap. 5 these are optimal values, granting good response in the low register. The estimated natural frequency of the soundboard is $\mathbf{F_{p0}} = 144$ Hz, resulting from suitable soundboard stiffness (49000 N/m) and vibrating mass (58 g). This leads to conclude that the soundboard is reasonably flexible, and not heavily braced. The vibrating surface is large (489 cm^2), because the bracing lets the soundboard vibrate above the soundhole, too. The surface/mass ratio (8.5) is considerable, and the coupling coefficient between soundboard and inner air is quite high (0.895). These parameters suitably agree with quality standards indicated in Sect. 5.5.

As for the analysis of the other instruments in this survey, we observe the basic resonances in the light of their decay time, illustrated in the former diagram. The decay time is exceptionally long (209 ms) at the resonance of the air $\mathbf{F_1} = 95$ Hz, and also quite long (87 ms) at the first basic resonance of the soundboard $\mathbf{F_2} =$ 200 Hz. This means that the resonator impedance in the low register is fairly high (see Sect. 2.2), basically owing to the values of vibrating mass and soundboard stiffness. The reader may want to get a deeper insight on this issue by referring back to the resonator model illustrated in Appendix 5.1.

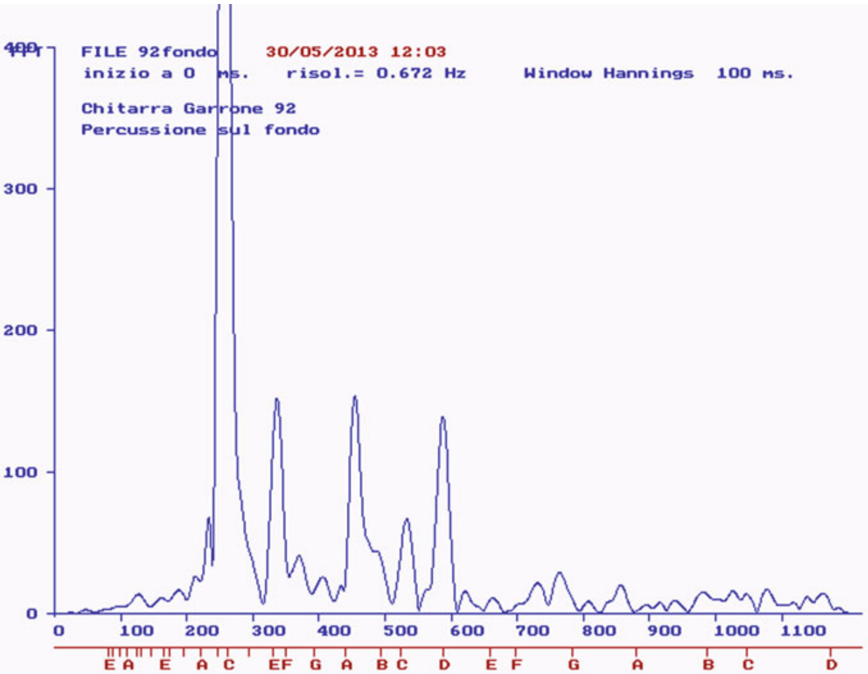
These observations lead to a double conclusion: optimal response of low pitched tones (at least up to G# on the third string) and, on the other hand, considerably long permanence of the basic resonances in the sound texture, as well as excellent sound projection. The same diagram shows how the decay time remains considerably long even far above the range of frequencies where the basic resonances are defined—at least up to 930 Hz (A# at the 18th fret on the first string); beyond this limit, the decay time stabilizes on lower values. We also notice that, at certain frequencies (approximately 400, 570, 590, 800 Hz), this value reaches very high levels. These are very important resonances for the instrument response, depending on the coupling between the resonator elements (soundboard to air and to back), which will be discussed hereafter. On the whole, the analysis of the decay times lets us presume that the instrument offers optimal sustain and a great sound projection within a very large band, spanning from the low register (concerning basic resonances) to the highest.

Let us now proceed with the evaluation of the instrument characteristics, examining the global response obtained on excitation of the soundboard at the bridge location, and the response on excitation of the back. The two are strictly connected. The response on percussion of the back is also reported hereafter.

The global response highlights a strong resonance at about 410 Hz, associated to vibration mode $\langle 0\ 1 \rangle$ of the resonator, whose origin and relevance we have previously discussed. As already pointed out, the decay time is long in correspondence with this resonance, which appears particularly prominent on percussion of the back as well because of the coupling soundboard—air—back illustrated in Sect. 6.1.

Between 410 Hz and 610 Hz (C# on the first string) we also notice a very dynamic response; the resonances manifesting in this frequency interval are supported by two important resonances of the back at 533 and 587 Hz, which reveal optimal coupling of the back with the air in the body and with the soundboard.

In high and highest registers, several resonances stand out with significant amplitude, covering the interval between 660 Hz (E at the 12th fret on the first string) and 930 Hz (A# at the 18th fret on the first string). At these frequencies, the back is not responsive: therefore, these resonances are due to upper vibration modes of the soundboard which, in this instrument, oscillates with considerable amplitude in its peripheral vibrating areas. We invite the reader to go back to FEM simulations of soundboards under equivalent conditions (Sect. 6.1). As pointed out in Sect. 6.2, such high pitched frequencies enhance the upper harmonics of the tones generated in highest registers, and are crucial for brightness and equilibrium of sound.



To complete the analysis of the response, we observe the resonances manifested between the basic resonance of the soundboard $\mathbf{F_2} = 200$ Hz and that of mode $\langle 0\ 1 \rangle$ at 410 Hz. This frequency band involves the main resonances of the back. The resonance in mode $\langle 0\ 0 \rangle$ comes at 237 Hz from the back and at 244 Hz from the soundboard, while the estimated natural frequency of the back in this mode is 202 Hz. The largest resonance obtained on percussion of the back appears at 259 Hz, corresponding to oscillation mode $\langle 0\ 1 \rangle$; it turns into a resonance at 261 Hz in the instrument global response, with the estimated natural frequency of the back at 231 Hz. Finally, the

resonance in mode ⟨0 2⟩ occurs at 336 Hz in the back and 330 Hz in the soundboard; the estimated natural frequency of the back is 310 Hz. See Sect. 5.6 and 5.7.

In conclusion, it is worth considering the third of octave graph, which sums up the former observations. Notice the excellent balance of the instrument in the intervals up to 1000 Hz (mid value), especially mid and upper registers (from 250 to 1000 Hz). The mean emission value between 80 and 1000 Hz is very high (about 15 dB). As stated in Sect. 1.6, it represents the mean level of sound in the frequency band involving fundamentals and harmonics relevant to sound quality. Therefore, this value represents the instrument *performance* within a large frequency range. The reader may take advantage in comparing the mean values of the thirds found in these guitars with levels resulting from different instruments, to shed light on their connection with specific characteristics of the response.

15.7 Conclusion

In this chapter we have examined six guitars built in the last 150 years (let us assume the Zontini model to be an accurate replica of the 1862 Torres guitar).

These remarkable instruments offer an overview of the evolution in guitar making, though our survey is not exhaustive, obviously: we would include many other historical instruments, if we had the chance to examine them.

Their evaluation was accomplished by means of the measurement and investigation criteria illustrated in the text (see Chaps. 5 and 6). Following indications, the reader will be able to apply the same principles to other modern or old-fashioned guitars, and identify their construction guidelines.

The following table reports main dimensions measured and evaluated on each instrument, offering direct comparison between parameters that are relevant for their quality. But we remind that these parameters must be considered and interpreted in the light of response diagrams, which cannot obviously be condensed in a single value. This is why comments are always supported by combined reference to diagrams: time response versus frequency, in connection with the decay time graph and thirds of octave response.

Investigation lets us cast a look on designing criteria. But we may naturally wonder what led these luthiers to certain options, instead of others. It would be unfair to think they were merely directed by empirical preferences: expertise and sensibility surely supported them in their effort to realize a personal concept of sound. Therefore, we approached these instruments with caution and utmost respect, even when actual data emerged from analyses raised doubts about the convenience of certain choices.

Obviously, luthiers of past times could not take advantage of research and manufacturing methods suggested in this book.

Recalling the concept expressed in the foreword of the text, we believe that everyone of us (including luthiers) can go beyond personal experience, blending latest knowledge and techniques with one's own insight and sensibility.

Guitar	*Basic Resonances* *[Hz]*	*Soundboard Natural Frequency [Hz]*	*Air Vol.* *[lt.]*	*Vibrating Mass* *[gr.]*	*Vibrating Surface* *[cm²]*	*Soundboard Stiffness* *[N / m]*	*Surface / Mass ratio* *[cm² / gr.]*	*Coupling Coeff.*	*Resonances due to the Back* *[Hz]*	*Back Natural Freq. [Hz]*	*Mean Sound Emission Level 80–1000 Hz [dB]*
Zontini – Torres	$F_1 = 84.5$ $F_h = 143$ $F_2 = 183$	$F_{p0} = 108$	10.7	69	416	31650	6.03	0.80	$F<00> = 299$ $F<01> = 343$ $F<02> = 420$	$F_1 = 301$ $F_2 = 341$ $F_3 = 426$	13.68
Fleta (1921)	$F_1 = 115$ $F_h = 143$ $F_2 = 230$	$F_{p0} = 186$	12	101	617	137229	6.12	0.86	$F<00> = 244$ $F<01> = 265$ $F<02> = 296$	$F_1 = 189$ $F_2 = 218$ $F_3 = 251$	12.51
Simplicio (1931)	$F_1 = 96$ $F_h = 129$ $F_2 = 193$	$F_{p0} = 144$	14	49	417	39884	8.56	0.86	$F<00> = 258$ $F<01> = 299$ $F<02> = 359$	$F_1 = 240$ $F_2 = 280$ $F_3 = 320$	13.94
Gallinotti (1974)	$F_1 = 104$ $F_h = 129$ $F_2 = 221$	$F_{p0} = 177$	14	74	571	91400	7.75	0.91	$F<00> = 239$ $F<01> = 278$ $F<02> = 340$	$F_1 = 215$ $F_2 = 231$ $F_3 = 316$	14.15
Ramirez (1982)	$F_1 = 109$ $F_h = 121$ $F_2 = 230$	$F_{p0} = 207$	16	70	468	120000	6.7	0.83	$F<00> = 244$ $F<01> = 265$ $F<02> = 325$	$F_1 = 200$ $F_2 = 240$ $F_3 = 292$	11.99
Garrone (2011)	$F_1 = 95$ $F_h = 129$ $F_2 = 200$	$F_{p0} = 147$	14	58	489	49000	8.50	0.895	$F<00> = 244$ $F<01> = 261$ $F<02> = 330$	$F_1 = 202$ $F_2 = 231$ $F_3 = 310$	14.86